Anti-disturbance Control for Unmanned Aerial Vehicles with NN Modeling

Bei Liu and Yang Yi
Yangzhou University, Yangzhou, China
Email: 1174622627@qq.com, yiyangcontrol@163.com

Abstract—This paper presents a novel anti-disturbance tracking control problem for Unmanned Aerial Vehicles (UAVs) systems. Different from the general external disturbances, this paper uses the neural network to describe the nonlinear disturbances. On this basis, a disturbance observer is designed to estimate the nonlinear disturbances. By integrating the estimation of disturbance with P-I control algorithm, a composite controller based on convex optimization theory is designed to ensure the UAVs system stability and convergence of the tracking error to zero. Finally, Matlab / Simulink is used to simulate a Unmanned Aerial Vehicles model, which helps to verify the feasibility and effectiveness of this algorithm.

Index Terms—anti-disturbance control, unmanned aerial vehicles, tracking control, neural network model

I. INTRODUCTION

In recent years, Unmanned Aerial Vehicles have received more and more attention because they have a lot of superior performance, such as: lowcost and high efficiency, strong security, autonomous hovering, convenience and flexibility. Because of these outstanding advantages, the Unmanned Aerial Vehicles is playing an increasingly important role in industrial production and national life. For example, agricultural monitoring, fire monitoring and rescuing, environmental pollution monitoring [1]-[3]. The Unmanned Aerial Vehicles, as a specific kind of aircraft, can produce unpredictable conditions during flight and even cause economic and property losses. Therefore, the requirements for the flying quality of UAVs are getting higher and higher. There is a need for some high-performance control algorithms to address the high performance requirements. New flight control algorithms are also emerging, including robust control, feedback linearization control, sliding mode variable structure control and so on.

As is well-known to all, the exogenous disturbance widely exist in almost all controlled systems, such as: motion control systems [4], missile systems [5], robotmanipulators [6] as well as flight control systems. Thus, the research of anti-disturbance control problem has attracted considerable attention of both academia and engineers. Many elegant approaches have been presented to resolve disturbance attenuation and rejection problems, such as: $H_{\infty}$ control [7], adaptive dynamical compensation [8], output regulator theory [9], sliding mode control [10] and disturbance-observer-control (DOBC) [11]-[14] theory. DOBC theory was studied in the second half of 1980s which have a simple structure and is easily applied. The essence of thought is that the disturbances can be estimated by disturbance-observer and compensated in the feed-forward channel immediately. However, in the most of DOBC results, the exogenous disturbances are assumed to be generated by a linear exogenous system. The DOBC theory is invalid when facing with some nonlinear disturbances.

All along, neural networks (NNs) are very powerful tools to approximate highly nonlinear and dynamic systems and has been of interest of many researchers during the past three decades [15]. Dynamic neural networks (DNNs), as a black-box identifier, have some memory ability and track nonlinear dynamics, which makes them much suited for modelling those irregular dynamics [16], [17].

Motivated by the above observation, this paper discusses the anti-disturbance control problem for Unmanned Aerial Vehicles systems by using neural network modelling. On this basis, a disturbance observer is designed to estimate the nonlinear disturbances. By integrating the estimation of disturbance with P-I control algorithm, a composite controller based on convex optimization theory is designed to ensure the UAV systems stability and convergence of the tracking error to zero.

In this paper, if not stated, matrices are assumed to have compatible dimensions. The identity and zero matrices are expressed by $I$ and $0$. The symbol $\text{sym}$ is defined as $\text{sym}(M) = M^T + M$.

II. DESCRIPTION OF UAV SYSTEMS

A. Simulation Models

The longitudinal equations of Unmanned Aerial Vehicles is considered here. The model usually is consists of four states and its dynamics can be expressed as follows
\[ V = \frac{F_i \cos \alpha - X + mg (-\cos \alpha \sin \theta + \sin \alpha \cos \theta)}{m} \]
\[ \alpha = \frac{q + (F_i \sin \alpha + mg \sin \alpha \sin \theta + \cos \alpha \cos \theta)}{mV} \]
\[ q = \frac{M \dot{I}_n}{I_n} \]
\[ \dot{\theta} = q \]

where \( V \) is airspeed, \( \alpha \) is angle of attack, \( q \) is pitch rate, \( \theta \) is pitch angle. Besides, \( m \) is quality of UAVs, \( X \) is aerodynamic force on the UAVs, \( g \) is the gravity, \( M \) is outside the moment, \( I_n \) is the moment of inertia. Besides, \( F_i \) is the engine thrust and
\[
\begin{align*}
F_i &= \rho n^2 D^4 C_{F_1}(J) \\
C_{F_1} &= C_{F_1} + C_{F_2} J + C_{F_3} J^2 \\
J &= V / (D \pi n)
\end{align*}
\]

where \( \rho \) is air density, \( n \) is engine speed, \( D \) is diameter of propeller, \( C_{F_1}(J) \) is dimensional coefficient of thrust, \( J \) is ratio of thrust coefficient.

