Novel Schönflies Motion Parallel Robot Driven by Differential Mechanism

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***Abstract*—Novel four-dof (three-translation and one-rotation) gantry type parallel robot driven by a differential belt and a differential screw is proposed in this paper. Single side timing belt was installed in the conventional research, which restricts arrangements of pulleys on the robot. The restriction of the arrangement of pulleys is eliminated by the double-sided timing belt. In order to eliminate the design limitation by using off-the-shelf spline-screw, our novel differential cylindrical drive by Gaudi-screw is installed in the moving part. Workspace of the robot is limited because no longer looped timing belt is off-the-shelf. Novel belt arrangement is proposed for enlarging the workspace by multiple looped timing belts. Linux CNC was customized for the first prototype.**

***Index Terms*—Parallel robot, Differential belt drive, Differential screw, Linux CNC**

I. Introduction

Two-dof planar translational motion generator by differential belt drive have been proposed such as H-Frame [1], H-Bot [2] and core XY [3]. These mechanisms are collectively called 'H-gantry' in this paper. Moving part is driven by two motors fixed on the base. The H-gantry is widely applied for XY-plotter and XY cutting machine. Recently, novel four-dof Schönflies Motion Generator (SMG), translations along x, y and z-axis and rotation around the z-axis, proposed by a combination of two H-gantry and screw-spline [4]. This mechanism is called Double-H SMG (Double H-Gantry Schönflies Motion Generator) in this paper. Double-H SMG is categorized into the parallel robot [5] because motors which are the heaviest mechanical element of robots fixed on the base. This characteristic enables high-speed motion of the moving part like as DELTA [6], the fastest SMG industrial robot for pick-and-place tasks. Double-H SMG is categorized into the Cartesian coordinate's robot. Kinematic and dynamics of Double-H SMG are simple comparing with DELTA of parallel link robot.

Single side timing belt was installed in the conventional H-gantry [1]-[3] as well as Double-H SMG [4]. However, arrangements of pulleys on the robot are restricted by the teeth side of the timing belt. In this paper, the restriction of the arrangement of pulleys is eliminated by the double-sided timing belt. Commercialized but specification limited spline-screw [7] [8] was installed in the moving part of conventional Double-H SMG. In order to eliminate the design limitation by using off-the-shelf spline-screw, our novel differential cylindrical drive by "Gaudi-screw" [9] [10] is installed in the moving part. In general, the looped timing belt is used for Double-H SMG [4] for the unlimited rotation around the z-axis. Workspace of the Double-H SMG is limited because no longer looped timing belt is off-the-shelf. In this paper, novel belt arrangement for Double-H SMG is proposed for enlarging the workspace by multiple looped timing belts. Linux CNC was customized for the first prototype.

II. H-Gantry

Fig. 1 illustrates H-Gantry, two-dof planar translational motion generator by a differential belt drive. Moving part moves along *xy* direction on the sliders S_1S_2 of the *x*-direction and the sliders S_3S_4 of the *y*-direction which is stacked on the slider S_1S_2. H-Gantry is categorized into '2T' mechanism in which the moving part generates two translational motions. The moving part is driven by a timing belt with two motors M_1 and M_2 via the fixed pulleys P_1-P_4 and moving pulleys P_5-P_8. When the two motors rotate with the same amount of angle and same direction, the moving part generates pure translational motion along the *y*-direction. When the two motors rotate with the same amount of angle but opposite direction, the moving part generates pure translational motion along the *x*-direction. Inverse displacement equation from the position of the moving part *x* and *y* to the angles of the two motors φ_1 and φ_2 is given as

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \frac{1}{r_m} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \quad (1)$$

In Eq. (1), r_m represents the common pitch radius of

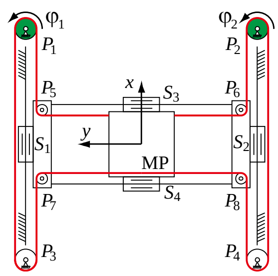

Figure 1. H-Gantry mechanism.

