Combined Effects of Rarefaction and Fractal Roughness on the Performance of Ultra-Thin Gas Film Journal Bearings

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***Abstract*—For the aerodynamic microbearings in micro-electro-mechanical systems, the gas film thickness is close to or less than the molecular mean free path. The combined effects of gaseous rarefaction and surface roughness on the performance characteristics of gas journal microbearing must be taken into account during the bearing design. The fractal geometry theory is used to characterize the homogeneous surface roughness on bearing surface, and the generalized modified Reynolds equation with different Poiseuille flow rates are derived and solved by using the partial derivative method and relaxation algorithm. The influences of Knudsen number, fractal dimension and bearing parameters on the load-carrying capacity, friction coefficient and attitude angle are investigated in detail. The results show that the Knudsen number has significant impact on the load-carrying capacity and friction coefficient of slip correction models. The roughness effect increases the load carrying capacity and friction coefficient while the corresponding attitude angles are decreased obviously.**

***Index Terms*—aerodynamic microbearings, rarefaction**

I. Introduction

Micro aerodynamic bearing supports rotating rotor by relying on the ultra-thin gas film between the journal and the bearing shell. It has many advantages of simple structure, high running speed, low friction power loss and wide working temperature range. It is one of the crucial parts in bearing-rotor system in micro-electro-mechanical systems (MEMS), inertial navigation system (INS), ultra-precision machine tool spindles and hard disk drives (HDDs) of microfluidic devices [1-5].

With the continuous and rapid development of micro fabrication technology, the MEMS applications are developed toward the trend of miniaturization, higher precision and lower energy consumption. The fluid mechanism of rarefied gas flow at very low spacing is different from that of at macroscales. The micro scale effect becomes important, and assumptions with smooth surface and continuum flows are no longer realistic. The classical theory of fluids such as Navier-Stokes equation is not applicable under this condition, thus the effects of surface roughness and gas rarefaction on the bearing performance should be comprehensively investigated [6, 7]. The real engineering surface of a mechanical part consists of a large number of distributed peaks and valleys. Owing to the continual decrease in the lubricating gas film thickness for smaller bearing structure, the asperity heights are of the same order of magnitude as the minimum radial clearance of bearing. In the view of the effect of gaseous rarefaction and surface roughness, many researchers like Burgdorfer [8], Hsia et al. [9] and Mitsuya [10], based on the slip flow boundary condition, developed the first-order, second-order and 1.5-order slip flow models to modify the compressible Reynolds equation for the analysis of ultra-thin gas film lubrication in the slider/disk interface. Applying the pressure and shear flow correctors, the average Reynolds equations for the isotropic surfaces and directional surfaces were derived by Patir and Cheng [11]. Fukui and Kaneko [12, 13] proposed a database of Poiseuille flow rate for a wide Knudsen number range and derived a generalized Reynolds equation including thermal creep flow and accommodation coefficient from linearized Boltzmann equation. Hwang et al. [14] established a new Poiseuille flow rate model with three adjustable coefficient, which provided a closer prediction of pressure distribution and load-carrying capacity with Boltzmann model. Wang et al. [15] investigated the combined effects of modified Poiseuille flow rate and Couette flow rate correctors on the performance of microbearings with arbitrary inverse Knudsen number and asymmetric accommodation coefficients by applying the modified molecular gas lubrication equation. The rarefaction effects on plane wedge head slider with rough surface was studied by Hsu et al. [16], by adopting the Christensen's stochastic roughness model and Hwang's slip flow model, they found that the longitude roughness structure increases the load-carrying capacity and gas film pressure, whereas the transverse roughness shows the opposite trend. The effect of first-order slip model and effective viscosity on the static characteristics of micro gas journal bearing was studied by Zhang et al. [17, 18], they identified that the numerical solution of slip

