Determining Optimum Partial Transmission Ratios of Mechanical Driven Systems Using a Chain Drive and a Two-step Bevel Helical Gearbox

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***Abstract*—This paper introduces a new study on determining optimum partial transmission ratios of mechanical drive systems using a chain drive and two-step bevel helical gearbox. The objective function of the optimum problem was the cross section dimension of the system. In the study, the design equation for pitting resistance of a gear set was investigated and equations on moment equilibrium condition of a mechanic system including a chain drive and a three helical gear units and their regular resistance condition were analyzed for solving the optimum problem. From the results of the problem, effective formula for calculation of the partial ratios of the chain drive and two-step bevel helical gearboxes were proposed. As the formulas are explicit, the partial ratios can be calculated accurately and simply.**

***Index Terms*—gearbox design, optimum design, helical gearbox, transmission ratio**

I. INTRODUCTION

In optimal gearbox design, one of the most important tasks is determination of the partial transmission ratios of the gearbox. This is because the size, the dimension, the mass, and the cost of the gearbox depend strongly on the partial ratios. Consequently, optimal calculation of the partial ratios of gearboxes has been subjected to many researches.

So far, there have been many studies on the optimum calculating the partial ratios of helical gearboxes. The partial ratios have been determined for two-step gearboxes [1, 2, 3, 4, 5], three-step gearboxes [2, 6, 7] and four-step gearboxes [2, 8, 9]. Also, the optimum partial transmission ratios have been predicted for different types of gearboxes such as for helical gearboxes [1, 2, 3, 5, 6, 7, 8], for bevel helical gearboxes [3, 4], for worm gearboxes [10, 11] and for planetary gearboxes [12, 13]. Besides, the optimum partial transmission ratios can be found by the following methods:

By graph method: in this method, the partial ratios were determined graphically. For example, for a three helical gearbox, the partial ratios u_1 of step 1 and u_2 of steps 2 are predicted from the total transmission ratio u_t (see Fig. 1) for getting the minimal mass of the gearbox. The transmission ratio of the third step u_3 is then calculated from the total ratio and the ratios of step 1 and 2: $u_3 = u_t / \left(u_1 \cdot u_2\right)$. This method has been used in many studies such as in [1], [2] and [3] for two, three and four-step helical gearboxes.

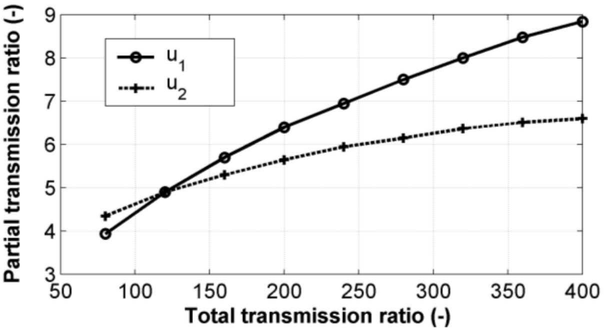

Figure 1. Transmission ratio of step 1 and 2 versus the total transmission ratio [1]

By "practical method": in this method, the partial transmission ratios are determined from practical data. Using this method, G. Milou et al. [3] noted that the weight of two-step helical gearbox will be minimum when the ratio a_{w2} / a_{w1} is from 1.4 to 1.6 (a_{w1} and a_{w2} are the center distances of the first and the second-step, respectively). From this, the tabulated optimal values of the partial ratios were proposed.

By model method: based on the results of optimization problems, models for determination of the partial ratios have been found for different objectives, such as for getting minimum cross section [5], for getting minimum mass of gears [7, 7, 9], for getting the minimal gearbox length of the gearbox [10, 14] or for minimum mass of the gearbox [15].

