The Geometrical Non-Linearity Influence of the Suspension of Micromechanical Gyroscope under the Conditions of Angular Vibration

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***Abstract* — This paper is a study of the geometrical non-linearity influence of the elastic suspension in micromechanical gyroscope under the conditions of angular vibration. The spatial non-linear oscillations of the elastic suspension were considered. A quasi-linear system of equations of Mathieu–Hill, describing the motion of sensitive element on vibrating conditions close to the main parametric resonance, was obtained. In this paper an averaged equation of motion and found stationary regimes of oscillations was built. The case of losing stability of oscillations in terms of the main parametric resonance was analyzed. And the angular vibration in the resonance case leads to additional deflection of gyroscope. It is shown that for certain parameters of the system the regime of plane stationary oscillations of the sensitive element of the MMG becomes unstable. The conditions for the appearance and stability of spatial stationary oscillations in the vicinity of the main resonance are investigated depending on the parameters of the system.**

***Index Terms* — micromechanical gyroscope, dynamic, non-linearity influence**

I. Introduction

Consider one of the constructive schemes micromechanical gyroscope (MMG) with an elastic suspension of a rigid body- sensitive element on sub-frame [1] (Fig. 1). Kinematic scheme of the gyroscope is two-stage of cardanic suspension of the sensitive element. According to the accepted classification, this is micromechanical gyroscope with an angular motion sensor (MMG-RR type) [2, 3].

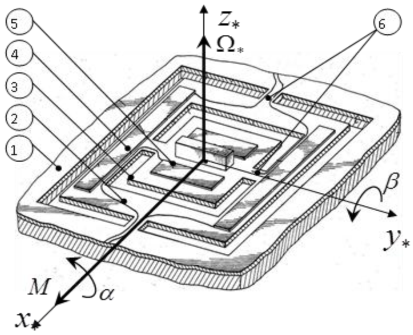

Figure 1. Micromechanical gyroscope RR-type.

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We assume that the sensor of MMG is balanced, point O center suspension and the center of mass coincide. Assume that the design provides infinite torsion bending stiffness.

II. Dynamic of Micromechanical Gyroscope

Consider a micromechanical gyroscope RR-type with angular motion of sensitive element, which was used within control system as mobile objects.

Let $Ox_*y_*z_*$ and $Oxyz$ are coordinate systems of basis and sensitive element of gyroscope, respectively (Fig.2). The relative position of the coordinate systems was presented by the following scheme:

$$Ox_*y_*z_* \xrightarrow[x_*]{\alpha} Ox_1y_1z_1 \xrightarrow[y_1]{\beta} Oxyz\ . \tag{1}$$

This scheme was indicated by upper the arrow - angle, and under the arrow - the axis around which rotates counterclockwise by this angle.

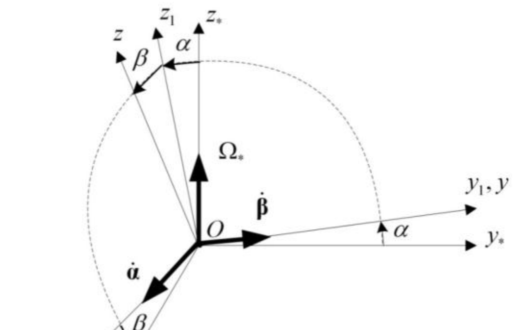

Figure 2. Kinematic scheme of sensitive suspension in micromechanical gyroscope RR-type.

The kinematic energy of the system is:

$$\begin{aligned} T = \frac{1}{2}\Big[& J_x\left(\dot{\alpha}\cos\beta - \Omega_*\cos\alpha\sin\beta\right)^2 + I_x\dot{\alpha}^2 + \\ & + J_y\left(\Omega_*\sin\alpha + \dot{\beta}\right)^2 + I_y\Omega_*^2\sin^2\alpha + \\ & + J_z\left(\Omega_*\cos\alpha\cos\beta + \dot{\alpha}\sin\beta\right)^2 + I_z\Omega_*^2\cos^2\alpha\Big], \end{aligned} \tag{2}$$

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where J_x, J_y, J_z - axial moments of inertia of the gyroscope sensor about *Ox*, *Oy*, *Oz* axes, respectively; I_x, I_y, I_z - axial moments of inertia of the intermediate frame about Ox_1, Oy_1, Oz_1, respectively. We assume centrifugal moments of inertia of the sensing element and frame are zero.