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model with effective viscosity are in better agreement with FK model. James White [19] developed a modified Reynolds equation by adopting the Poiseuille flow rate of Fukui and Kaneko for high Knudsen number and certain types of striated rough surfaces in which the multiple-scale analysis and a rotational transformation were performed on the lubrication equation with averaged roughness effects. Jao et al. [20] examined the influence of surface roughness and anisotropic slips on hydrodynamic lubrication of journal bearings, they described the lubricant flow in rough bearing surface by the product of flow factors and flow in nominal film thickness, the results identified that the existence of boundary slip reduces the effect of surface roughness. Kalavathi et al. [21] reported a generalized Reynolds equation for finite porous slider bearing with both longitudinal and transverse roughness, the authors showed the surface roughness enhances the pressure distribution and load-carrying capacity while the permeability parameter reduces the load. Yang et al. [22] employed the 50 dynamic coefficients to characterize the dynamic behavior of the spiral-grooved opposed-hemisphere gas bearing. By considering the tilting and coupling motion besides the translational motion of the bearings, the results showed that the tilting motion has remarkable influence on the synchronous response and natural frequency. Quiñonez [23] utilized the linear perturbation approach and Fourier transformation to obtain the characteristics of wide exponential land slider bearings with roughness parameters, and the results were in good agreement with the cases of sinusoidal and single Gaussian dent.

In this paper, the coupling effects of gas rarefaction and surface roughness on the lubrication characteristics of self-acting gas-lubricated microbearings are studied. The three-dimensional surface roughness topography is characterized by the Weierstrass-Mandelbrot (W-M) function, and the modified Reynolds equations considering different slip correction models as well as the effect of surface roughness are solved by the partial derivative method and relaxation iteration algorithm. The load-carrying capacity, friction coefficient and attitude angle of gas journal microbearing are compared and discussed.

II. Characterization of Fractal Rough Surface

Mandelbrot first introduced the concept of fractal theory by researching the coastal geomorphology in 1967 [24], he reported that the most machined surface can be described by the W-M function with properities of randomicity, multiscale, self-similarity and self-affine in nature. Unlike the traditional characterization parameters of surface roughness such as variance, skewness and kurtosis, which are sensitive to the sampling lengths and resolution of measuring equipment, the fractal roughness parameters are independent of the measurement scale and can provide all the surface topography information of rough surface. W-M function is given as

$$h_r(x,y) = L\left(\frac{G}{L}\right)^{D_f-2} \cdot \sqrt{\frac{\ln\gamma}{M}} \cdot \sum_{m=1}^{M}\sum_{n=0}^{n_{\max}} \gamma^{(D_f-3)n} \times \left(\cos\phi_{m,n} - \cos\left\{\frac{2\pi\gamma^n\sqrt{x^2+y^2}}{L}\cdot\cos\left[\tan^{-1}\left(\frac{y}{x}\right)-\frac{\pi m}{M}\right]+\phi_{m,n}\right\}\right) \tag{1}$$

where $h_r(x,y)$ is the height of rough surface, x and y are the measure distances in the vertical and horizontal positions, respectively. L is the sampling length. D_f is the fractal dimension, varying from 2 to 3 in three-dimensional surface topography. G is the scaling constant that relates to the roughness profile. γ is the scaling parameter (γ>1) which determines the spectral density and self-similarity property, it is equal to 1.5 for a normal distribution surface profiles. M is the number of overlapped ridges on the surface, m and n are the frequency index, $n_{\max}$=int[log(L/L_s)/logγ], $\Phi_{m,n}$ is the random phase, L_s is the cut-off length that depends on the cut-off wavelength of resolution in measuring equipment. The schematic diagram of a three-dimensional fractal surface topography is illustrated in Fig. 1.

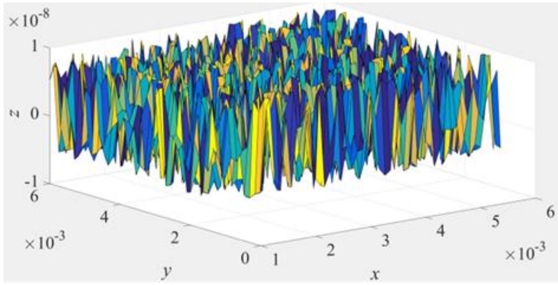


Figure 1. Simulation of a three-dimensional fractal surface topography.

