Non-holonomic Constrains for a Mobile Manipulator

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***Abstract*—Currently, there is a great scientific and technological interest in the development of mobile manipulators. It can be evidenced not only in the numerous scientific publications but also in research to develop applied technologies in this field. One of the challenges that has generated great interest is to obtain the Non-holonomic constrains of a mobile manipulator. Therefore, the main objective of this paper is to contribute to the knowledge to obtain the Non-holonomic constrains and therefore the coupled kinematic and dynamic model of a mobile manipulator.**

***Index Terms*—anthropomorphic, constraints, differential, manipulator, mobile, model, non-holonomic**

I. Introduction

Robotic manipulators are usually installed on a fixed base with a limited workspace. However, due to the diversity of tasks that currently arise, it is necessary to expand the work spaces, which is why it is common to add conveyor belts, transport devices and in some cases, choose to use Mobile Manipulators (MM). MM have many potential applications due to the capabilities, some of them are present in manufacturing, assembly, construction, explosives neutralization, nuclear reactors maintenance, and planets exploration [1].

A MM consists of one or several manipulators assembled on top of a mobile platform or mobile robot [2]. This has a considerably larger workspace than a fixed manipulator and greater manipulation than mobile robots. Furthermore, it combines the manipulation capacity offered by fixed-base manipulators and the mobility offered by mobile platforms. By being able to perform both tasks simultaneously, the MM has advantages over stationary manipulators, such as a greater work space and greater autonomy by being wireless.

In general, a MM can perform different types of tasks, of which locomotion and manipulation are highlighted. Therefore, the kinematic and dynamic modeling of the base and the manipulator arm are usually carried out independently even with different methodologies [3]–[5] and sometimes the coupling is not performed due to the complexity in the modeling regarding the non-holonomic restrictions [6]–[8]. In this paper, the procedure to obtain the non-holonomic restrictions of a mobile and the results of the movement are presented.

Some examples of the possible configurations that can be implemented are the locomotion with wheels or paws and in the manipulation with arms of open or closed kinematics, making it necessary to determine the type of articulation and the number of DOF. A fundamental work for the modeling of mobile robots with wheels is presented in [7], [9], where the mathematical model of four types of configurations is determined. A kinematic modeling scheme for mobile manipulators is presented in [10], [11], where the model of the mobile platform and the manipulator are determined separately. In [4] a method to couple the kinematic model of the mobile base with the one of the stationary manipulator is presented, but the mobile and the arm are modeled by different methods. In general, the Euler-Lagrange method is used as main tool to obtain the model, and the Lagrange multipliers to incorporate the non-holonomic constraints [12]; however, exist a depend on the kinematic model to incorporate the non-holonomic constraints. Some practical cases can be consulted in [13]–[16].

This article is organized as follows, section 2 presents the configuration of the MM to be modeled; in section 3 the non-holonomic constraints is obtained of the differential type mobile-robot. Section 5 shows the implementation and results obtained. Finally, the conclusions and the bibliographical references used are presented.

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II. Conceptual Design of the MM

The selected robot has an anthropomorphic type manipulator, three links and three degrees of freedom. Each one of the degrees of freedom is given by an articulation of the rotational type. For the conceptual design of the mobile platform, a type (2.0) robot [6] was chosen. This platform has versatility of movement, ease in modeling and in the action of control. The robot has two fixed wheels and mobility is achieved by having a different speed on each wheel, which is called differential locomotion. The robot has a third passive wheel that is used to give stability and serve as a point of support. The conceptual design of the MM is presented in Fig. 1.

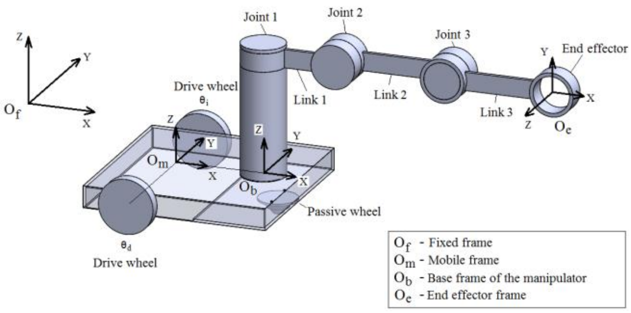

Figure 1. Conceptual design of the MM driven by differential system.

III. Non-Holonomic Constraints

For the mobile platform driven by a differential system as the one shown in Fig. 2. The kinematics of the mobile platform is determined in the reference system O_b, because there is the base of the manipulator. The following notation is used in the equations for ease of handling: $s_m = \sin\theta_m$, $c_m = \cos\theta_m$ being θ_m the angular position of the mobile robot.

