Fault-Tolerant Sliding Mode Control of a Quadrotor UAV with Delayed Feedback

Safi Ullah and Kamran Iqbal
University of Arkansas at Little Rock, Little Rock, Arkansas, USA
Email: sxullah@ualr.edu, kxiqbal@ualr.edu

Fahad Mumtaz Malik
College of Electrical and Mechanical Engineering, NUST, Islamabad, Pakistan
Email: malikfahadmumtaz@cem.e.nust.edu.pk

Abstract—A quadrotor unmanned aerial vehicle (UAV) controller distributes the pitch, roll and yaw commands to individual propellers. This paper explores fault-tolerant control of a quadrotor UAV using delayed feedback and Divided State Feedback Control (DSFC). Initially, a Sliding Mode Controller (SMC) for the quadrotor UAV is designed to obtain sustained performance in the presence of actuator faults. The SMC performance deteriorates considerably in the presence of delayed sensory feedback from the UAV. A DSFC is then used to restore effectiveness of the device controller. The proposed control structure delivers improved stabilization, robustness and transient response in the presence of actuator faults. Computer simulations are presented to illustrate the effectiveness of our hybrid control scheme.

I. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) aka drones have become popular in recent years for a variety of applications such as security, payload delivery, military and traffic surveillance, photography, wild-life monitoring, etc. It has also become a fascinating area of research for control engineers due to the challenges involved. A quadrotor UAV has six degrees of freedom (6DOF) that represent three angular and three translational motions i.e. roll, pitch, and yaw, and x, y, z, respectively. Quadrotor consists of four propellers that are mounted on the corners of an X-shaped frame. The position and orientation of the UAV in space is controlled due to the challenges involved. A quadrotor UAV has six degrees of freedom (6DOF) that represent three angular and three translational motions i.e. roll, pitch, and yaw, and x, y, z, respectively. Quadrotor consists of four propellers that are mounted on the corners of an X-shaped frame. The position and orientation of the UAV in space is controlled by controlling the speed of the propellers. Since we have four inputs controlling 6DOF, the UAV represents an under actuated electromechanical system.

Quadrotor UAV models have been used by numerous researchers as a promising candidate plant for the experimentation and testing of control algorithms [1]. Diverse control techniques for the most part intended for UAVs are feedback linearization [2], [3], back-stepping control [4], and sliding mode control (SMC) [5], [6].

Some different techniques are implemented on a linearized model of quadrotor and a comparison of the obtained results has been presented [7]. In this paper, SMC has been picked as controller due to its robustness to the model uncertainties, parametric vulnerabilities and external aggravations. This sliding mode controller is capable of making the quadrotor reach and stay within the desired altitude with desired rotations.

Fault Tolerant Control System (FTCS) is a control framework with the capacity to endure faults automatically and proceed with its intended operation in case of a failure in some of its segments [8]. FTCSs are classified into two major categories, i.e. Passive FTCS and Active FTCS. The Passive FTCS can only handle pre-defined faults with the controller tuned to fixed gains whereas in case of AFTC systems, the fault is detected, diagnosed, and estimated, and the controller is reconfigured online [5], [9], [10], [11]. The popular approaches toward fault-tolerant control includes SMC [5], [12], [13], [14], Model predictive Control (MPC) [15] which also allow operation under fault free conditions. A large measure of existing exploration has tended to the FTCS for quadrotor UAVs but still there exists considerable room for improvement. Some of these efforts addressing the fault tolerance issue include hybrid switching fault-tolerant control [12], Adaptive observers for the magnitude estimation of complex and time-varying actuator faults [14], Terminal SMC for robust control operations in unstructured environments [6], [16], and the non-linear SMC observer used as Fault Diagnosis and Identification (FDI) unit for the online detection and estimation of fault magnitudes [17]. The type of fault addressed in this paper is the Loss of Effectiveness (LOE) in the thrust of actuators which is actually the most common type of actuator faults. Passive FTC is implemented for a pre-defined set of actuator faults.

Divided state feedback (DSF) involves sensory feedback with time delays, and DSF control, which entails control with feedback delays, is capable of delivering an improved stabilization and transient response [18]. It is worth noting that time delays dependably exist in genuine control frameworks because of estimations by means of sensors, and so forth. The
history of “Delayed Control” is quite old and results have been obtained to accomplish execution for the frameworks comparable to conceivable [19]. Further, practical implementation of the control technology invariably involves time lags due to sensing, communication and processing times. These delays need to be appropriately addressed in the control design. The history of the “Delayed Control” case is quite old and controller delays were used to advantage [19]. This and other investigations, e.g., [20], concluded that a system with or without process delays could be stabilized with satisfactory performance by a controller with time delay provided the time delay was kept bounded [18]. In this paper, DSF control integrated with passive FTC is applied to a quadrotor subject to its potential value in delivering an improved stabilization and transient response as compared to other conventional control techniques.

