The Mathematical Model of Motion of Particles in Vibrating Conveyors

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***Abstract*— The modes of movement of a material point on vibrating inclined surface, characteristic for oscillating vibratory and inertial conveyors are considered. The mathematical model of movement is developed, taking into account both the influence of inertial forces and forces of gravity, and aerodynamic forces. The stated algorithm of determination of parameters of movement of a material point allows to analyze its peculiarities, to detect optimum parameters of movement.**

***Index Terms*— Oscillating inertial conveyors, oscillating vibrating conveyors, continuous movement of the point, harmonic oscillations, friction force**

I. Introduction

Vibration in technology usually has undesirable effect [1, 2]. However, there is a whole class of machines and mechanisms that have the basic principle of the vibration process [3, 4]. Machines and devices with vibrating working surfaces are widely used in industry. If we consider the design of modern conveyors, they are also different [5]. Machines that use vibrating surfaces are very diverse-vibratory conveyors, oscillating inertia and vibration conveyors, concentration tables, jigging machines with a movable sieve and others. The use of inertial and vibratory conveyors allows efficient and economical to solve the problem of moving solids.

The expediency of their application and efficiency is not in doubt. In connection with the widespread use of vibration machines, the problem of studying the movement of a material point on a vibrating inclined plane with the action of an additional force field is very relevant. To design the most efficient and cost-effective machine, it is necessary to find the optimal parameters of the oscillation process.

The principle of operation of vibration machines is based on vibration motion. It is known that the vibrational motion consists in the directed motion of the material particles under the action of inertia, gravity and friction forces [6, 4]. In this paper we consider the effect of aerodynamic forces of the moving air flow on the nature of the particle motion. This is the qualitative difference between the proposed mathematical model and the previously described one.

Manuscript received August 2, 2018; revised July 18, 2019.

In the given work differential equations of vibration movements particles on a gutter record and analyze both in a stationary system and in a movable system of coordinates taking into account aerodynamic influence.

II. Definition of a Task

Consider the plane inclined to the horizon at an angle α, batch the rectilinear harmonic oscillations with same frequency ω in the direction along the angle β with the plane.

We will adopt a movable coordinate system xOy, rigidly connected with the vibrating plane and the stationary coordinate system $\xi O\eta$. They are shown in the figure 1. The specified particle has the force of gravity mg, the force of dry friction F, normal reaction N, resistance forces F_x and F_y caused by the action of vertical and horizontal V_x, V_y air flows. The solution of the problem of control of the frequency of perturbing oscillations does not cause problems [7, 8, 9].

The particle motion equations in the projections on the movable axes are as follows:

$$\left.\begin{aligned} m\left(\ddot{x}+\ddot{\xi}\right) &= -mg\sin\alpha - F_x + F \\ m(\ddot{y}+\ddot{\eta}) &= -mg\cos\alpha - F_y + N \end{aligned}\right\} (1)$$

where: $\ddot{\xi}$ and $\ddot{\eta}$ – projections of moving the plane on the axis of the stationary coordinate system $\xi O\eta$ will be

$$\left.\begin{aligned} \xi &= A\cos\beta\sin\omega t \\ \eta &= A\sin\beta\sin\omega t \end{aligned}\right\} (2)$$

where A – amplitude, ω – frequency of oscillation of a plane, angles α and β lie within

$$-\frac{\pi}{2} \le \alpha \le \frac{\pi}{2},\ 0 \le \beta \le \frac{\pi}{2}.$$

Calculate projection of inertia force $m\ddot{\xi}$ and $m\ddot{\eta}$ in relative motion using (2).

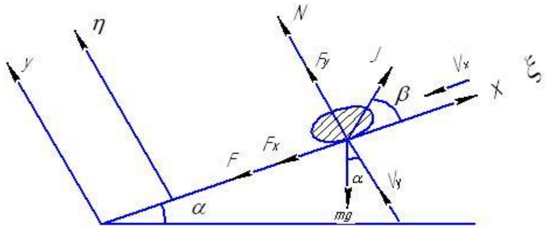

Figure 1. Scheme of motion of a material particle on the vibrating surface.

