A Comparative Study on the Transmission Loss of Helmholtz Resonator and Quarter, Half, Conical Half-Wave Resonator Using Acoustic Analysis Model

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***Abstract*—This study deals with the Helmholtz resonator and the half- and quarter-wave, conical half-wave resonator, which are noise-absorbing structures for reducing noise. Each resonator was made into an analysis model with the same resonance frequency set. This analysis model is used to predict the performance of each resonator before proceeding with the actual experiment. According to the user's request, the band to reduce the noise is set, and the resonator is modeled accordingly. The ducts were used to position the resonator in the middle of the duct, and the performance of each resonator was compared when the same noise was applied. It can be helpful to design before resonator production by confirming by using analysis model whether noise reduction occurs well in the band that wanted to reduce noise.**

***Index Terms*— Helmholtz resonator, half wave resonator, quarter wave resonator, acoustic analysis**

I. Introduction

A typical sound absorbing structure used to reduce noise is a Helmholtz resonator. The Helmholtz resonator usually consists of a cavity and neck. The resonance frequency is determined by the geometrical information of the cavity and the neck to reduce the noise by matching the resonance frequency to the narrow band to reduce the noise. The resonance frequency formula of the resonator presented by Helmholtz is determined by the volume of the cavity, the cross-sectional area and the length of the neck. Another sound absorbing structure has half, conical half, quarter-wave resonators. The principle of this wave resonator is canceled by the sound of the opposite phase of the specific band in the tube, and the sound absorption is made. The half-wave resonator should be open on both sides, and the quarter-wave tube should be open on one side and closed on the other. This wave resonator can be absorbed even if it does not have a very large volume like Helmholtz. So the wave resonator will be widely used in future industrial applications.

H. K. Ryu et al. designed a Helmholtz resonator for duct noise reduction considering the shape information and compared the experimental and analytical results [1]. In this paper, the analytical model is verified by using Helmholtz noise test results using the duct of this paper. The half-wave resonator confirmed the acoustic attenuation performance through experiments and analysis in several papers [2, 3, 4]. N. YE has carried out research to reduce noise of compressor by installing 1/4 wavelength resonator in centrifugal compressor [5]. G.J. Sreejith et al. conducted a study comparing and varying parameters such as independent distance, nozzle pressure ratio, and cone angle between a conical resonator and a cylindrical resonator [6]. Also C.H. Sohn et al. compared the absorption performance of Helmholtz, quarter-wave and half-wave resonators experimentally for the same resonance frequency [7]. In this paper, we developed analysis models that can compare the performance of each resonator using an analysis model to design a resonator to meet the needs of the user.

II. Resonator

A. Helmholtz Resonator

Fig.1 is a Helmholtz resonator having a volume V with a cross-sectional area of S and length of L. $p_0(t)$ is the pressure change value outside the neck due to the air mass flowing in one second at the volume V, and $p_i(t)$ is the pressure change value within the volume V. Ultimately, finding the Helmholtz resonance frequency is to obtain the frequency at which the pressure change ($p_0(t)$) of the neck part of the resonator becomes minimum. When it resonates, it becomes an antinode. There is almost no pressure change in the part that becomes antinode, and the air flow rate is maximum. On the other hand, the inner nodes of the air have the least

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amount of adoption, but the changes of the pressure and the pressure of the air are the maximum. A mass of Q in a second will increase the density (ρ) of volume V.

$$Q = V\frac{\partial \rho}{\partial t} \tag{1}$$

In this case, since the airflow will occur by harmonic motion when resonance occurs, the density at the atmospheric pressure is $\rho = \rho_0 + \rho' e^{jwt}$.

$$\frac{\partial \rho}{\partial t} = \frac{Qe^{jwt}}{V} = \rho' iwe^{jwt} \tag{2}$$

This change in density causes a change in pressure in volume V. From the definition of sound velocity,

$$p_i = c^2\rho' = \frac{c^2 Q}{Viw} \tag{3}$$

Apply the Newton's force formula to the air mass in the neck of the resonator.

$$F_0 - F_i = S(p_0 - p_i) = \rho_0 SL\frac{\partial u}{\partial t} = iwLQ \tag{4}$$

Equation(3) is substituted in to Equation(4) and rearranged in to Equation(5).