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the pulleys P_1 and P_2 which are fixed to the rotational axis of the motor. By inverting Eq. (1), forward displacement equation from the angle of the two motors to the position of the moving part is derived as

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{r_m}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} \tag{2}$$

III. DOUBLE-H SMG

A. Modified H-Ganty

Double-H SMG, four-dof Schönflies Motion Generator translations along x, y and z-axis, and rotation around the z-axis, is comprised of two *modified H-Gantry* and differential screw drive. Instead of fixing the end of the belt on the moving part as conventional H-Gantry, the looped-belt is winded around the newly located pulley P_v of pitch radius r_v on the moving part as shown in Fig. 2 (a). Inverse displacement equation from the position x, y and angle γ_1 of the pulley on the moving part to the angles of two motors φ_1 and φ_2 is given as

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \end{bmatrix} = \frac{1}{r_m} \begin{bmatrix} 1 & 1 & -r_v \\ -1 & 1 & -r_v \end{bmatrix} \begin{bmatrix} x \\ y \\ \gamma_1 \end{bmatrix} \tag{3}$$

Fig. 2 (b) represents the same mechanism of Fig. 2 (a) with its base is rotated 90 degrees around the z-axis. Inverse displacement equation from the position x, y and angle γ_2 of the pulley on the moving part to the angles of two motors φ_3, φ_4 is given as

$$\begin{bmatrix} \varphi_3 \\ \varphi_4 \end{bmatrix} = \frac{1}{r_m} \begin{bmatrix} -1 & 1 & -r_v \\ -1 & -1 & -r_v \end{bmatrix} \begin{bmatrix} x \\ y \\ \gamma_2 \end{bmatrix} \tag{4}$$

B. Double-H 2T2R mechanism

As shown in Fig. 3, when the two modified H-Gantries are stacked as each pulley on the moving part shares the same rotational axis, the mechanism generates two-translational motions (x, y) and individual two-rotational (2T2R) motions (γ_1, γ_2) along the z-axis. In Fig. 3, the moving part of each modified H-Gantry is combined to one, and each pulley is located above and below the combined moving part. Inverse displacement equation from the position and angles of the pulleys on the moving part to the angles of four-motors φ_1- φ_4 is given as

$$\begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{bmatrix} = \frac{1}{r_m} \begin{bmatrix} 1 & 1 & -r_v & 0 \\ -1 & 1 & -r_v & 0 \\ -1 & 1 & 0 & -r_v \\ -1 & -1 & 0 & -r_v \end{bmatrix} \begin{bmatrix} x \\ y \\ \gamma_1 \\ \gamma_2 \end{bmatrix} \tag{5}$$

By inverting Eq. (5), forward displacement equation from the angle of the four motors to the position and angles of the pulleys on the moving part is derived as