III. Governing Equations and Solution Method

The physical configuration of a rough gas microbearing is shown in Fig. 2. The gas lubricant is assumed to remain isoviscous, isothermal and laminar. The dimensionless modified Reynolds equation including gas rarefaction effects and surface roughness appears as

$$\frac{\partial}{\partial\varphi}\left(QPH^3\frac{\partial P}{\partial\varphi}\right)+\frac{\partial}{\partial\lambda}\left(QPH^3\frac{\partial P}{\partial\lambda}\right)=\Lambda\frac{\partial(PH)}{\partial\varphi}+2\Lambda\frac{\partial(PH)}{\partial T} \tag{2}$$

where $P=p/p_a$, $H=h/c$, $\varphi=x/R$, $\lambda=z/R$ are the dimensionless gas film pressure, gas film thickness and coordinates in the circumferential and axial direction, p_a is the ambient pressure, c is the radius clearance, R is the radius of journal, p is the gas film pressure. h is the local film thickness and it is made up of the nominal smooth film thickness h_0=1+εcosφ and the random roughness h_r measured from the nominal level, ε is the eccentricity ratio of gas journal microbearings. $\Lambda=6\mu\omega R^2/(p_a c^2)$ is the gas bearing number, μ is the viscosity coefficient, ω is the rotating angular velocity of journal, Q is the Poiseuille flow rate ratio which can be express as

First order slip model [8]:

$$Q_1=1+6K_n \tag{3}$$

Second order slip model [9]:

$$Q_2=1+6K_n+6K_n^2 \tag{4}$$

Fukui and Kaneko (FK) Poiseuille flow rate model [13]:

$$\begin{cases} Q_p = \dfrac{D}{6} + 1.0162 + \dfrac{1.0653}{D} - \dfrac{2.1354}{D^2} \\ (\mathrm{D} \geq 5) \\ Q_p = 0.13852D + 1.25087 + \dfrac{0.15653}{D} - \dfrac{0.00969}{D^2} \\ (0.15 \leq \mathrm{D} < 5) \\ Q_p = -2.22919D + 2.10673 + \dfrac{0.01653}{D} - \dfrac{0.0000694}{D^2} \\ (0.01 \leq \mathrm{D} < 0.15) \\ Q = \dfrac{Q_P}{Q_{con}}, \quad Q_{con} = \dfrac{D}{6} \end{cases} \tag{5}$$

Boltzmann model [25]:

$$Q_B=1+0.10842K_n+9.3593/K_n^{-1.17468} \tag{6}$$

where $D = \sqrt{\pi}/(2K_n)$ is the inverse Knudsen number, $K_n=\lambda_0/h$, λ_0 is the molecular mean free path, $\lambda_0=65\times10^{-9}$m.

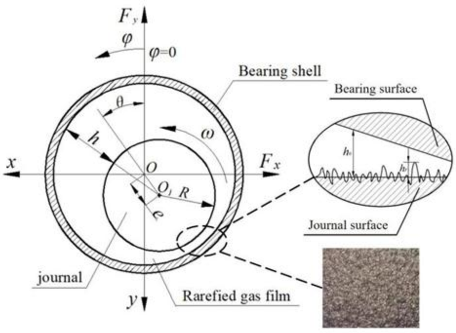


Figure 2. Schematic illustration of a gas lubricated journal microbearings with fractal rough surface.

Equation (2) is a two-dimensional nonlinear partial differential equation (PDE), which is difficult to solve analytically except some specific situations. Hence the partial differential method is convenient to use for air bearing analysis. By using mathematical transformation $PH=S$, $(PH)^2=S^2=\Pi$ [26, 27] and converting the Reynolds equation to the ellipse-type partial differential equation $-\nabla\cdot(c\nabla u)+au=f$. The modified Reynolds equation can be rewritten as:

$$\begin{aligned} &-(\frac{\partial^2\Pi}{\partial\varphi^2}+\frac{\partial^2\Pi}{\partial\lambda^2})+\frac{2\Pi}{H}(\frac{\partial^2 H}{\partial\varphi^2}+\frac{\partial^2 H}{\partial\lambda^2})+\frac{2\Pi}{QH}(\frac{\partial Q}{\partial\varphi}\frac{\partial H}{\partial\varphi}+\frac{\partial Q}{\partial\lambda}\frac{\partial H}{\partial\lambda})= \\ &-\frac{1}{H}(\frac{\partial H}{\partial\varphi}\frac{\partial\Pi}{\partial\varphi}+\frac{\partial H}{\partial\lambda}\frac{\partial\Pi}{\partial\lambda})+\frac{1}{Q}(\frac{\partial Q}{\partial\varphi}\frac{\partial\Pi}{\partial\varphi}+\frac{\partial Q}{\partial\lambda}\frac{\partial\Pi}{\partial\lambda})- \\ &\frac{2\Lambda}{QH}\frac{\partial S}{\partial\varphi}-\frac{4\Lambda}{QH}\frac{\partial S}{\partial T} \end{aligned} \tag{7}$$