This paper is organized as follows. In section II, the nonlinear dynamical model, the rotor model, and the linear state-space model of a quadrotor UAV are presented. In section III, passive FTC based on SMC is designed in the presence of actuator faults. In section IV, DSF control is integrated with passive FTC and SMC and simulation results are presented to show the viability of our design. Finally, conclusions are stated in section V.

II. SYSTEM MODEL

A. Nonlinear Dynamic Model

The nonlinear dynamical model of the quadrotor UAV (see Fig. 1) is derived using Euler-Lagrange terminology and is comprised of the following equations [1].

$$\begin{align*}
\ddot{\Phi} &= \dot{\Psi} (I_y I_z) / I_z - (J_r I_r) \dot{\Omega} + (U_2 / I_z) \\
\ddot{\Psi} &= \dot{\Phi} (I_2 I_z) / I_y - (J_r I_r) \dot{\Omega} + (U_3 / I_y) \\
\ddot{x} &= (\cos \Phi \sin \theta \cos \psi + \sin \Phi \sin \psi) \left( \frac{U_1}{m} \right) \\
\ddot{y} &= (\cos \Phi \sin \theta \sin \psi - \sin \Phi \cos \psi) \left( \frac{U_1}{m} \right) \\
\ddot{z} &= -g + (\cos \Phi \cos \theta) \left( \frac{U_1}{m} \right)
\end{align*}$$

(1)

where $\Phi$, $\theta$, $\Psi$ and $x$, $y$, $z$ are the roll, pitch, and yaw angles, and the linear positions, respectively, with respect to the inertial frame of reference. The inputs $U_2$, $U_3$, $U_4$ control the rotational subsystem, and, combined with $U_1$, form the control inputs for the translational subsystem. These inputs are defined as,

$$\begin{align*}
U_1 &= b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\
U_2 &= b(\Omega_4^2 - \Omega_2^2) \\
U_3 &= b(\Omega_3^2 - \Omega_1^2) \\
U_4 &= d(\Omega_2^2 + \Omega_4^2 - \Omega_3^2 - \Omega_1^2)
\end{align*}$$

(2)

where,

- $b$=thrust co-efficient
- $d$=drag co-efficient
- $\Omega=\Omega_2 + \Omega_4 - \Omega_1 - \Omega_3$
- $\Omega=$Speed of rotation of each propeller

B. Rotor Model

The rotor model [1] consists of DC motor equations, given as:

$$\begin{align*}
J \frac{d\omega}{dt} &= U - R \omega - k \omega_m \\
J \frac{d\omega_m}{dt} &= r_m - \tau_d
\end{align*}$$

(3)

The dynamics of the DC-motor are approximated as:

$$J \frac{d\omega_m}{dt} = -\frac{k^2}{R} \omega_m - \tau_d + (k_m / R) U$$

(4)

where $\omega_m=$Angular Speed of motor, $U=$Motor Input, $\tau_d=$Motor Load, $r_m=$Motor Torque, $k_m=$Torque constant, $R=$Motor Resistance, $k_e=$Back emf constant, $J=$Motor Inertia.

C. State Space Representation

The state-space representation of the quadrotor UAV consists of 12 states (six positions and six velocities, three each for translation and rotation). The state variables $x_1$, $x_3$, and $x_5$ represent the $x$, $y$, and altitude position while $x_7$, $x_9$, and $x_{11}$ represent the roll, pitch, and yaw angles respectively.