As above analysis, till now, there have been many studies on optimum determination of partial transmission

Manuscript received September 8, 2018; revised July 9, 2019.

doi: 10.18178/ijmerr.8.5.708-712

of gearboxes. However, all of the above studies were carried out on mechanical driven systems which are not use a belt or a chain drive. Recently, there have been several studies on calculating the partial ratios of the systems which use a V-belt [16, 17] or a chain drive [18] and a two-step gearbox. However, previously, there have not been studies for a system using a chain drive and a two-step bevel helical gearbox. This paper introduces a study for optimum calculation of partial ratios for driven systems using a chain drive and a two-step helical gearbox in order to get the minimum system cross-sectional dimension.

II. THEORETICAL BASIS

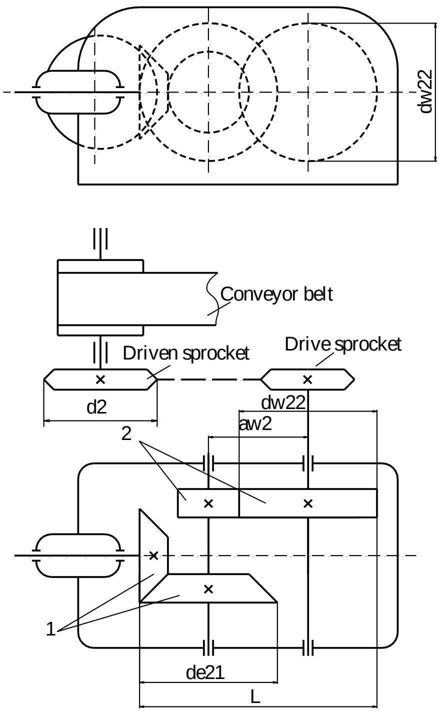

Figure 2. Calculation scheme

For a two-step bevel helical gearbox (Fig. 2), the cross-sectional dimension is minimum when [1]:

$$d_{e21} = d_{w22} \tag{1}$$

From (1) and from Fig. 2, it can be seen that, for a mechanical system which use a chain drive and a two-step helical gearbox (Fig. 2), the cross-sectional dimension is minimum when:

$$d_{e21} = d_{w22} = d_2 \tag{2}$$

In the above equations, d_{e21} and d_{w22} are driven diameters of two steps of the gearbox; d_2 is the pitch diameter of the driven sprocket of the chain drive.

For a two-step helical gearbox, the condition (1) is guaranteed when [4]:

$$u_2 = 1.32 \cdot u_g^{1/3} \tag{3}$$

In which, u_2 is transmission ratio of the low speed unit of the gearbox; u_g is the transmission ratio of the gearbox; u_g can be calculated by:

$$u_g = u_1 \cdot u_2 = \frac{u_t}{u_c} \tag{4}$$

where, u_1 is transmission ratio of the high speed unit of the gearbox; u_t is the total transmission ratio of the system; u_c is the transmission ratio of the chain drive.

From above analysis, for finding the optimum partial ratios of the systems in order to get the minimum system cross section, it is necessary to determine the diameters d_2 and d_{w22}.

A. Determining the Driven Diameter d_{w22}

The following equation is used as the design equation for pitting resistance of the third step of the gearbox [19]:

$$\sigma_{H2} = Z_{M2} \cdot Z_{H2} \cdot Z_{\varepsilon 2} \cdot \sqrt{\frac{2 \cdot T_{12} \cdot K_{H2} \cdot \sqrt{u_2 + 1}}{b_{w2} \cdot d_{w12}^2 \cdot u_2}} \leq [\sigma_{H2}] \tag{5}$$