Equation (1) will take the following form:

$$\left.\begin{aligned} m\ddot{x} &= mA\omega^2\cos\beta\sin\omega t - \\ &-mg\sin\alpha - F_x + F \\ m\ddot{y} &= mA\omega^2\sin\beta\sin\omega t - \\ &-mg\cos\alpha - F_y + F \end{aligned}\right\} \quad (3)$$

III. Mathematical Model

In machines that implement vibration movement, it is possible to move the part on the vibrating surface with or without separation from the surface. When moving particles on the surface of the complaint must be considered at the design stage the presence of friction between the particle and the surface. Known materials and technologies for their application to the vibrating surface, providing minimal wear of the working part of the surface [10]. Let us consider in more detail and get the equation of motion of the particle.

At the continuous movement of a particle on a vibrating plane in the tilt inertial conveyor The coordinate y equals 0, and the force of friction F is determined by the ratio:

$$F = -fN, \text{ if } \dot{x} > 0$$
$$F = fN, \text{ if } \dot{x} < 0 \quad (4)$$
$$F = sign\dot{x} fN$$

From the equation (3) we will find a normal reaction

$$N(t) = mg\cos\alpha - mA\omega^2\sin\beta\sin\omega t - F_y \quad (5)$$

Vibrating conveyor is characterized by the fact that the movement of particles occurs without interruption from the vibrating gutter, i.e. on condition of positive reaction $N(t) > 0$. In this case, from (5) it is possible to define a phase angle at which the particle will slide on the vibrating surface without interruption.

$$\sin\omega t < \frac{g\cos\alpha - K_y}{A\omega^2\sin\beta} = z_0 \qquad (6)$$

$$K_y = \frac{3C_yV_y^2}{4\rho_r d};\ K_x = \frac{3C_xV_x^2}{4\rho_r d}$$

where: C_x and C_y - the coefficient of aerodynamic resistance, ρ_r - the density of the particle.

From expressions (5) and (6) it follows: if the condition is executed

$$z_0 = \frac{g\cos\alpha - K_y}{A\omega^2\sin\beta} \geq 1 \ (7)$$

normal reaction $N(t)$ at any time is positive and the particle ranked on the vibrating plane remain on the plane. In the phase angles interval

$$\omega t_0 < \omega t < \pi - \omega t_0 \ (8)$$

where $\omega t_0 = \arcsin z_0$, a particle previously on a vibrating gutter, breaks away from it and moves independently. The conditions of separation are as follows:

$$\sin\omega t_0 = \frac{g\cos\alpha - K_y}{A\omega^2\sin\beta} = z_0, \ (9)$$

or in another form

$$z_0 = \frac{g\cos\alpha - \frac{3C_yV_y^2}{4\rho_r d}}{A\omega^2\sin\beta} \leq 1. \quad (10)$$

To swinging the conveyor for calculating a number of practically important parameters, it is necessary to find out the character of the mode of particle movement at the specified values of basic parameters. The established regime is characterized by periodically repeating moments of transition from one stage of movement to another -from sliding forward - to flight and from flight - to sliding back, etc.

Some qualitative representation about the character of possible established movements of a particle, it is possible to get, having broken all axis of time on two kinds of intervals.

The interval *I* is characterized by the fact that the particle moves without interruption from the vibrating plane. The value of the normal reaction $N(t)$ determined by the equation (5). It is positive and in this case the following inequality (6) of the unbreakable movement is performed. This mode is typical for vibrating swinging conveyors.

The interval *II* is characterized by a separation of a particle from a vibrating plane. Inequality (10) is performed within this interval. The presence of the type *II* interval in motion is necessary for inertial oscillating conveyors.

The time limits $t = t_0^{(k)} (k = 0,1,2 \ldots)$ of the first and second intervals are from the equation

$$\sin\omega t_0^{(k)} = z_0, \ (11)$$

obtained from the equation (9) to determine the moments of separation of the particle from the gutter.

The interval splitting is as follows:

If the particle having a zero transverse velocity $\dot{y}$ is on a vibrating plane, a moment of time t belonging to the interval *I*, then this particle remains on the vibrating plane some finite period of time t_0 before the moment of separation, determined from the equation (9).

If the particle with zero velocity $\dot{y}$ gets on the vibrating plane at a time, belonging to interval *II*, the particle instantly breaks off from a vibrating plane. Intervals *I* exist at all possible values of parameters of the system, as for intervals of *II* type, they take place at performance of inequality (10).