$$p_0 = \left[\frac{c^2}{Viw} + \frac{iwL}{S}\right] Q = \left[w^2 - \frac{c^2 S}{VL}\right] \frac{iLQ}{wS} \tag{5}$$

Here, the frequency at which the pressure change (sound pressure) $p_0(t)$ outside the neck is minimized can be obtained. That is, when the frequency at which the square brackets are set to 0 is obtained, the value becomes the resonant frequency of the Helmholtz resonator. The resonant frequency of the Helmholtz resonator is Equation (6).

$$f = \frac{c}{2\pi}\sqrt{\frac{S}{VL}} \tag{6}$$

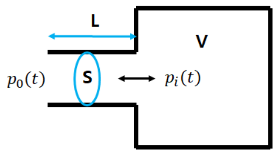


Figure 1. Helmholtz resonator.

B. *Quarter-wave Resonator*

Fig.2 shows a quarter-wave tube with one side open and the other side closed. This wave tube uses the principle that the sound is reflected from the closed tube and the sound is formed in the opposite phase at the frequency corresponding to the quarter wavelength to lower the sound pressure. A cylindrical column with a closed end produces a resonant standing wave at fundamental frequencies and odd harmonics. Closed ends are restricted to nodes in a wave, and open ends are anti-nodes. This creates a fundamental mode in which the wavelength is four times the column length. Due to the constraints of the closed end, the column does not produce even harmonics. The resonance frequency of the quarter-wave tube is calculated by the equation (7).

$$f_1 = \frac{c}{2L},\quad f_3 = \frac{3c}{4L},\quad f_5 = \frac{5c}{4L} \tag{7}$$

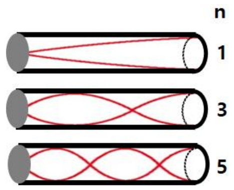


Figure 2. Quarter-wave resonator.

C. *Half-wave Resonator*

Fig.3 shows the half-wave tube with both open. The half-wave tube has an integral multiple of the resonance, unlike the quarter-wave resonator. The formula for determining the resonant frequency of a half-wave tube is similar to the formula for a quarter-wave tube resonator but with some differences. The sound speed v is the same, but the length of the tube is given by L, just like a quarter-wave resonator, but the resonance is divided by two and the resonance is doubled like a quarter of a meter with the same length. And the multiplier is not like (2n-1) like quarter-wave, but only n, resonance is much more concentrated than quarter-wave.

$$f_1 = \frac{c}{2L},\quad f_2 = \frac{c}{L},\quad f_3 = \frac{3c}{2L} \tag{8}$$

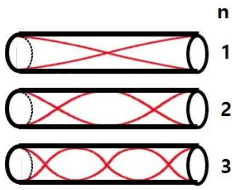


Figure 3. Half-wave resonator.

D. *Conical half-wave Resonator*

Fig.4 shows the conical half-wave tube. The conical half-wave resonator will produce the same fundamental frequency as an open half-wave resonator of the same length. And it will also produce all harmonics n. The formula for obtaining the resonant frequency is the same as that of the half-wave resonator.

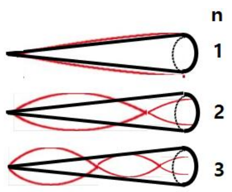
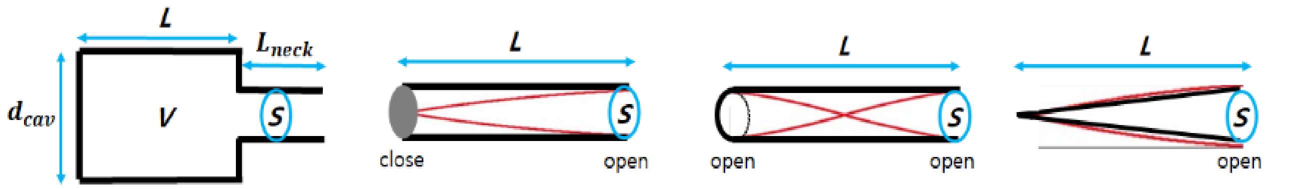


Figure 4. Conical-wave resonator.