The corresponding coefficient of differential equation are:

$$\begin{cases} c=1, \quad a=\dfrac{2}{H}(\dfrac{\partial^2 H}{\partial\varphi^2}+\dfrac{\partial^2 H}{\partial\lambda^2})+\dfrac{2}{QH}(\dfrac{\partial Q}{\partial\varphi}\dfrac{\partial H}{\partial\varphi}+\dfrac{\partial Q}{\partial\lambda}\dfrac{\partial H}{\partial\lambda}) \\ f=-\dfrac{1}{H}(\dfrac{\partial H}{\partial\varphi}\dfrac{\partial\Pi}{\partial\varphi}+\dfrac{\partial H}{\partial\lambda}\dfrac{\partial\Pi}{\partial\lambda})+\dfrac{1}{Q}(\dfrac{\partial Q}{\partial\varphi}\dfrac{\partial\Pi}{\partial\varphi}+\dfrac{\partial Q}{\partial\lambda}\dfrac{\partial\Pi}{\partial\lambda})- \\ \dfrac{2\Lambda}{QH}\dfrac{\partial S}{\partial\varphi}-\dfrac{4\Lambda}{QH}\dfrac{\partial S}{\partial T} \end{cases} \tag{8}$$

In the steady state analysis of bearing characteristics, the gas film pressure is independent of time, so the governing equation can be simplified as follows:

$$\begin{aligned} &-(\frac{\partial^2\Pi}{\partial\varphi^2}+\frac{\partial^2\Pi}{\partial\lambda^2})+\frac{2\Pi}{H}(\frac{\partial^2 H}{\partial\varphi^2}+\frac{\partial^2 H}{\partial\lambda^2})+\frac{2\Pi}{QH}(\frac{\partial Q}{\partial\varphi}\frac{\partial H}{\partial\varphi}+\frac{\partial Q}{\partial\lambda}\frac{\partial H}{\partial\lambda})= \\ &-\frac{1}{H}(\frac{\partial H}{\partial\varphi}\frac{\partial\Pi}{\partial\varphi}+\frac{\partial H}{\partial\lambda}\frac{\partial\Pi}{\partial\lambda})+\frac{1}{Q}(\frac{\partial Q}{\partial\varphi}\frac{\partial\Pi}{\partial\varphi}+\frac{\partial Q}{\partial\lambda}\frac{\partial\Pi}{\partial\lambda})-\frac{2\Lambda}{QH}\frac{\partial S}{\partial\varphi} \end{aligned} \tag{9}$$

The pressure boundary conditions for the very low clearance gas bearing with homogeneous roughness are

$$\begin{cases} P\Big|_{\varphi,\lambda=\pm\frac{B}{2R}}=1, \\ P\Big|_{\varphi=0,\lambda}=P\Big|_{\varphi=2\pi,\lambda}, \\ \dfrac{\partial P}{\partial\varphi}\Big|_{\varphi=0,\lambda}=\dfrac{\partial P}{\partial\varphi}\Big|_{\varphi=2\pi,\lambda} \end{cases} \tag{10}$$

The non-dimensional aerodynamic forces in the horizontal and vertical directions appear as:

$$\begin{cases} \bar{F}_x = p_a R^2\displaystyle\int_{-\frac{B}{2R}}^{\frac{B}{2R}}\int_0^{2\pi}(P-1)\sin\varphi d\varphi d\lambda \\ \bar{F}_y = p_a R^2\displaystyle\int_{-\frac{B}{2R}}^{\frac{B}{2R}}\int_0^{2\pi}(P-1)\cos\varphi d\varphi d\lambda \end{cases} \tag{11}$$

The corresponding attitude angle of microbearing is given as:

$$\theta=\arctan\left(\frac{\bar{F}_x}{\bar{F}_y}\right) \tag{12}$$

The dimensionless load-carrying capacity can be obtained by integrating the dimensionless gas film pressure.

$$C_L=\frac{W}{p_aRB}=\frac{R}{B}\int_{-\frac{B}{2R}}^{\frac{B}{2R}}\int_0^{2\pi}(P-1)\cos\varphi d\varphi d\lambda \tag{13}$$

where B is the width of bearing.