The corresponding nonlinear state-space representation is given as:

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= (U_1 / m)(\cos x_7 \sin x_9 \cos x_{11} + \sin x_7 \sin x_{11}) \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= (U_1 / m)(\cos x_7 \sin x_9 \cos x_{11} - \sin x_7 \cos x_{11}) \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= (U_1 / m)(\cos x_7 \cos x_9 - g) \\
\dot{x}_7 &= x_8 \\
\dot{x}_8 &= [(J_y - J_z)x_{10}x_{12} - J_r \omega x_{10} + U_2] \left( \frac{1}{J_x} \right) \\
\dot{x}_9 &= x_{10} \\
\dot{x}_{10} &= [(J_z - J_x)x_6x_{12} - J_r \omega x_8 + U_3] \left( \frac{1}{J_y} \right) \\
\dot{x}_{11} &= x_{12}
\end{align*}$$
\[ x_{12} = [(J_x - J_y) x_\theta x_{10} + C_d U_4] \left( \frac{1}{J_z} \right) \]

### III. FAULT-TOLERANT SLIDING MODE CONTROL

#### A. Sliding Mode Control

In this section, an SMC to control the translational and rotational motions of a quadrotor UAV is designed. We consider the nonlinear dynamic equation for the altitude, which is given as:

\[ \ddot{z} = -g + (\cos \phi \cos \theta) \frac{U_1}{m} \]

where \( U_1 \) is the control input. We next define a sliding surface \('S'\) such that the system tracks the desired trajectory, i.e. \( z_d(t) = z(t) \).

\[ S_z = \dot{e}_z + \lambda_z e_z + k_p \int e_z \]

where \( e_z = z_d - z \) is the altitude error, and the integral term is added to minimize the steady-state error and the fault effect.

\[ \dot{S}_z = \dot{e}_z + \lambda_z e_z + k_p \int e_z \]

The stability of the controller can be investigated via Lyapunov methods. In particular, choosing the Lyapunov function as \( V = \frac{1}{2} S_z^2 \) results in a negative definite time derivative \( (\dot{V} = S_z \dot{S}_z < \eta |S_z|) \). Accordingly, the control effort \( U_1 \) is selected as:

\[ U_1 = \frac{m}{\cos(\phi)\cos(\theta)} [g + \dot{z}_d + \lambda_z e_z + k_p \text{sign}(S) \ k_p e_z] \]

In order to satisfy the reachability condition, i.e. to force system trajectories to reach and stay on the sliding manifold, \( S_z = 0 \) in finite time ( \( S_z \dot{S}_z < -\eta \ |S_z| \) ), a discontinuous term is added to \( U_1 \), where \( \text{sign}(S) \) is defined as:

\[ \text{sign} (S) = \begin{cases} 1 & \text{if } s > 0 \\ 0 & \text{if } s = 0 \\ -1 & \text{if } s < 0 \end{cases} \]

The main drawback of SMC is the chattering effect produced by the discontinuous ‘sign(S)’ term in the controller. In order to overcome this problem, we may replace the sign function with a saturation function. Under this modified control, the system is guaranteed to reach and stay on the manifold \( S_z = 0 \). The same steps are then followed to derive the other control efforts, i.e. \( U_2, U_3, U_4 \) and \( U_x, U_y \), where \( U_x \) and \( U_y \) are the control efforts required for \( x \) and \( y \) positions.

\[ \text{Sat}(S) = \left\{ \begin{array}{ll} \text{sign}(S) & \text{if } |S| > \rho \\ (S/\rho) & \text{if } |S| < \rho \end{array} \right. \]

In the above, \( \rho \) represents a boundary layer around the sliding surface ‘S’. The remaining control efforts designed on same lines are given as:

\[ U_2 = \frac{J_z}{I} [(J_y - J_z) \dot{\theta} \dot{\psi} + J_z \dot{\phi} + \Phi_d + \lambda_z e \phi + K_p \text{sat}(S) + k_p e \phi] \]

\[ U_3 = \frac{J_z}{I} [(J_x - J_z) \dot{\phi} \dot{\psi} + J_z \dot{\theta} + \Phi_d + \lambda_z e \theta + K_p \text{sat}(S) + k_p e \theta] \]

\[ U_4 = \frac{J_z}{I} [(J_y - J_z) \dot{\theta} \dot{\psi} + J_z \dot{\phi} + \Phi_d + \lambda_e e \phi + K_p \text{sat}(S) + k_p e \phi] \]

#### B. Actuator Faults and Passive Fault Tolerant Control

SMC is a nonlinear robust controller which is capable of stabilizing the control of a quadrotor UAV in the presence of external disturbances as far as the system is operating under normal conditions. The question which arises here is that what would happen if a fault occurs? Two of the most common faults that occur on these types of nonlinear dynamical systems are the sensor faults and actuator faults. The type of fault addressed in this paper is the Loss of Effectiveness (LOE) in the thrust of actuators, which is a common type of actuator fault. Such faults may occur due to actuator wear out, mainly the bearing, or the damage to a rotor in the case of collision.