III. Acoustic Analysis

A. *Acoustic Resonator Model*

In this study, the target resonance frequency of Helmholtz resonator, quarter-wave, half-wave, and conical half-wave resonators is 400 Hz. Each resonator sectional area S is the same as a circle with a radius of 7 mm and the thickness is equal to 1 mm. The Helmholtz resonator was modeled by measuring the cavity height L from the Equation (6) for obtaining the resonant frequency by fixing the cavity cross-sectional area to 1600 mm^2 and the neck length to 12 mm. In the quarter-wave resonator, the length L was obtained from Equation (7) with one side closed and the other side open. The half-wave resonator is open on both sides, and the conical half-wave resonator has one side open and the other side closed with a cone. The resonance frequency of the two resonators can be found from Equation (8). Each resonator is shown in Fig. 5 and the geometric specifications of resonator is shown in Table I.



(a) Helmholtz resonator (b) Quarter-wave resonator (c) Half-wave resonator (d) Conical half-wave resonator

Figure 5. Resonator model.

TABLE I. GEOMETRIC SPECIFICATIONS OF RESONATOR

Resonator	Resonance frequency [Hz]	L [mm]	S [mm^2]	thickness [mm]	L_{neck} [mm]	d_{cav} [mm]
Helmholtz	400	150	49π	1	25	40
Quarter-wave		214			x	x
Half-wave		428			x	x
Conical Half-wave		428			x	x

B. Acoustic Analysis

In order to verify the acoustic analysis model, the experimental data obtained by H.K. Ryu [1] was compared with the analysis results. In the before study, it was found that sound absorption occurs in the resonance frequency band of the resonator by installing a resonator in the middle of the duct. The effectiveness of the acoustic analysis model using the duct was verified by comparing the acoustic analysis results with the experimental results. Therefore, the acoustic analysis model using the duct was decided to be used in this study. The acoustic analysis model using the modeled duct for verification is shown in Fig. 6 and the results of the analysis are shown in Fig. 7.

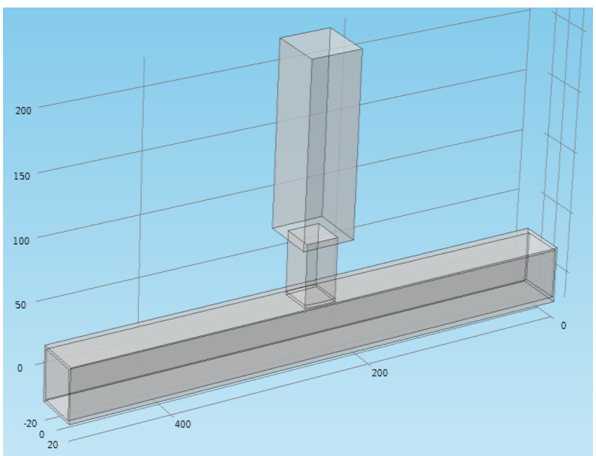


Figure 6. Helmholtz resonator connected to the duct

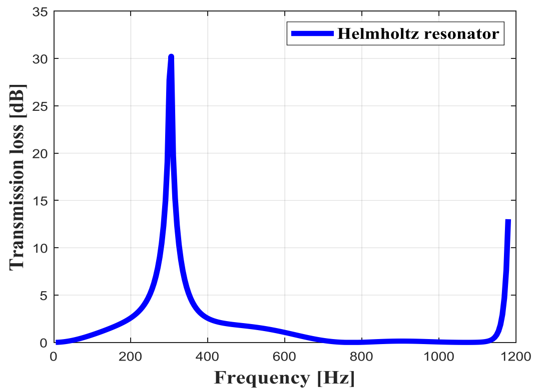
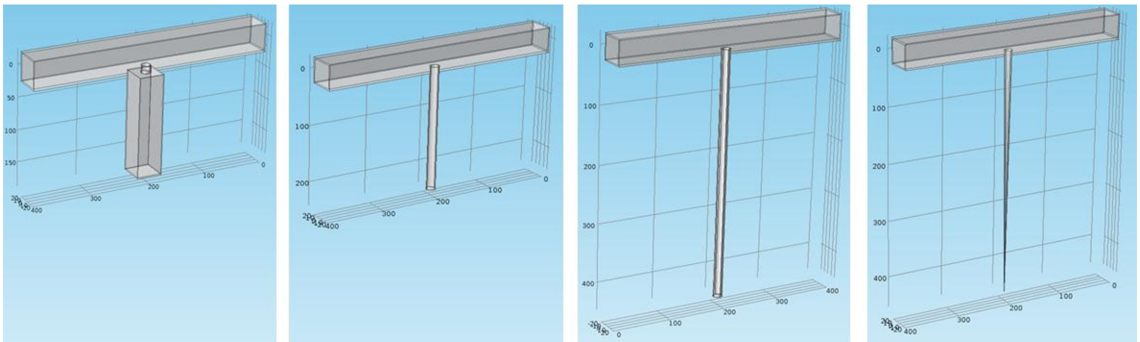