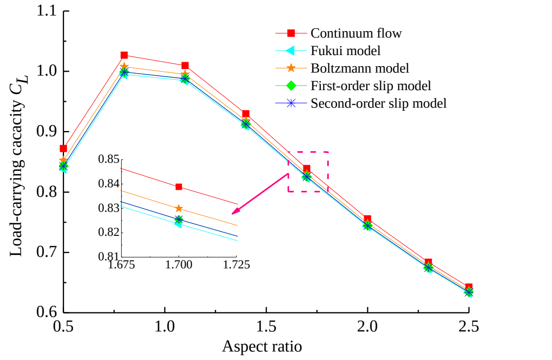
The friction coefficient of the bearing acting on the journal is calculated by

$$F_b=-\int_{-\frac{B}{2R}}^{\frac{B}{2R}}\int_0^{2\pi}(\frac{\Lambda}{6}\frac{1}{H}+\frac{H}{2}\frac{\partial P}{\partial\varphi})d\varphi d\lambda \tag{14}$$

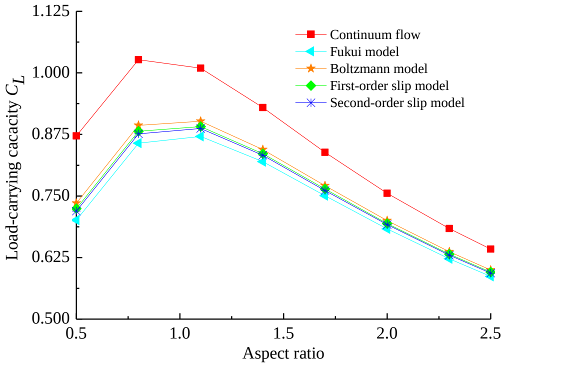
IV. Results and Discussion

A. Rarefaction Effects on the Load-carrying Capacity and Friction Coefficient

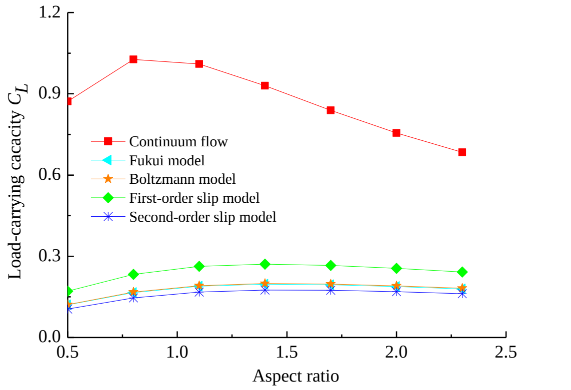
Fig. 3 depicts the variation of dimensionless load-carrying capacity C_L of different models as a function of aspect ratio B/D for K_n varying from 0.0054 to 2.1667. The results show that C_L increases to a maximum value and then decreases dramatically with the increase in B/D. Increasing the Knudsen number decreases the dimensionless load-carrying capacity as comparison with the continuum assumption with nonslip boundary conditions. The difference between the pressure distributions of the first and second order slip cases is not very much for the smaller K_n values. At higher values of Knudsen number, both C_L values of slip correction models are significantly lower than that of continuum flow in the aerodynamic microbearings, and the load-carrying capacity is almost unaffected by aspect ratio. The numerical results of FK model coincide with the Boltzmann model which lies between the first order slip model and second order slip model.