The fault can be modelled in the system as:

\[ F_i = \eta \ast b \ast \omega^2 \]

\[ T_i = \eta \ast k \ast \omega^2 \]

Where 'b' and 'd' are the thrust and drag co-efficient respectively.

\[ \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \end{bmatrix} = \begin{bmatrix} 0 & -1/2 & b/4d & 1/4 \\ 0 & 1/2 & b/4d & 1/4 \\ -1/2 & 0 & -b/4d & 1/4 \\ 1/2 & 0 & -b/4d & 1/4 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \end{bmatrix} \]

where 'b' and 'd' are the thrust and drag co-efficient respectively.

#### IV. DIVIDED STATE FEEDBACK CONTROL

DSF Control is a novel control technique that comprises of control with feedback delays. Technically, it is the establishment of partitioned sampling that yields...
the proposed feedback [1]. The block diagram above depicts a general overview of DSF control, see Fig. 3. Let us consider a framework [18] with variable time delays

$$\dot{x}(t) = \sum_{i=0}^{n} A_i x(t - \tau_i^s(t)) + Bu$$  \hspace{1cm} (17)

where $x \in \mathbb{R}^d$, $\tau_i^s(t) \in [0, \tau^s]$, $g = 1, 2, ..., n$ are the delays present in the system. The parameters defined for stability criterion [18] are

$$b = 2\hat{f}\|P\|, \quad \hat{f} = \psi r, \quad L_F = \sum_{i=0}^{n} ||A_i|| + d||BK||$$

$$\psi = \sum_{i=1}^{n} \sum_{p=0}^{n} ||A_i A_p|| + 2d||BK|| \sum_{i=1}^{n} ||A_i|| + d||BK|| ||A_0|| + d^2||BK||^2$$  \hspace{1cm} (18)

A. Control Law

The DSFC law for (18) is defined as:

$$u(t) = K \sum_{k=1}^{d} K_k(t) x(t - \tau_k^c(t))$$  \hspace{1cm} (19)

The stability of the closed-loop system is determined via the following result [18].

Corollary [18]: Given a deterministic delayed system (17), design the DSFC law as (19), if the upper headed $\tau$ for the time delays, with $0 \leq \tau < (\frac{1}{2\theta})$ is sufficiently little to such an extent that $c > 0$, where $\theta = 2L_F$, and the lattice $P$ is sure answer for the Riccati-Ito network [18], with $F_j = 0, j = 1,2, ..., r$ then the harmony $x=0$ (17) is globally asymptotically stable in mean square.

Time delays generally lead to oscillations, however, DSF control defines bounds on the state feedback delays such that the system delivers an improved stabilization and transient state response [18].

B. Computer Simulations

We present computer simulation results to illustrate the viability of our hybrid control design. Fig. 4 demonstrates the trajectory comparisons when the system is required to track a position of {10m, 10m} along the x and y directions, respectively. The figure shows a comparison between the results generated by applying a conventional passive fault-tolerant SMC and passive fault-tolerant SMC integrated with the DSF Control, see Fig. 4. The simulation results clearly demonstrate that the DSF control integrated with passive fault-tolerant SMC delivered an improved response as compared to the case without DSF control. The DSF control has its potential incentive for speeding up the response of systems [18] and has outperformed the conventional passive fault-tolerant sliding mode control. Fig. 5 shows the control efforts required for the accomplishment of the task.

TABLE I. DSF CONTROL TIME DELAYS

<table>
<thead>
<tr>
<th>States</th>
<th>Optimal Delays (sec)</th>
<th>Inferior Delays (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-position</td>
<td>0.25</td>
<td>&gt; 0.25</td>
</tr>
<tr>
<td>y-position</td>
<td>0.35</td>
<td>&gt; 0.3</td>
</tr>
<tr>
<td>Altitude</td>
<td>0.1</td>
<td>&gt; 0.1</td>
</tr>
<tr>
<td>Roll angle</td>
<td>0.045</td>
<td>&gt; 0.045</td>
</tr>
<tr>
<td>Pitch angle</td>
<td>0.045</td>
<td>&gt; 0.045</td>
</tr>
<tr>
<td>Yaw angle</td>
<td>0.005</td>
<td>&gt; 0.005</td>
</tr>
<tr>
<td>x-position rate</td>
<td>0.01</td>
<td>&gt; 0.01</td>
</tr>
<tr>
<td>y-position rate</td>
<td>0.01</td>
<td>&gt; 0.01</td>
</tr>
<tr>
<td>Altitude rate</td>
<td>0.01</td>
<td>&gt; 0.01</td>
</tr>
<tr>
<td>Roll rate</td>
<td>0</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>Pitch rate</td>
<td>0</td>
<td>&gt; 0</td>
</tr>
<tr>
<td>Yaw rate</td>
<td>0</td>
<td>&gt; 0</td>
</tr>
</tbody>
</table>

A detailed mathematical procedure for computation of bounds on time delays is presented in [18], [19]. Since our system and control are nonlinear, we have determined the values of time delays on the basis of simulation results. Table I above shows the range of time delays incorporated in our design.