Figure 7. Transmission loss analysis result.

The information of the duct used in the acoustic analysis model is shown in Table II. Duct has an inlet and an outlet. At the inlet, noise equivalent to 1Pa (about 94 dB) is applied, and the outlet has the condition of impedance. It can be seen that each resonator is installed in the middle of the duct and is absorbed in the noise corresponding to the resonance frequency of the resonator. Each resonant acoustic model is showed in Fig. 8.

TABLE II. INFORMATION OF DUCT

Duct area [mm^2]	40 x 40
Duct length [mm]	300
Duct thickness [mm]	5
Inlet condition	1 Pa (94 dB)
Outlet condition	Impedance



(a) Helmholtz resonator (b) Quarter-wave resonator (c) Half-wave resonator (d) Conical half-wave resonator

Figure 8. Resonator analysis model

IV. Acoustic Results

The analytical results were obtained by comparing the acoustic power of the inlet and outlet. The acoustic power is expressed by Equation (9) multiplied by the intensity and cross-sectional area. An intensity is the power delivered in a specific direction per unit area. The transmission loss, which represents the noise reduction, can be calculated from Equation (10).

$$W = I \times S = \frac{p^2}{2\rho c} S \tag{9}$$

$$TL = 10log_{10}\frac{W_{in}}{W_{out}} \tag{10}$$

Each of the four resonators was designed with the same resonance frequency of 400 Hz based on the theoretical equation for obtaining the resonance frequency. The comparison data is the transmission loss calculated from Equation (9) and (10). From the analysis results, it can be confirmed that the resonator modeled based on the theory of resonance frequency suffers from the frequency band of 400 Hz that was targeted. The Fig. 9 is the transmission loss results and Fig. 10 is a sound pressure level surface results at 400Hz. Based on the transmission loss peak, the Helmholtz resonator has the largest transmission loss, the second is the quarter-wave resonator, the third is the half-wave resonator and the worst performing resonator is the conical half-wave resonator.

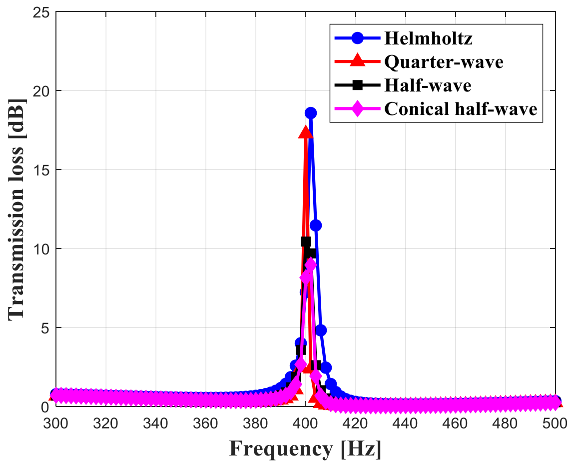
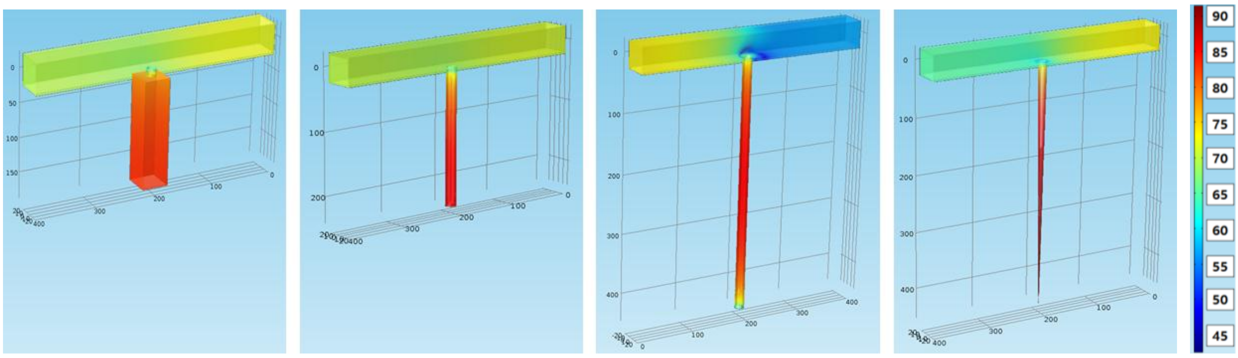