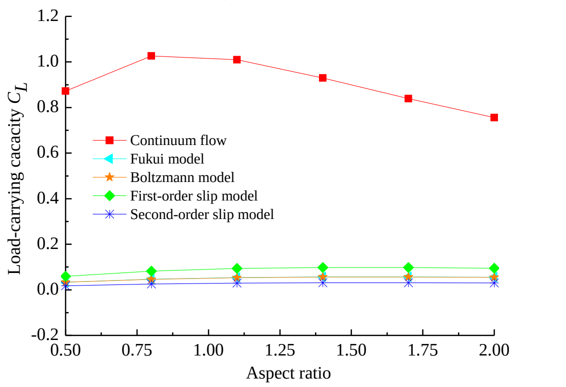
(a) K_n=0.0054



(b) K_n=0.0325



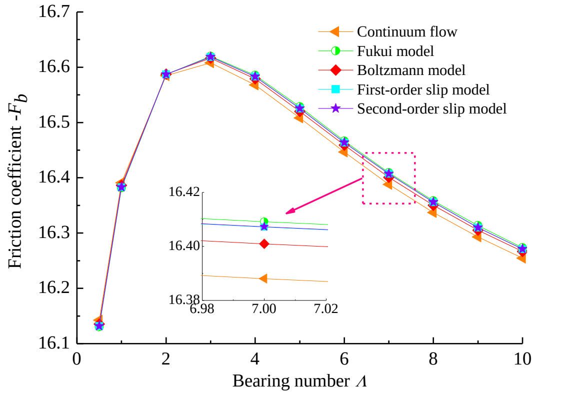
(c) K_n=0.65



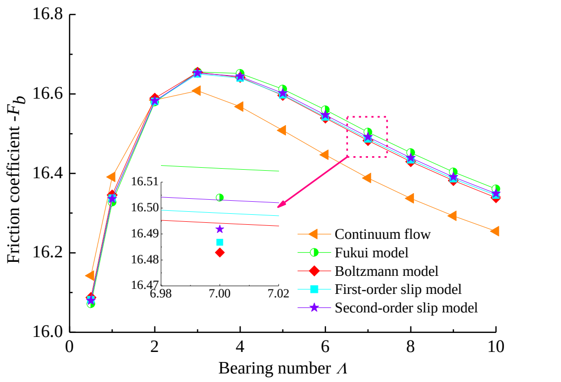
(d) K_n=2.1667

Figure 3. Dimensionless load-carrying capacity versus aspect ratio for different model with several Knudsen number (Λ=3, ε=0.5).

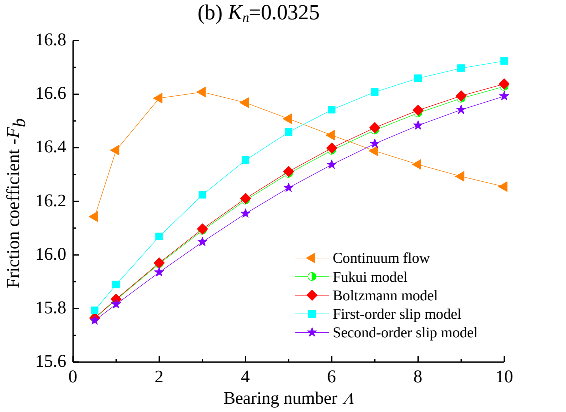
Fig. 4 compares the friction coefficients of different models with bearing number for several Knudsen numbers. It is observed that the friction coefficients of gas microbearing firstly increase and then decrease as the bearing number increases when K_n is equal to 0.0054 and 0.0325. In comparison with the continuum flow condition, the gaseous rarefaction effects increase the values of friction coefficient at bearing numbers about Λ>2. The friction coefficients exhibit a near-linear increasing trend with respect to bearing number as the Knudsen number increases, the calculation results of friction coefficient from FK and Boltzmann slip models are significantly close, and the first order slip model has higher calculation and the second order slip model predicts the minimum friction coefficient among them.