On the power electrodes to excite vibrations, the AC-voltage is applied and the moment of electrostatic actuator appeared:

$$M = M_0 \sin \omega_0 t \;, \tag{3}$$

where M_0 and ω_0 - amplitude and frequency of the forced moment.

The potential energy of the system is illustrated:

$$P = \frac{1}{2} c_\alpha \alpha^2 + \frac{1}{2} c_\beta \beta^2 - M\alpha \;, \tag{4}$$

where M – the forced moment, c_α, c_β - the torsion stiffness's by respective coordinates.

Dissipative function of dissipative energies is:

$$\Phi = \frac{1}{2} k_\alpha \dot{\alpha}^2 + \frac{1}{2} k_\beta \dot{\beta}^2 \;, \tag{5}$$

where k_α, k_β - damping coefficients while rotating the outer and inner frames about axes Ox_* and Oy, respectively.

The system of Lagrange's equations is:

$$\begin{cases} \dfrac{d}{dt}\dfrac{\partial T}{\partial \dot{\alpha}} - \dfrac{\partial T}{\partial \alpha} = Q_\alpha, \\ \dfrac{d}{dt}\dfrac{\partial T}{\partial \dot{\beta}} - \dfrac{\partial T}{\partial \beta} = Q_\beta, \end{cases} , \tag{6}$$

where Q_α, Q_β - generalized forces:

$$Q_\alpha = -\frac{\partial P}{\partial \alpha} - \frac{\partial \Phi}{\partial \dot{\alpha}}, \quad Q_\beta = -\frac{\partial P}{\partial \beta} - \frac{\partial \Phi}{\partial \dot{\beta}} \;. \tag{7}$$

The system of equations of gyroscope motion is obtained:

$$\begin{cases} \left(I_x + J_z \sin^2\beta + J_x \cos^2\beta\right)\ddot{\alpha} + \\ +\left(J_z - J_x\right)\dot{\alpha}\dot{\beta}\sin 2\beta + \dfrac{1}{2}\left(J_z - J_x\right)\dot{\Omega}\sin 2\beta \cos\alpha - \\ -\left(J_y - \left(J_z - J_x\right)\cos 2\beta\right)\dot{\beta}\Omega_* \cos\alpha - \\ -\dfrac{1}{2}\left(I_y - I_z + J_y - J_z\cos^2\beta - J_x \sin^2\beta\right)\Omega_*^2 \sin 2\alpha + \\ +k_\alpha \dot{\alpha} + c_\alpha \alpha + M = 0, \\ J_y \ddot{\beta} - \dfrac{1}{2}\left(J_z - J_x\right)\dot{\alpha}^2 \sin 2\beta + J_y \dot{\Omega}\sin\alpha - \\ -\left(\left(J_z - J_x\right)\cos 2\beta - J_y\right)\Omega_* \dot{\alpha}\cos\alpha + \\ +\dfrac{1}{2}\left(J_z - J_x\right)\Omega^2 \sin 2\beta \cos^2\alpha + k_\beta \dot{\beta} + c_\beta \beta = 0. \end{cases} \tag{8}$$