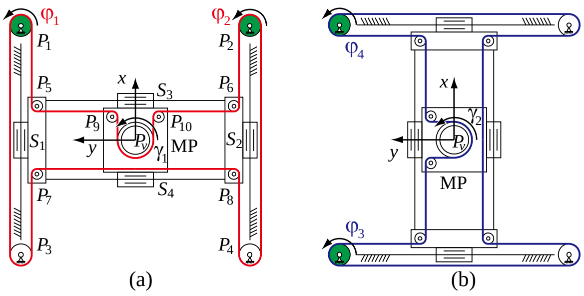


Figure.2 modified G-Gantry

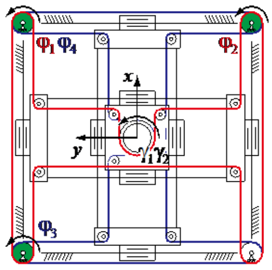


Figure. 3 Double-H 2T2R mechanism

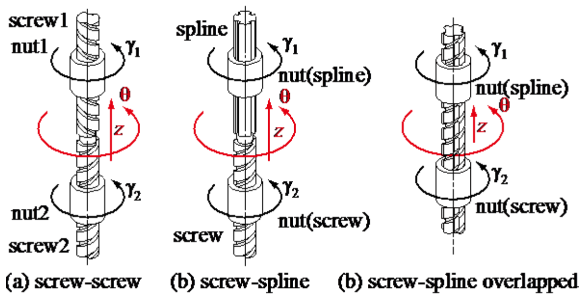


Figure. 4 Differential screw drive mechanisms

$$\begin{bmatrix} x \\ y \\ \gamma_1 \\ \gamma_2 \end{bmatrix} = \frac{r_m}{2} \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ -1/r_v & -1/r_v & 1/r_v & -1/r_v \\ -1/r_v & 1/r_v & -1/r_v & -1/r_v \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \varphi_3 \\ \varphi_4 \end{bmatrix} \tag{6}$$

C. Differential Screw Drive

Differential screw drive shown in Fig. 4 (a) installed in the moving part for converting from two angles γ_1, γ_2 of nuts to cylindrical displacement (one-translation and one-rotation) z, θ, thus the Double-H 2T2R mechanism is upgraded to Double-H SMG (3T1R) as shown in Fig. 5. Each end of the screw with differential lead l_1 and l_2 is connected to each other. Inverse displacement equation from z, θ of screw-set to the angle of two nuts γ_1, γ_2 is given as

$$\begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} = \begin{bmatrix} -2\pi/l_1 & 1 \\ -2\pi/l_2 & 1 \end{bmatrix} \begin{bmatrix} z \\ \theta \end{bmatrix} \tag{7}$$

By inverting Eq. (7), forward displacement equation from the angle of the nuts to the displacements of the screw-set is derived as

$$\begin{bmatrix} z \\ \theta \end{bmatrix} = \frac{1}{l_1 - l_2} \begin{bmatrix} \frac{l_1 l_2}{2\pi} & -\frac{l_1 l_2}{2\pi} \\ l_1 & l_2 \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \end{bmatrix} \tag{8}$$

When the two screws share an identical lead ($l_1 = l_2$), or one of the leads be zero, the matrix in Eq. (8) becomes singular and the differential screw drive mechanism does not work.

D. *Double-H SMG (3T1R)*

Fig. 4 represents the Double-H SMG, 3T1R motion generator of three-translations along x, y, and z-axis and one-rotation around the z-axis. Origin of the reference coordinate frame of the end-effector is set at the central position of the screw-set. Position and angle around the z-axis of the reference coordinate are collected into vector $\boldsymbol{x}$, and angles of the four motors are collected into vector $\boldsymbol{q}$. The inverse displacement equation from $\boldsymbol{x}$ to $\boldsymbol{q}$ is given by Eq. (5) and (7) as

$$\begin{aligned} \boldsymbol{q} &= \boldsymbol{A}^{-1}\boldsymbol{x} \\ \boldsymbol{q} &= \begin{bmatrix} \varphi_1 & \varphi_2 & \varphi_3 & \varphi_4 \end{bmatrix}^T \\ \boldsymbol{x} &= \begin{bmatrix} x & y & z & \theta \end{bmatrix}^T \\ \boldsymbol{A}^{-1} &= \frac{1}{r_m} \begin{bmatrix} 1 & 1 & (2\pi r_v)/l_1 & -r_v \\ -1 & 1 & (2\pi r_v)/l_1 & -r_v \\ -1 & 1 & (2\pi r_v)/l_2 & -r_v \\ -1 & -1 & (2\pi r_v)/l_2 & -r_v \end{bmatrix} \end{aligned} \tag{9}$$

By inverting Eq. (9), forward displacement equation from $\boldsymbol{q}$ to $\boldsymbol{x}$ is derived as