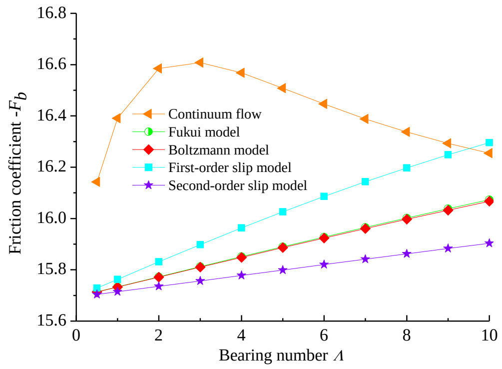
(a) K_n=0.0054



(b) K_n=0.0325



(c) K_n=0.65



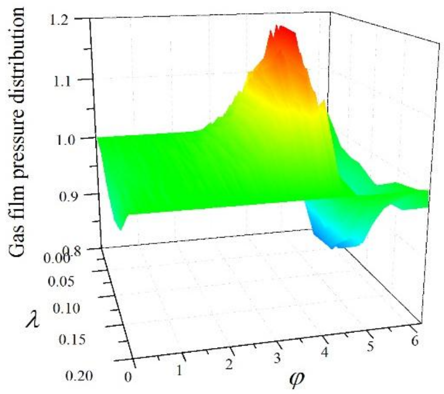
(d) K_n=2.1667

Figure 4. Variation of friction coefficient with bearing number for different values of Knudsen number (ε=0.6, B/D=1.4).

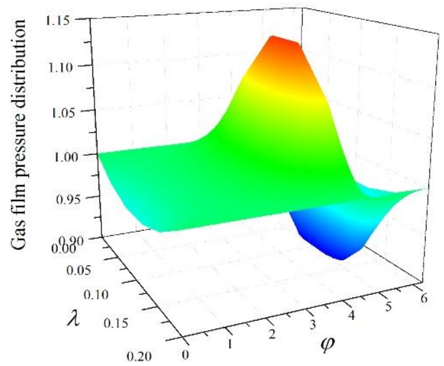
B. Coupling Effects of Surface Roughness and Rarefaction on the Load-carrying Capacity and Friction Coefficient

In order to elucidate the coupling effects of gas rarefaction and surface roughness on the lubrication performance of gas journal microbearings, the static characteristics are presented and discussed in comparison with smooth surface bearings in this part.

Fig. 5 presents the variation of pressure distributions P with rough and smooth bearing surface for fixed values of ε=0.6 and Λ=20. An important observation exhibited by Figs. 5(a) and 5(b) is that the gas film pressure distribution becomes larger with consideration of the roughness effect. The random surface roughness makes the gas pressure of microbearing unpredictable.



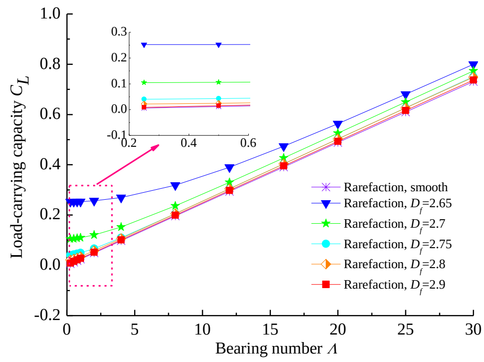
(a) ε=0.6, Λ=20, D_f=2.3, and G=1×10^{-10}m



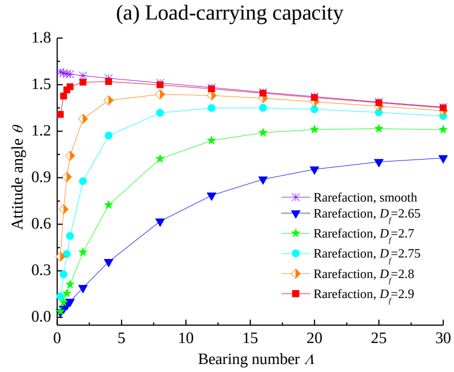
(b) ε=0.6, Λ=20, smooth

Figure 5. Pressure distributions of the gas-lubricated micro bearing for rough and smooth bearing surface.

Fig. 6 describes the roughness effect on the load-carrying capacity and attitude angle of gas-lubricated slider bearings for ε=0.8, B/D=0.2, K_n=1.083 and G=1.36×10^{-10}m. The smaller fractal dimension D_f represent the higher roughness of the surface asperities. In Fig. 6(a), the load capacity C_L increases linearly as Λ increases, and the higher surface roughness results in a larger load capacity. This is due to the higher roughness decreases the film thickness that leads to the restriction in the rarefied gas flow, thus the rough bearing surface magnifies C_L to some extent. It can be seen that the attitude angles decrease with increasing fractal dimension by comparison with the smooth case in Fig. 6(b), the attitude angle will reach a certain value when the bearing number is greater than 15. The reason is that the increased roughness effect can the increase content of the rarefied gas flow.