Giving the smallness of generalized coordinates α and β, rewrite the Lagrange's equations (8) with precision of fourth order of coordinates:

$$\begin{cases} \left(I_x + J_x\right)\ddot{\alpha} + \left(J_z - J_x\right)\ddot{\alpha}\beta^2 + 2\left(J_z - J_x\right)\beta\dot{\alpha}\dot{\beta} - \\ -\left(J_x + J_y - J_z\right)\dot{\beta}\Omega + \left(J_z - J_x\right)\beta\dot{\Omega} + \\ +\left(I_z - I_y + J_z - J_y\right)\alpha\Omega^2 + k_\alpha\dot{\alpha} + c_\alpha \alpha + M = 0, \\ J_y \ddot{\beta} + \left(J_x - J_z\right)\beta\dot{\alpha}^2 + \left(J_x + J_y - J_z\right)\dot{\alpha}\Omega + \\ +\left(J_z - J_x\right)\beta\Omega^2 + J_y \dot{\Omega}\alpha + c_\beta \beta + k_\beta \dot{\beta} = 0. \end{cases} \tag{9}$$

The system of equations (9) describes a quasi-linear oscillations of a micromechanical gyroscope sensor on a moving basis.

Normalization of the system of equations. It is convenient to go to the dimensionless variables, by which we can reduce the number of system parameters, and investigate the nonlinear effects associated by finite amplitude oscillation of sensor, vibration of base and manufacturing errors.

We assume that there are two oscillation frequencies ω_1 and ω_2 of gyroscope sensor:

$$\omega_2 = \sqrt{\frac{c_\beta}{J_y}} = \omega, \quad \omega_1 = \sqrt{\frac{c_\alpha}{J_x + I_x}} = \omega\left(1 - \varepsilon\sigma\right) \tag{10}$$

where σ - dimensionless parameter, characterizing the small residual difference between oscillation frequencies.

Also consider the small difference in energy dissipation of generalized coordinates α and β:

$$\varepsilon\gamma_1 = \frac{k_\alpha}{\sqrt{c_\alpha\left(J_x + I_x\right)}}, \quad \varepsilon\gamma_2 = \frac{k_\beta}{\sqrt{c_\beta J_y}} \;, \tag{11}$$

where γ_1, γ_2 - normalized damping coefficients of the order of unity.

The dimensionless time τ, normalized generalized coordinates x, y and angular velocity ν are obtained by the following relationships:

$$\tau = \omega t, \;\; \alpha = \sqrt{\varepsilon}\sqrt{j_1}\,x, \;\; \beta = \sqrt{\varepsilon}\sqrt{j_2}\,y, \\ \varepsilon\nu = \sqrt{j_1}\sqrt{j_2}\,\frac{\Omega_*}{\omega}, \tag{12}$$

where j_1, j_2 - dimensionless parameters that characterize the mass-geometric characteristics of the gyroscope:

$$j_1 = \frac{J_x + J_y - J_z}{J_x + I_x}, \; j_2 = \frac{J_x + J_y - J_z}{J_y}, \; \xi = \frac{1}{j_2}, \\ j = \frac{\left(J_x - J_z\right) j_1}{J_y}, \; m\varepsilon^{3/2} = \frac{M}{c_\alpha}\sqrt{\frac{J_x + I_x}{J_x + J_y - J_z}}, \tag{13}$$

where j and ξ - normalized parameters of gyroscope.

Giving substitutions (12), (13), equation of motion (9) takes the following form:

$$\begin{cases} \ddot{x}+x=\varepsilon\left(-\gamma_1\dot{x}+\nu\dot{y}+\left(1-\xi\right)\dot{\nu}y+\right. \\ \qquad \left. +2\sigma x+jy\left(xy+2\dot{x}\dot{y}\right)-m\right), \\ \ddot{y}+y=\varepsilon\left(-\gamma_2\dot{y}-\nu\dot{x}-\xi\dot{\nu}x-jy\dot{x}^2\right). \end{cases} \quad (14)$$

Hereinafter, the dot over letters denotes differentiation with respect to the dimensionless time τ.

III. The Geometrical Non-Linearity Influence Of The Elastic Suspension

We pose the problem of studying the dynamics of small free oscillations of a micromechanical gyroscope sensor on a vibrating base, taking into account the geometric nonlinearity of elastic suspension. The greatest practical interest is the case of the main parametric resonance when the frequency of the angular vibration closes to double the frequency of free oscillations of the system.