The calculation of the speeds in X and Y of the point O_b is presented in (1) and (2).

$$\dot{x}_b = vc_m - v_T s_m \tag{1}$$

$$\dot{y}_b = vs_m + v_T c_m \tag{2}$$

where v_T is the tangential velocity at that point due to the angular velocity $w = \dot{\theta}_m$. Equations (1) and (2) can be expressed in terms of w by means of the relationship $v_T = wd$ and in matrix form as presented in (3).

$$\begin{bmatrix} \dot{x}_b \\ \dot{y}_b \\ \dot{\theta}_m \end{bmatrix} = \begin{bmatrix} c_m & -d\,s_m \\ s_m & d\,c_m \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} \tag{3}$$

In Fig. 3, shows that the velocity of each wheel v_r (right wheel) and v_l (left wheel) is perpendicular to the rotation axis, so the velocity $\dot{\theta}_m$ is produced by the composition of the linear velocities of each of the wheels and the linear velocity at the center of mass $v_{cm} = v$ will be equally perpendicular to those of the wheels' shafts.

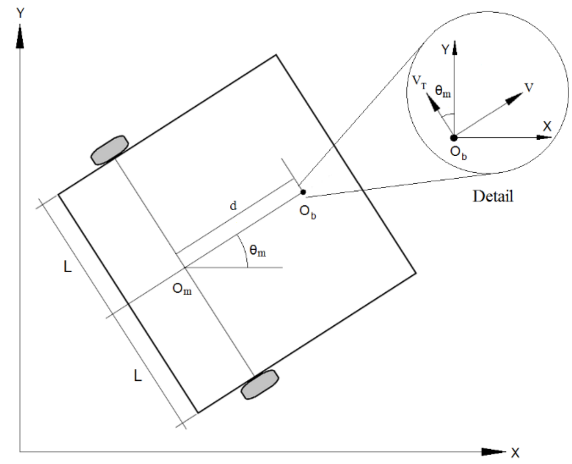

Figure 2. Robot speeds in Ob

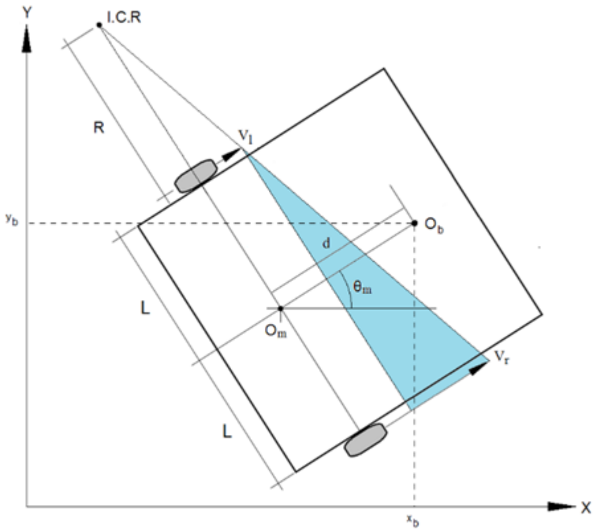

Figure 3. Robot speeds in ICR

As the Instant Center of Rotation (ICR) is the full mass of the mobile, the linear speed of the mobile is the product of the angular velocity of the mobile $\dot{\theta}_m$ and position of O_m in respect of ICR, as presented in (4).

$$v = \dot{\theta}_m (R + L) \tag{4}$$

where 2L is the length and width of the mobile and R is the distance from ICR to the nearest wheel. Therefore, the speed of each wheel is calculated by means of (5) and (6).

$$v_l = \dot{\theta}_m R \tag{5}$$

$$v_r = \dot{\theta}_m (R + 2L) \tag{6}$$

From Fig. 2.b, we can deduce (7) by means of the similarity between triangles, which relates the linear velocities of each wheel with respect to the point O_m.

$$\frac{v - v_l}{v_r - v_l} = \frac{L}{2L}$$

$$2(v - v_l) = v_r - v_l$$

$$v = \frac{v_r + v_l}{2} \tag{7}$$

Also, through Fig. 3, you can determine the relationship between the speed of each wheel at the time of producing a turn. In this way, the result is the difference in the speeds of the wheels in respect of the width of the platform 2L and is presented in (8).

$$\dot{\theta}_m = \frac{r(\dot{\theta}_r - \dot{\theta}_l)}{2L} \quad (8)$$

Expressing (7) and (8) in terms of $\dot{\theta}_r$ and $\dot{\theta}_l$ we have (9) and (10), by means of the expressions $v_r = r\dot{\theta}_r$ and $v_l = r\dot{\theta}_l$, where r is the radius of the wheel.

$$v = \frac{r(\dot{\theta}_r + \dot{\theta}_l)}{2} \quad (9)$$

$$\dot{\theta}_m = \frac{r(\dot{\theta}_r - \dot{\theta}_l)}{2L} \quad (10)$$

In matrix form, it is presented in (11).