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{q}$$

$$\boldsymbol{A} = \begin{bmatrix} \frac{r_m}{2} & -\frac{r_m}{2} & 0 & 0 \\ 0 & 0 & \frac{r_m}{2} & -\frac{r_m}{2} \\ 0 & -\frac{l_1 l_2 r_m}{2\pi r_v (l_1 - l_2)} & \frac{l_1 l_2 r_m}{2\pi r_v (l_1 - l_2)} & 0 \\ -\frac{r_m}{2r_v} & -\frac{(l_1 + l_2) r_m}{2(l_1 - l_2) r_v} & \frac{(l_1 + l_2) r_m}{2(l_1 - l_2) r_v} & -\frac{r_m}{2r_v} \end{bmatrix} \tag{10}$$

IV. Beyond the Design Limitations

A. *The Arrangement of the Moving Part*

Single-side timing belt was installed in the conventional Double-H SMG [4]. However, single-side timing belt restricts the arrangements of pulleys on the robot. As shown in Fig. 6 (a), moving part is located outside the slider when using the single-side timing belt. The limitation is eliminated by using double-side timing belt as sown in Fig. 6 (b). The moving part can be located inside the slider, thus the load applied to the MP can be effectively supported by the slider.

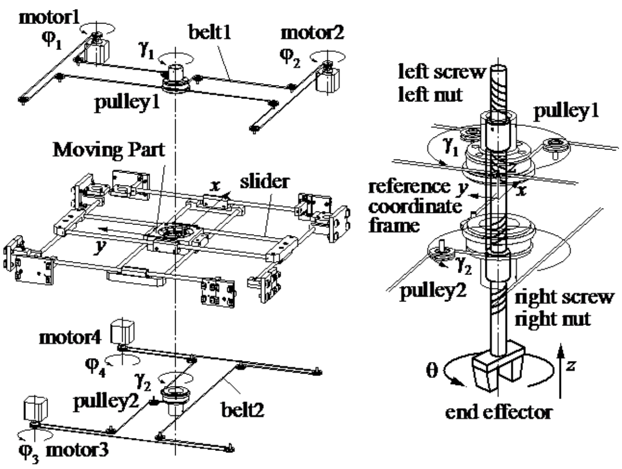
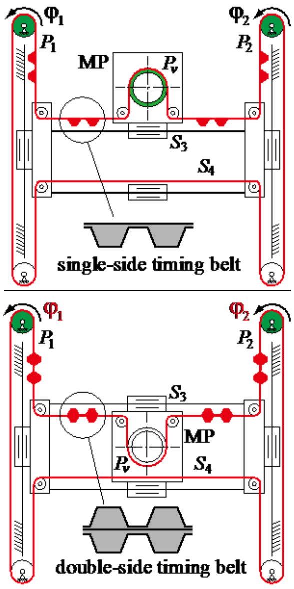


Figure. 5 Double-H SMG (3T1R)



(a) (b)