First, consider the case of still hard $(\sigma=0)$ elastic suspension sensor and an isotropic energy dissipation $(\gamma_1=\gamma_2=\gamma)$ then the equation of small oscillations of the micromechanical gyroscope can be written in such a way:

$$\begin{cases} \ddot{x}+x=\varepsilon\left(-\gamma_1\dot{x}+\nu\dot{y}+\left(1-\xi\right)\dot{\nu}y+\right. \\ \qquad \left. +jy\left(xy+2\dot{x}\dot{y}\right)-m\right), \\ \ddot{y}+y=\varepsilon\left(-\gamma_2\dot{y}-\nu\dot{x}-\xi\dot{\nu}x-jy\dot{x}^2\right). \end{cases} \quad (15)$$

Let the angular velocity of the base varies according to the rule [5]:

$$\Omega_*=\Omega_0+\Omega_1\sin 2\omega_0 t, \quad (16)$$

where Ω_0 - given constant angular velocity of the base; Ω_1 and $2\omega_0$ - amplitude and frequency angular vibration of the base.

Taking into account (12) the expression (16) for normalized angular velocity of the base will be:

$$\nu=\nu_0+\nu_1\sin 2\mu\tau, \quad (17)$$

where ν_0 - dimensionless constant of angular velocity of the base, ν_1 - normalized amplitude of angular vibrating velocity of the base, μ - dimensionless vibrating frequency of the base:

$$\varepsilon\nu_0=\sqrt{j_1}\sqrt{j_2}\frac{\Omega_0}{\omega}, \varepsilon\nu_1=\sqrt{j_1}\sqrt{j_2}\frac{\Omega_1}{\omega}, \mu=\frac{\omega_0}{\omega}. \quad (18)$$

In view of the law for the angular velocity of the vibration base (17) equations (15) are a nonlinear system of differential equations of Mathieu – Hill:

$$\begin{cases} \ddot{x}+x=\varepsilon\left(-\gamma\dot{x}+\left(\nu_0+\nu_1\sin 2\mu\tau\right)\dot{y}+jy\left(xy+2\dot{x}\dot{y}\right)+\right. \\ \qquad \left. +2\mu\left(1-\xi\right)\nu_1 y\cos 2\mu\tau-m_0\sin\mu\tau\right), \\ \ddot{y}+y=\varepsilon\left(-\gamma\dot{y}-\left(\nu_0+\nu_1\sin 2\mu\tau\right)\dot{x}-\right. \\ \qquad \left. -2\mu\xi\nu_1 x\cos 2\mu\tau-jy\dot{x}^2\right). \end{cases} \quad (19)$$

The dynamics of the system in the slow variables. Next, we consider the problem of the dynamics (19) in the resonance case, when the angular vibrating frequency closes to double the frequency of free oscillations of the system, i.e. [7,10]:

$$\mu-1=\varepsilon\lambda, \quad (20)$$

where λ - frequency detuning of system $(\lambda\sim 1)$.

For the perturbed system (19) apply the averaging method Krylov-Bogolyubov and present it to the standard form by the transition from the variables to the slow variables Van der Pol from the formulas :

$$\begin{aligned} &x=q_1\cos\mu\tau+p_1\sin\mu\tau, \dot{x}=-q_1\mu\sin\mu\tau+p_1\mu\cos\mu\tau, \\ &y=q_2\cos\mu\tau+p_2\sin\mu\tau, \dot{y}=-q_2\mu\sin\mu\tau+p_2\mu\cos\mu\tau. \end{aligned} \quad (21)$$