It follows from Equation (5) that:

$$[T_{12}] = \frac{b_{w2} \cdot d_{w12}^2 \cdot u_2}{2 \cdot (u_2 + 1)} \frac{[\sigma_{H2}]^2}{K_{H2} \cdot (Z_{M2} \cdot Z_{H2} \cdot Z_{\varepsilon 2})^2} \tag{6}$$

where, b_{w1} is the face width (mm) and d_{w11} is the pitch diameter of the first step; they are calculated by the following equations:

$$b_{w2} = \psi_{ba2} \cdot a_{w2} = \psi_{ba2} \cdot d_{w12} \cdot (u_2 + 1)/2 \tag{7}$$

$$d_{w12} = \frac{d_{w22}}{u_2} \tag{8}$$

Substituting Eqs. (7) and (8) into Equation (6), we get

$$[T_{12}] = \frac{\psi_{ba2} \cdot d_{w22}^3 \cdot [K_{02}]}{4 \cdot u_2^2} \tag{9}$$

in which

$$[K_{02}] = \frac{[\sigma_{H2}]^2}{K_{H2} \cdot (Z_{M2} \cdot Z_{H2} \cdot Z_{\varepsilon 2})^2} \tag{10}$$

From Equation (9) the pitch diameter d_{w22} can be calculated by

$$d_{w22} = \left(\frac{4[T_{12}]u_2^2}{\psi_{ba2}[K_{02}]} \right)^{1/3} \tag{11}$$

B. Determining the Driven Sprocket Diameter d_2

For a chain drive, the pitch diameter of the driven sprocket is calculated by [19]:

$$d_2 = \frac{d_1}{u_c} \tag{12}$$

where, d_1 is the pitch diameter of the drive sprocket, d_1 is determined by the following equation [19]:

$$d_1 = \frac{p}{\sin(\pi / z_1)} \tag{13}$$

In which,

z_1 - the number of teeth in the drive sprocket; Based on the tabulated data from [19], the following equation was found for calculating z_1 (with R^2 =0.995):

$$z_1 = 32.4 - 2.4 \cdot u_c \tag{14}$$

p - the chain pitch (mm); p is determined based on the design power capacity P which can be expressed as follows [19]:

$$P = P_1 \cdot k \cdot k_z \cdot k_n \tag{15}$$

where, P_1 is the power rating (kW) which can be calculated by:

$$P_1 = \frac{T_1 \cdot n_1}{9.55 \cdot 10^6} \tag{16}$$

With n_1 is the revolution of the drive sprocket (rpm):

$$n_1 = \frac{n_m}{u_g} \tag{17}$$

$$T_1 = T_o \cdot \eta_c \cdot \eta_b \tag{18}$$

In the equations (17) and (18), η_c is efficiency of chain drive (η_c =0.95-0.97 [19]); η_b is efficiency of a pair of bearings (η_b =0.99-0.995 [19]); T_1 is the torque on the drive (Nmm); T_0 is the output torque (Nmm).

k, k_z and k_n are coefficients which are determined by the following equations:

$$k = k_d \cdot k_p \cdot k_c \cdot k_{adj} \cdot k_{\text{lub}} \cdot k_{con} \tag{19}$$

$$k_z = \frac{25}{z_1} \tag{20}$$

$$k_n = \frac{n_{01}}{n_1} \tag{21}$$

In the above equations, k_d is effect of shock factor; k_p is effect of position of the drive; k_c is effect of center distance; k_{adj} is effect of possibility of adjusting the center distance; k_{lub} is effect of lubrication; k_{con} is effect of operating conditions; n_{01} is tabulated number of teeth of the drive sprocket.

III. Determining Partial Transmission Ratios

For determining the partial transmission ratios of the system, a computer program was conducted. The aim of the program is to find the optimum values of the transmission ratio of the chain drive u_c which satisfies the condition (2). The chosen programming language was Matlab. The following input values of the program were chosen: u_t =10÷80; u_g =5÷70; To=70,000÷2,000,000 (Nmm); k_d =1; k_p =1; k_c =1; k_{adj} =1; k_{lub} =1; k_{con} =1.25.