Figure 9. Transmission loss results.



(a) Helmholtz resonator (b) Quarter-wave resonator (c) Half-wave resonator (d) Conical half-wave resonator

Figure 10. Sound pressure level results at 400 Hz.

The wavelength resonator has an integral multiple or an odd multiple of the resonance. The quarter-wave resonator is a structure with one wall closed to generate resonant standing waves at fundamental frequency and odd harmonics. Due to the constraints of the closed end, the column does not produce even harmonics. On the other hand, the half-wave resonator is open on both sides, producing both odd and even harmonics. The conical half-wave resonator is open on one side and closed on the other by a cone, but it produces all the resonance standing waves of integer multiples like the half-wave resonator. The results of the acoustic analysis wave resonator model are shown in Fig. 11. The resonance wave of the wave tube can be confirmed.

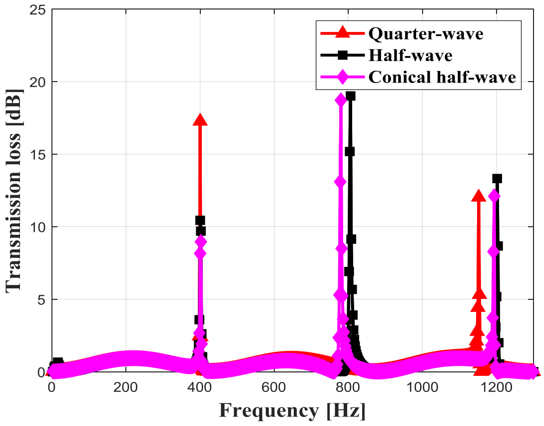


Figure 11. Transmission loss harmonics of wave resonator.

V. Conclusion

In this study, we used a sound analysis model that is verified by comparing with the resonator experimental data using a duct. Helmholtz, quarter-wave, half-wave, and conical half-wave resonators were installed at the center of the duct to compare the noise reduction of resonators. Each resonator was modeled by equally choosing the frequency to reduce the noise using the resonant frequency formula. In this acoustic analysis model, a resonator is located at the center of the duct, and when the resonance frequency of the resonator is equal to the noise applied, the resonance is performed by the resonator. As a result of analysis under the same conditions, it was found that the sound absorption was best at 400 Hz which was the target resonance frequency, followed by quarter-wave, half-wave, and conical half-wave resonator.

In the quarter-wave, half-wave and conical half-wave resonators, we can see that the resonant frequencies of the odd-numbered resonator and the integer-wave resonator appear in addition to the fundamental frequency. The quarter-wave tube has an odd number of resonance waves, the half-wave and the conical half-wave tube has an integer number of resonance waves.

Using this analytical model, the performance of a resonator can be predicted by analyzing the resonator before it is fabricated and tested. Therefore, we can effectively reduce the cost and time to build and experiment. Furthermore, by using the analytical model, it is possible to move the sound absorption frequency band according to the user's demand or maximize the noise reduction amount through the optimum design. In the next study, we plan to develop and study a resonator to reduce tire cavity noise using Helmholtz, wavelength resonators.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Chul Hyung Lee conducted the research and wrote the paper; Myeong Jae Han analyzed the data; Tae Won Park reviewed the paper. Young Sik Kim and Kyoung Duck Shin confirmed the results of the analysis; all authors had approved the final version.

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