The DSF Control problem can be viewed as an optimality problem as investigation shows an initial improvement, followed by deterioration of performance with an increase in time delays. We have found the optimum values of time delays for improved results. The DSF control thus improves the stability and transient response of the system, by making use of delays in the feedback path. The bounds on the values of the delays satisfy the stability criteria [18].

Figure 4. Comparison of DSF Control with Passive FTC based on SMC: (a) Roll Angle; (b) Pitch Angle; (c) X-Displacement; (d) Y-Displacement
This paper considered the fault-tolerant control of a nonlinear model of quadrotor UAV in the presence of control and/or sensory delays. An SMC provided good tracking performance under actuator fault conditions. In particular, we implemented a passive FTC that was designed for a pre-defined set of actuator faults. The FTC performance was observed to deteriorate in the presence of communication and sensory delays. Finally, a DSF control integrated with passive Fault-Tolerant SMC demonstrated improved stabilization, robustness and transient response in computer simulations.

V. CONCLUSION

This paper considered the fault-tolerant control of a nonlinear model of quadrotor UAV in the presence of control and/or sensory delays. An SMC provided good tracking performance under actuator fault conditions. In particular, we implemented a passive FTC that was designed for a pre-defined set of actuator faults. The FTC performance was observed to deteriorate in the presence of communication and sensory delays. Finally, a DSF control integrated with passive Fault-Tolerant SMC demonstrated improved stabilization, robustness and transient response in computer simulations.

CONFLICT OF INTEREST

There is no conflict of interest from any author for publishing this work. The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

AUTHOR CONTRIBUTIONS


ACKNOWLEDGEMENT

The first author is very much thankful for the funding support by UA little rock Graduate School, George W. Donaghey college of Engineering and Information Technology in form of research travel grant.

REFERENCES


Safi Ullah was born at Lahore, Pakistan. He obtained his BS in Electrical Engineering from The University of Lahore in 2014, and his Masters in Electrical Engineering from the EME College, National University of Sciences and Technology, Rawalpindi, Pakistan in 2016. He is currently pursuing his PhD degree in Electrical and Computer Engineering at University of Arkansas at Little Rock, Little Rock, Arkansas, USA. He has previously worked as Sr. Research Officer at University of Engineering and Technology, Lahore, Pakistan. His research interests include control systems, power electronics, renewable energy, dc control drives and statistical signal processing.

Fahad Muntaz Malik received his Bachelors in Mechatronics Engineering in year 2004. He did his MS and PhD in Electrical Engineering in 2006 and 2009 respectively. Since 2009 he is a member of the faculty of Electrical Engineering Department at National University of Sciences and Technology, Islamabad, Pakistan. His research interests are ariel and ground robotics and control systems.

Kamran Iqbal was born at Multan, Pakistan. He obtained his BE in aeronautical Engineering from NED University, Karachi in 1980, and his MS and PhD in electrical engineering from the Ohio State University, Columbus, Ohio in 1980 and 1992. He has held academic appointments at several institutions including College of Aeronautical Engineering, GIK Institute of Engineering Science and Technology, Northwestern University, California State University at Fullerton, University of California, Irvine, and University of Arkansas at Little Rock where he currently serves as Professor of Systems Engineering. His research interests include neuro-mechanics of human movement, myoelectric control of prostheses, biomedical signal and image processing, cyber physical systems, electrical energy, and machine learning. He has authored two books and over one hundred peer-reviewed research papers. Dr. Iqbal is a member of IEEE, IET (UK), BMES, ASEE, IASTED, and Sigma Xi (past president of Central Arkansas Chapter). He is an editor of Journal of Engineering (Hindawi).

Copyright © 2020 by the authors. This is an open access article distributed under the Creative Commons Attribution License (CC BY-NC-ND 4.0), which permits use, distribution and reproduction in any medium, provided that the article is properly cited, the use is non-commercial and no modifications or adaptations are made.