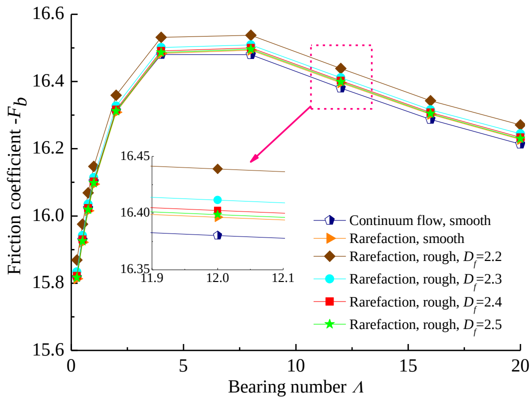
(a) Load-carrying capacity



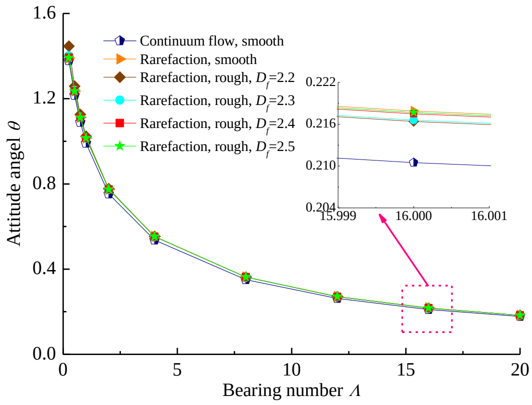
(b) Attitude angle

Figure 6. Load-carrying capacity and attitude angle of the gas journal bearing as a function of bearing number with different fractal dimension.

Fig. 7 shows the variations of friction coefficient with respect to bearing number for fractal dimension varying from 2.2 to 2.5. It is observed that the surface roughness has a marginal effect on $-F_b$ as compared to the smooth bearing surface. The friction coefficients become larger with decreasing fractal dimension, the reason for these changes is that the increased asperity heights which signifies the thinner nominal film thickness, and the effect of high speed shearing is enhanced dramatically. The gas flow in circumferential direction is constricted through the asperities. The attitude angles of slip correction models are slightly higher than that of continuum flow model, and the larger the asperities in rough surface, and the attitude angles show a slightly more gradual decrease with decreasing fractal dimension.



(a) Friction coefficient



(b) Attitude angle

Figure 7. The friction coefficient and attitude angle of the gas journal bearing versus the bearing number for different fractal dimension (ε=0.6, B/D=0.75, K_n=0.0065 and G=1.36×10^{-10}m).

V. Conclusion

On the basis of the fractal geometry theory and different slip correction factors, the effects of gas rarefaction and homogeneous surface roughness on lubrication characteristics of gas journal microbearings are studied. The modified Reynolds equations with different Poiseuille flow rates are solved by the partial derivative method and relaxed iterative algorithm, the influence of Knudsen number, fractal dimension and bearing geometry parameters on the load carrying capacity and friction coefficient are analyzed and discussed.

The effect of gas rarefaction results in an increased friction coefficient and an decreased load-carrying capacity as compared with the case of a continuum flow. At higher Knudsen numbers, the first order slip model predicts the maximum dimensionless load capacity and static friction coefficient, whereas the second order slip model has the minimum calculations. The results of Fukui model are in good agreement with Boltzmann solutions with increasing Knudsen number. The gas film pressure distribution increases for rough bearing at the same bearing number and eccentricity ratio, which exhibits an increment of load-carrying capacity. The attitude angle increases with the larger fractal dimension compared with smooth case.

Conflict of Interest

We declare that we do not have any commercial or associative interest that represents a conflict of interest in connection with the work submitted.

Author Contributions

Conceptualization, L.Y. and Y.W.; methodology, Y.W.; software, T.X.; validation, Y.W.; formal analysis, Y.W.; investigation, Y.W.; resources, Y.W.; data curation, T.X.; writing-original draft preparation, Y.W.; writing-review and editing, Y.W.; supervision, L.Y.; project administration, L.Y.; funding acquisition, L.Y

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