If $z_0 > 1$ the inequality (7) will not be executed at any values t, interval *II* is absent. The particle moves without interruption, constantly remaining on a vibrating plane.

However, in these cases, there may be established regimes with tossing at the elastic bump of the particle on the vibrating surface. The increase of the vertical component of the velocity V_y or coefficient K_y in the equation (10) decreases z_0, In this case, the interval *II* is extended. Taking into account that in flight normal reaction and force of friction of sliding is equal 0, differential equations for interval *II* are the following type:

$$\left.\begin{aligned}\ddot{x} &= A\omega^2 \cos\beta \sin\omega t - g\sin\alpha - K_y\\ \ddot{y} &= A\omega^2 \sin\beta \sin\omega t - g\cos\alpha + K_y\end{aligned}\right\} \quad (12)$$

To determine the initial velocity of the separation, it is necessary to consider the character of the bump particle with a vibrating chute.

Often believe that bump occur almost instantaneously. It is assumed that in the process of bump the particles change the components of the velocity, and the bump on the motion of the plane is neglected.

To preserve the commonality of results we take a blow as absolutely inelastic. For this purpose, enter the recovery factor $R = \frac{\dot{y}_b}{\dot{y}_0} = 0$. Here $\dot{y}_b$ and $\dot{y}_0$ - projection velocity of the particle on the axle y after and to the bump respectively.

Concerning the law of change of longitudinal component t as a result of bump there is a different assumption. The most common nature of the change is measured by the instantaneous friction coefficient λ.

$$1 - \lambda = \frac{\dot{x}_b}{\dot{x}_0},$$

where $\dot{x}_b$ and $\dot{x}_0$ – the velocity of the particle is after and to the bump $0 \le \lambda \le 1$. In the future we shall count $\lambda = 1$ [6]. The significant change in the coefficient of instantaneous friction is connected with the fact that the actual laws defining the behavior of real bodies at bump are very complex and varied and practically cannot be described in mathematical form.

Placing the origin at the point of separation of the particle and integrating the equation (10) under the initial conditions: where the flight start time, and the projection velocity of the particle at the time of separation: $t - t_0^* = 0$, $x(t_0^*) = 0$, $\dot{x}(t_0^*) = \dot{x}_0^*$, $y(t_0^*) = 0$, $\dot{y}(t_0^*) = \dot{y}_0^*$, where t_0^* – the time of the flight, $\dot{x}_0^*$ and $\dot{y}_0^*$ – the projections of the particle velocity at the time of separation $t = t_0^*$.

Let's find the velocity projection. It should be noted that the time of the beginning of the flight t_0^* does not coincide with the boundary of intervals t_0, because the separation of the particle from the plane can occur immediately after its fall on the plane, such is the case with an elastic bump. For our case, at an absolutely inelastic bump, the moment of particle bumping t_b – can belong to both interval *I* and interval *II*. And therefore moments of time t_0 and t_0^* may be different. The initial conditions are used to determine the velocity of the particle in the flight, i.e. at the moment of separation t_0^* – of the velocity are equal $\dot{x}_0^*$ and $\dot{y}_0^*$.

$$\left.\begin{aligned}\dot{x} &= -A\omega\cos\beta\,(\cos\omega t - \cos\omega t_0^*) + \dot{x}_0^* - (g\sin\alpha + K_y)(t - t_0^*)\\ \dot{y} &= -A\omega\sin\beta\,(\cos\omega t - \cos\omega t_0^*) + \dot{y}_0^* - (g\sin\alpha - K_y)(t - t_0^*)\end{aligned}\right\} \quad (13)$$

By integrating the equation (13), we will find the trajectory of the particle during the flight in the movable coordinate system, taking into account the initial conditions. In this case, at the time t_0^* the projection of the movement $x(t_0^*)$ and $y(t_0^*)$ the particles are equal to zero.