Near the main parametric resonance (20) equations (19) after the transition to the slow variables according to the formulas (21) take the form [4,6]:

$$\begin{cases} \dot{q}_1=\frac{\varepsilon}{8}\left(-4\gamma q_1-8\lambda p_1+4\nu_0 q_2-4\nu_1 p_2+\right. \\ \qquad \left. +jp_1\left(q_2^2+p_2^2\right)-2jq_2\left(p_2q_1-p_1q_2\right)-4m_0\right), \\ \dot{p}_1=\frac{\varepsilon}{8}\left(-4\gamma p_1+8\lambda q_1+4\nu_0 p_2-4\nu_1 q_2-\right. \\ \qquad \left. -jq_1\left(q_2^2+p_2^2\right)-2jp_2\left(p_2q_1-p_1q_2\right)\right), \\ \dot{q}_2=\frac{\varepsilon}{8}\left(-4\gamma q_2-8\lambda p_2-4\nu_0 q_1-4\nu_1 p_1+\right. \\ \qquad \left. +jp_2\left(q_1^2+p_1^2\right)+2jq_1\left(p_2q_1-p_1q_2\right)\right), \\ \dot{p}_2=\frac{\varepsilon}{8}\left(-4\gamma p_2+8\lambda q_2-4\nu_0 p_1-4\nu_1 q_1-\right. \\ \qquad \left. -jq_2\left(q_1^2+p_1^2\right)+2jp_1\left(p_2q_1-p_1q_2\right)\right). \end{cases} \quad (22)$$

Stationary oscillations of a nonlinear system. Analytically integrate the system of nonlinear differential equations (22) for arbitrary initial conditions is not possible. This can be done by numerical integration methods.

For applications, it is important to study the stationary mode of sensor MMG vibrations. To determine the stationary oscillations we find the singular points of the system (20), equating to zero the right-hand side of equations (20).

The basic properties of stationary vibrations of the system (22) can be studied analytically in the case of small damping coefficient $(\gamma=0)$ and the lack of the uniform angular velocity $(\nu_0 = 0)$. In this case, the solution of the nonlinear algebraic equations:

$$\begin{cases} -8\lambda p_1 - 4\nu_1 p_2 + jp_1\left(q_2^2 + p_2^2\right) - \\ \quad -2jq_2\left(p_2 q_1 - p_1 q_2\right) - m_0 = 0, \\ 8\lambda q_1 - 4\nu_1 q_2 - jq_1\left(q_2^2 + p_2^2\right) - \\ \quad -2jp_2\left(p_2 q_1 - p_1 q_2\right) = 0, \\ -8\lambda p_2 - 4\nu_1 p_1 + jp_2\left(q_1^2 + p_1^2\right) + \\ \quad +2jq_1\left(p_2 q_1 - p_1 q_2\right) = 0, \\ 8\lambda q_2 - 4\nu_1 q_1 - jq_2\left(q_1^2 + p_1^2\right) + \\ \quad +2jp_1\left(p_2 q_1 - p_1 q_2\right) = 0, \end{cases} \tag{23}$$

can be found by numerical methods.