Figure. 6 Single-side and double-side timing belt drive

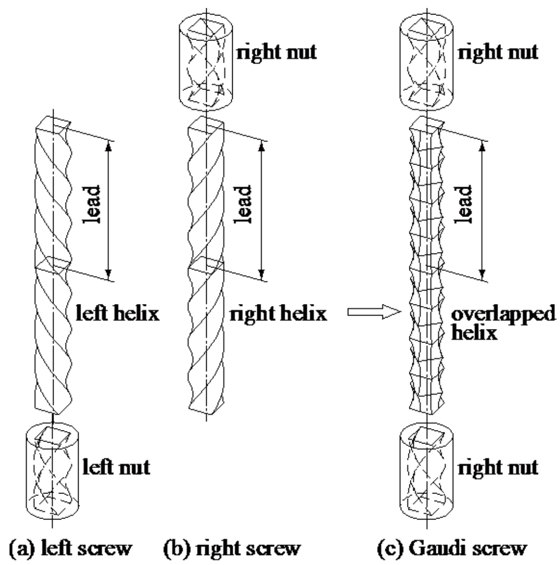


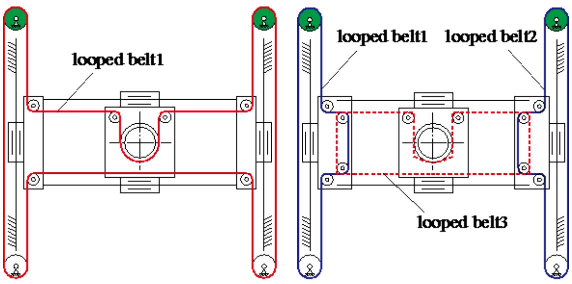
Figure. 7 Gaudi-screw

B. *Novel Differential Screw Mechanism*

Commercialized but specification limited spline-screw shown in Figs. (b) and (c) were installed in the moving part of conventional Double-H SMG. In order to eliminate the design limitation by using off-the-shelf spline-screw, our novel differential screw drive by "Gaudi-screw" [9] [10] shown in Fig. 7 (c) is installed in the moving part. Arbitral specification of the screw such as lead, total length can be designed by using the Gaudi-screw.

C. *Multiple Belt Drive*

Workspace of the Double-H SMG is limited because no longer looped timing belt is off-the-shelf. A novel belt arrangement for Double-H SMG is proposed for enlarging the workspace by multiple looped timing belts as shown in Fig. 8 (b).



(a) single looped-belt (b) multi looped-belt

Figure. 8 Expand the workspace by multi looped-belt

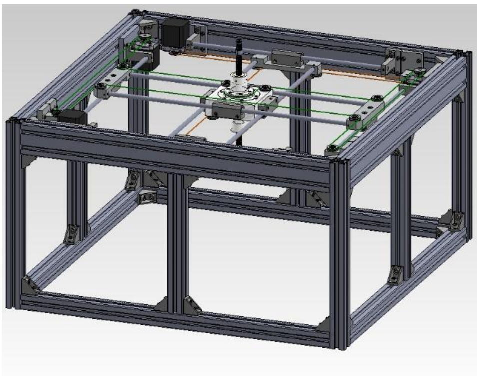


Figure. 9 3D design of double-H SMG

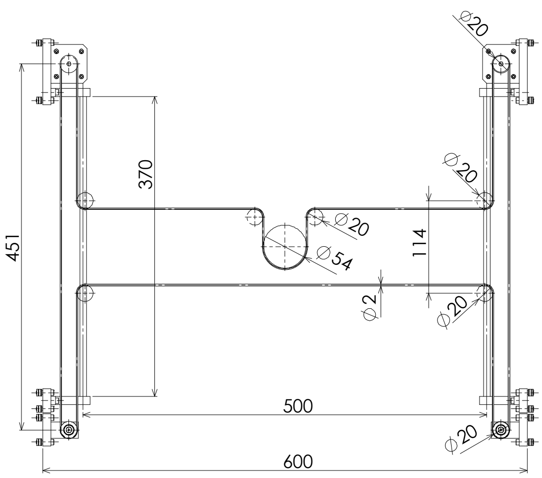


Figure. 10 Dimensions of the belt drive and pulleys

V. PROTOTYPE

A. *Design of the First Prototype*

The first prototype of one looped belt is designed as shown in Fig. 9. Dimensions of the prototype are shown in Fig. 10. Gaudi-screw of 50 mm lead and 8mm × 8mm square of the cross-section is embedded in the moving part. Figs. 11 (a) and (b) represents the entire picture of the prototype and the close-up image around the Gaudi-screw inside the moving part, respectively.

Double-side looped timing belt 2790-D3GT-6, 2790 mm of total length, 3 mm of the pitch of the tooth and 6 mm of the width of the belt, by Gates Unitta Asia Company is installed in the prototype. The belt is the longest in the series of 3 mm pitch, the workspace of the prototype is limited by the length of the looped belt. Indeed, there exist up to 3850 mm long looped-belt but has a long pitch (14 mm) and wide width (40 mm), which brings less positional resolution and larger size of the pulley. The proposed design of the multiple looped-belt will expand the workspace of the Double-H SMG.