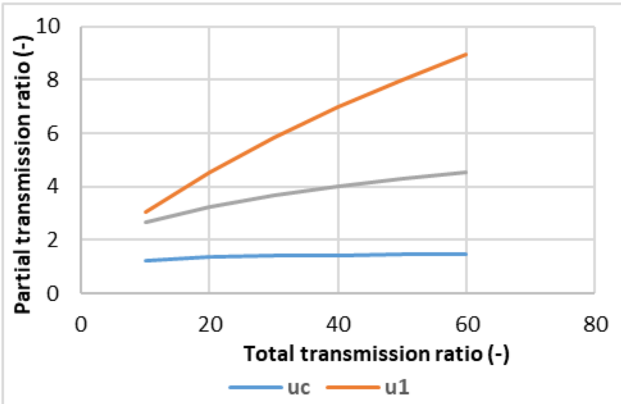

Figure 3. Partial transmission ratios versus total transmission ratio with different output torque

Fig. 3 shows the relation between the partial transmission ratio and the total transmission ratio (the graph was built with the output torque T_{out} =1000,000 Nmm). It is observed that with the increase of the total transmission ratio the transmission ratio of the chain drive is nearly not changed. Besides, the transmission ratio of the helical gear set is increased lightly, whereas the transmission ratio of the bevel gear set increase very fast. The reason of that is the torque on the output shaft of the chain drive is the largest whereas it is smaller on the output shaft of the helical gear set and much smaller on the output shaft of the bevel gear set.

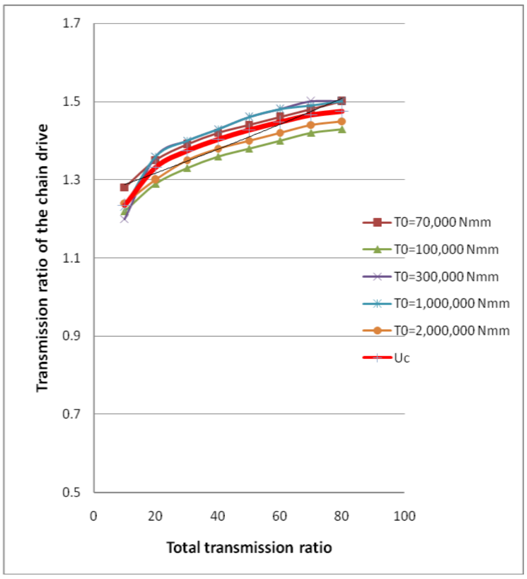

Figure 4. Transmission ratio of the chain drive versus total transmission ratio with different output torque

It was found that the partial transmission ratios of the bevel gear set and the helical gear set do not depend on the output torque of the system. Opposite, the transmission ratio of the chain drive depends slightly on the output torque. The relation between the optimum transmission ratio of the chain drive and the total transmission ratio of the system with different output torque is described in Fig. 4. It was found that the optimum values of the transmission ratio of the chain drive depend strongly on the total transmission ratio (Fig. 4). Besides, as above analysis, the output torque does not affect much on the transmission ratio of the chain drive. Consequently, the influence of the output torque can be replaced by choosing average values of the transmission ratio of the chain drive (the line of u_c (Fig. 4)). Additionally, the following equation was found for calculating the optimum transmission ratio of the chain drive (with R^2 =0.99):

$$u_c = 1.024 \cdot u_t^{0.084} . \tag{22}$$

After determining the transmission ratio of the chain drive by Equation (22), the partial ratios u_1 and u_2 of the two-step bevel helical gearbox can be easily found from Equations (3) and (4).

IV. CONCLUSIONS

The minimum system cross-sectional dimension of mechanical drive systems using a chain drive and a two-step helical gearbox can be obtained by optimum splitting the total transmission ratio of the system.

An equation for calculation of the optimum transmission ratio of the chain drive was proposed. Also, the partial transmission ratios of the two-step helical gearbox can be calculated in order to get the minimum cross-sectional dimension of the system.

The partial transmission ratios of the chain drive and the gearbox of the system can be determined accurately and simply by using explicit equations.

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