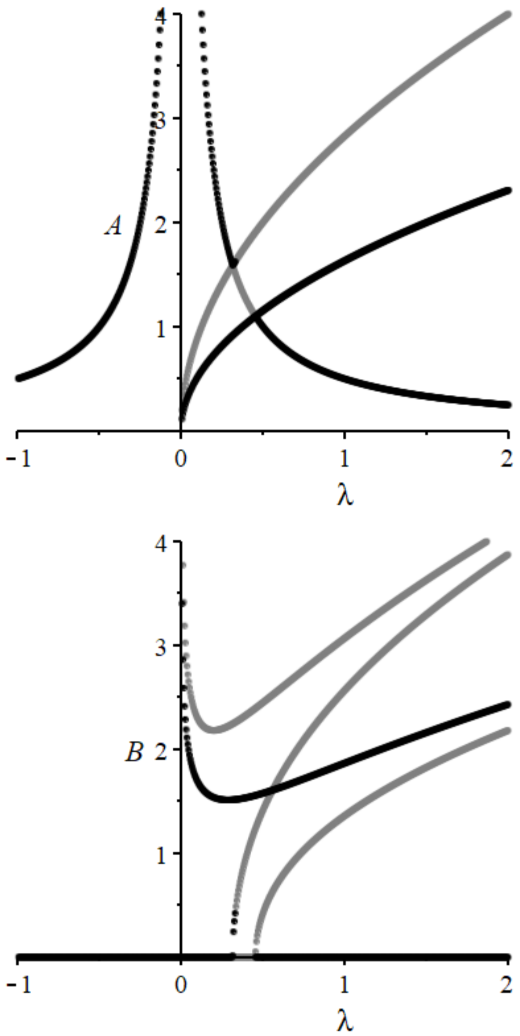

Figure 3. The amplitude-frequency characteristics of the forced stationary oscillations when $\nu_1 = 0$.

The amplitude-frequency characteristics $A(\lambda), B(\lambda)$ (Fig. 3, 4,) are a family of parabolas, where in the doubled value of frequency detuning λ must be greater than the angular amplitude of vibration of a base ν_1.

Lyapunov stable stationary oscillation amplitudes are marked in Fig. 3, 4, dark and unstable - a light background. Note that the presence of rotational vibration to the base $(\nu_0 \neq 0)$ leads to the additional branch on the amplitude-frequency characteristic [8, 9, 10].

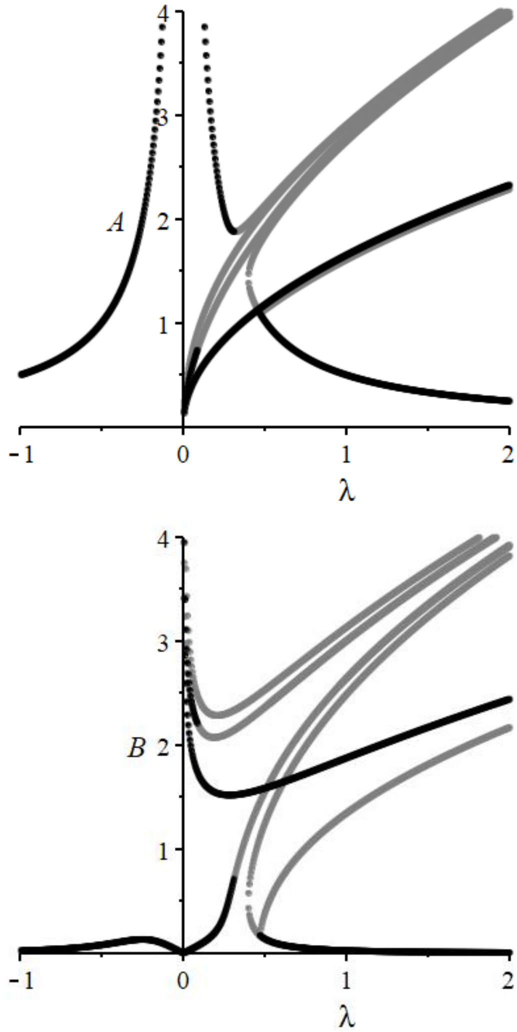

Figure 4. The amplitude-frequency characteristics of the forced stationary oscillations when $\nu_1 = 0.1$.

IV. Conclusion

A new mathematical model of the sensor vibrations, which takes into account the geometric nonlinearity of the elastic suspension under conditions of a rotational vibration, was developed. Equations of oscillations are system of nonlinearity differential equations Mathieu-Hill. Using the averaging method of Krylov-Bogolyubov to investigate the dynamics of a gyroscope in the slow variables. As shown that the angular vibration, occurring at the non-resonant frequencies within the framework of the assumptions, does not affect the accuracy of the gyroscope. And the angular vibration in the resonance case leads to additional deflection of gyroscope. It is

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shown that for certain parameters of the system the regime of plane stationary oscillations of the sensitive element of the MMG becomes unstable. The conditions for the appearance and stability of spatial stationary oscillations in the vicinity of the main resonance are investigated depending on the parameters of the system.

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