B. *CNC Controller by Linux CNC*

NEMA 17 stepper motors are installed in the first prototype. Linux CNC [11] is customized for the first prototype. Forward and inverse displacement equations of the first prototype are given as

$$
\begin{aligned}
&\boldsymbol{q} = \boldsymbol{A}^{-1}\boldsymbol{x}\\
&\boldsymbol{x} = \boldsymbol{A}\boldsymbol{q}\\
&\boldsymbol{A}^{-1} = \frac{1}{r_m}\begin{bmatrix} 1 & 1 & (2\pi r_v)/l & -r_v \\ -1 & 1 & (2\pi r_v)/l & -r_v \\ -1 & 1 & -(2\pi r_v)/l & -r_v \\ -1 & -1 & -(2\pi r_v)/l & -r_v \end{bmatrix}\\
&\boldsymbol{A} = \frac{r_m}{2}\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & l/(2\pi r_v) & -l/(2\pi r_v) & 0 \\ -1/r_v & 0 & 0 & -1/r_v \end{bmatrix}
\end{aligned}
\tag{11}
$$

In Eq. (11), $l > 0$ represents the pitch of the right helix ($l_1 = 1$) and the left helix ($l_2 = -1$).

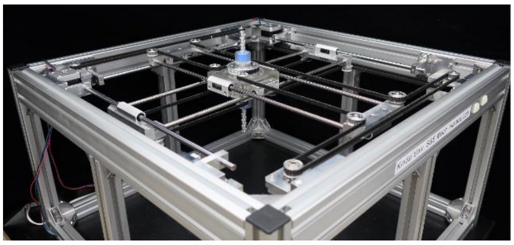
HAL (Hardware Abstraction Layer) component functions 'int kinematicsForward()' and 'int kinematicsInverse()' were installed into Linux CNC using Eq. (11). Machine simulation with Vismach [12] was installed into Linux CNC. Screen shots after customizing Linux CNC for the prototype of Double-H SMG is shown in Fig. 11.

VI. CONCLUSIONS

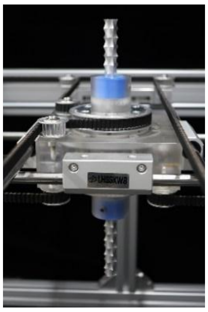
Novel Schönflies motion parallel robot driven by a differential belt and a differential screw was proposed.

- The restriction of the arrangement of pulleys is eliminated by the double-sided timing belt.

- Novel differential cylindrical drive by Gaudi-screw is installed in the moving part.
- Novel belt arrangement is proposed for enlarging the workspace by multiple looped timing belts.
- Linux CNC was customized for the first prototype.

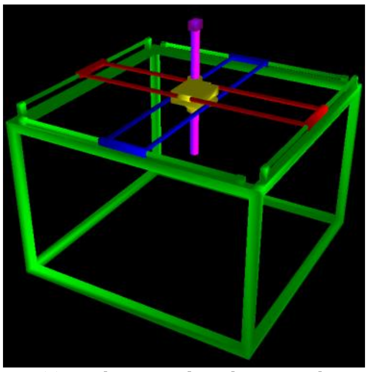


(a) entire image

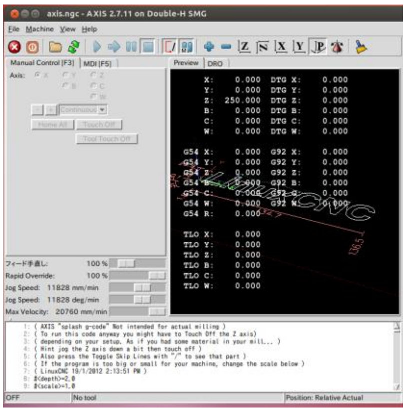


(b) closed-up image around the Gaudi-screw inside the moving part

Figure. 11 First prototype of Double-H SMG



(a) machine simulator by vismach



(b) Linux CNC

Figure. 12 Screen shots of Linux CNC

ACKNOWLEDGMENT

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