The equation of the particle motion trajectory takes the following form:

$$\left.\begin{aligned}x = A\omega\cos\beta\,(\mathrm{t} - t^*)\cos\omega t^* + \dot{x}_0^*(t - t_0^*) -\\ -\mathrm{A}\cos\beta(\sin\omega t - \sin\omega t_0^*) -\\ -\frac{(t - t_0^*)^2}{2}(g\sin\alpha + K_y),\\ y = A\omega\sin\beta\,(\mathrm{t} - t^*)\cos\omega t^* + \dot{y}_0^*(t - t_0^*) -\\ -\mathrm{A}\sin\beta\,(\sin\omega t - \sin\omega t_0^*) -\\ -\frac{(t - t_0^*)^2}{2}(g\sin\alpha - K_y)\end{aligned}\right\} \quad (14)$$

The motion equation (14) includes the velocity of the airflow V_x and V_y. Consequently, the nature of the particle movement will depend on both the magnitude and the direction of the aerodynamic forces associated with them. At sufficiently large vertical velocities the particle can break away from a vibrating plane and is carried out by an air stream, and the horizontal component carries out a particle to the left. It is especially characteristic for light particles.

To analyze the particle movement in the vibrating tilt conveyor with additional forces F_x and F_y to determine the average velocity of the displacement, it is necessary to determine the phase angle of the bump ωt_b.

The equation (14) defines movement only until the particle falls on the plane. The moment of bumping t_b is the closest to t_0^* the root of the equation.

$$y(t_b) = A\omega\sin\beta\,(t_b - t_0)\cos\omega t_0^* + \dot{y}_0^*(t_b - t_0^*) -$$
$$-\frac{(t_b - t_0^*)^2}{2}(g\sin\alpha - K_y) -$$
$$-\mathrm{A}\sin\beta\,(\sin\omega t_b - \sin\omega t_0^*) = 0$$

Denote accordingly $\omega t_0^* = \delta_0^*$, $\omega t_b = \phi_b$, $b_0^* = \frac{\dot{y}_0^*}{A\omega\sin\beta}$, and $\Delta_\phi = (\phi_b - \delta_0^*)$ then the last equation takes the form

$$(\sin\phi_b - \sin\delta_0^*) - (\cos\delta_0^* - b_0^*)\Delta_\phi + z_0\frac{\Delta_\phi}{2} = 0 \quad (15)$$

We assume that as a result of bump the transverse component of velocity $\dot{y}$ is drawn to zero. It is possible to assume, that at movement of small particles in an air environment there is a condition which makes a blow absolutely inelastic with a coefficient of recovery $R = 0$.

If there is no slippage at the moment of bump, we will change the longitudinal component of velocity $x = 1$, $\dot{x}_b = 0$. Given the nature of the particle hitting the vibrating chute, the velocity of the particle at the time of separation will be equal to the velocity of the vibrating chute, and the relative velocities in the movable system xOy are equal to zero. In this case, the equation will look like this:

$$(\sin\phi_b - \sin\delta_0^*) - \cos\delta_0^*\,(\phi_b - \delta_0^*) + z_0\frac{\phi_b - \delta_0^*}{2} = 0$$

The peculiarity of the obtained equation is that it is necessary to take into account the resistance forces included in the expression z_0.

To calculate the movement of the particle S_b during the flight, use the formulas (14) and (15). Suppose that the blow is absolutely inelastic R=0 and there is no slippage at the moment of impact λ =1. As a result, we get the formula

$$S_b = \frac{1}{2\omega^2}\left[g\frac{\cos(\alpha+\beta)}{\sin\alpha} - K_x - \frac{K_y}{tg\beta}\right](\phi_b - \delta_0^*)^2$$

IV. Conclusion

A mathematical model of the motion of a material point along a vibrating inclined surface is formed. When creating the model, it is taken into account that the movement of the material point along the oscillating inclined surface is formed under the action of a system of forces taking into account the forces of inertia, gravity and aerodynamic forces.

The developed mathematical model allows us to investigate the nature of the mode of motion, the transition from one stage of movement to another, taking into account the action of an additional force field. The studies have shown that the movement of particles in the tilt conveyor depends both on the forces of resistance, the frequency of oscillations of the plane, the angle of inclination of the plane and the angle of vibration.

Depending on the type of swinging conveyor (vibratory or inertial) the described algorithm of motion simulation allows to determine the optimal parameters of the system. The calculation of the particle motion process allows to determine the design parameters of the conveyor, which have a major impact on the amount of movement of the particle on the work surface.

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