$$\begin{bmatrix} v \\ w \end{bmatrix} = \begin{bmatrix} \frac{r}{2} & \frac{r}{2} \\ \frac{r}{2L} & -\frac{r}{2L} \end{bmatrix} \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} \quad (11)$$

Finally, the product of the matrices (3) and (11) is made and the result is presented in (12).

$$\begin{bmatrix} \dot{x}_b \\ \dot{y}_b \\ \dot{\theta}_m \\ \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} = \begin{bmatrix} \frac{r}{2L}(Lc_m - ds_m) & \frac{r}{2L}(Lc_m + ds_m) \\ \frac{r}{2L}(Ls_m + dc_m) & \frac{r}{2L}(Ls_m - dc_m) \\ \frac{r}{2L} & -\frac{r}{2L} \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\theta}_r \\ \dot{\theta}_l \end{bmatrix} \quad (12)$$

IV. Results

To validate the obtained model, a simulation was carried out in Simulink of Matlab®. In this simulation the Simscape Multibody™ tool was used to loading the mechanic model of the MM designed in SolidWorks®. The inclusion of mechanical design allows the simulation to be more real and reliable, since the physical properties (mass, inertia, etc.) of each robot body are considered.

The values of the physical parameters that are used in the simulation, and that come from the design made in SolidWorks, are presented in Table I.

Table I. Values of the Physical Parameters Used for the Simulation.

Symbol	Value
L [m]	0.250
d [m]	0.270
r [m]	0.075

The values of the movement parameters that are used in the simulation are presented in the Table II.

Table II. Values of the Movement Parameters Used for the Simulation.

	Symbol	Value
Execution time [s]	t	3.6
Angular speed of the right wheel [rad/s]	θ_r	56.635533
Angular speed of the left wheel [rad/s]	θ_l	45
Initial condition in X [m]	-	0
Initial condition in Y [m]	-	-2.1931978
Initial condition in θ_m [rad]	-	0

The results of the simulation are presented in Fig. 4, this is the angular position of the mobile or θ_m during the simulation time and Fig. 5 is the trajectory in the plane X-Y with radius $R = 2.19\ m$. In this way it is verified the results of the non-holonomic constraints are correct.

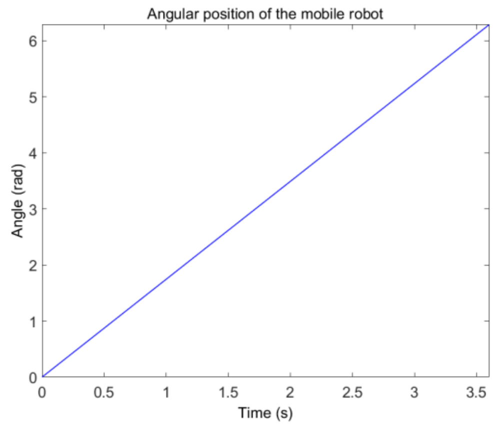

Figure 4. Angular position θ_m of the mobile robot.

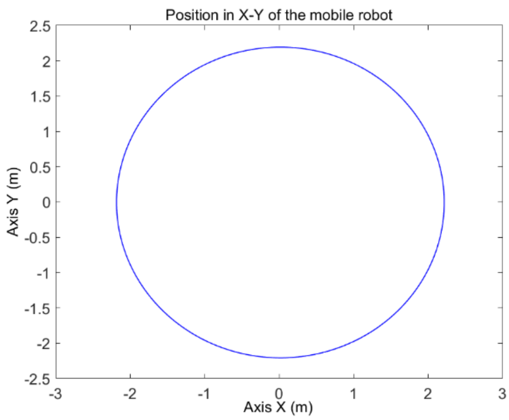

Figure 5. Trajectory in the plane X-Y of the mobile robot

V. Conclusions

The results of the simulation show that the mobile robot did not execute lateral movements evidencing that non-holonomic constraints are correct.

The emphasis of this paper has been set on obtaining precise and complete equations of motion, which includes the simultaneous rotating, the non-holonomic constraints associated with the movement of the mobile platform.

As a future work, it is desired to develop the control of the coupled dynamic model of the MM. The advantage of having the coupled dynamic model of MM lies in the fact that coordinated movements of the base can be programmed with the manipulator and in turn avoid inappropriate positions that involve overturning or collisions with the same structure. Other future work is the implementation of optimization techniques to guarantee the minimum energy consumption.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Vladimir Prada Jiménez conducted the research, analyzed the data and wrote the paper; Paola A. Niño-Suárez and Mauricio F. Mauledoux-Monroy reviewed the content and writing.; all authors had approved the final version.

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