### B. Problem State

Since the mathematical model of UAVs systems have nonlinear characteristics, it is difficult to design a suitable controller and it needs to be linearized. Selecting the UAV system, we can get
\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) \\
y(t) &= Cx(t) + Du(t)
\end{align*}
\]

where \( x(t) = [V, \alpha, q, \theta]^T \) as the states of the UAVs system, \( u(t) = \delta \) as the controlled input, \( y(t) = V \) as the output. \( A, B, C, D \) are the coefficient matrices with appropriate dimensions and given as follows
\[
A = \begin{bmatrix}
-0.0088 & -0.0105 & 0 & -0.0409 \\
-0.0915 & -0.4917 & 1 & 0 \\
-0.0294 & -2.5464 & -0.8966 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]
\[
B = \begin{bmatrix}
0 \\
-0.1011 \\
-7.7307 \\
0
\end{bmatrix}
\]
\[
C = \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}, \quad D = 0
\]

Consider the UAVs nonlinear system with different exogenous disturbances
\[
\begin{align*}
\dot{x}(t) &= Ax(t) + Bu(t) + d(t) + \dot{d}_1(t) \\
y(t) &= Cx(t) + Du(t) + d(t) \\
y_1(t) &= C_1 x(t) + D_1 (u(t) + d(t))
\end{align*}
\]

where \( y_1(t) \) is the measurement output, \( d(t) \) and \( \dot{d}_1(t) \) are two different types of disturbances. On the one hand, \( d(t) \) is supposed to be generated by an exogenous neural network model with adjustable expressed by
\[
\begin{align*}
\dot{z}(t) &= M \dot{\theta} + \Psi (\dot{\theta}) \\
\dot{d}(t) &= N \dot{\theta}
\end{align*}
\]

where \( z(t) \) is the state of the neural network model. \( M \) and \( N \) are known coefficient matrices. \( \Psi \) represents the optimal model parameter and \( \sigma(z(t)) \) is the designed known basis function of neural network.

On the other hand, \( \dot{d}_1(t) \) is assumed to be norm bounded and \( \| \dot{d}_1(t) \| \leq 1 \) is assumed to satisfy in order to simply the analysis process.

### III. Design of Composite Controller

To guarantee the tracking performance of the UAVs system, we introduce a new state variable
\[
\tilde{x}(t) := [x^T, t \int e^T(t) d\tau]^T
\]

where the tracking error \( e(t) \) is defined as \( e(t) = y(t) - y_d \), \( y_d \) is the reference output. Then the augmented system can be established as
\[
\begin{align*}
\dot{x}(t) &= \tilde{A}x(t) + \tilde{B}(u(t) + d(t)) + \tilde{B}_d \dot{d}_1(t) \\
y(t) &= \tilde{C}_d x(t) + \tilde{D}_d (u(t) + d(t)) \\
y_1(t) &= \tilde{C}_d x(t) + \tilde{D}_d \dot{d}_1(t)
\end{align*}
\]

where
\[
\tilde{A} = \begin{bmatrix}
A & 0 \\
C & 0
\end{bmatrix}, \quad \tilde{B} = \begin{bmatrix}
B \\
0
\end{bmatrix}, \quad \tilde{B}_d = \begin{bmatrix}
B_i \\
0
\end{bmatrix}, \quad \tilde{D} = D, \quad \tilde{D}_d = D_i
\]

To realize the dynamical tracking for system output, the PI-type control input can be constructed as
\[
u_{\dot{d}}(t) = k_x x(t) + K_{i} \int_0^t e(\tau) d\tau
\]

where matrices \( K_x \) and \( K_i \) are controller gains to be determined later on.

In order to estimate the model disturbance \( d(t) \), a nonlinear disturbance observer based on neural network is formulated as
\[
\begin{align*}
\dot{\hat{d}}(t) &= N \hat{\theta} \\
\dot{z}(t) &= v(t) - L \tilde{x}(t) \\
v(t) &= (M + L \hat{\theta}) \left[ y(t) - L \tilde{x}(t) \right] - L \tilde{\theta}(t) \\
&\quad - \tilde{B} (u(t) - \tilde{C}_d y_1(t)) + \tilde{W}(t) \sigma(z(t))
\end{align*}
\]

where \( \hat{d}(t) \) and \( \dot{z}(t) \) are the estimates of \( d(t) \) and \( \dot{z}(t) \), respectively. \( L \) is the observer gain to be determined later. \( v(t) \) is the designed auxiliary variable.
Define $e_i(t) = z(t) - \hat{z}(t)$. Comparing (5) and (9) yields

$$\dot{e}_i(t) = (M + L\hat{B}N)e_i(t) + \hat{W}\sigma(\hat{z}(t)) + W^T(\sigma(z(t)) - \sigma(\hat{z}(t)))$$

(10)

where $\sigma(*)$ is a sigmoid function and satisfies

$$(\sigma(z(t)) - \sigma(\hat{z}(t)))^T(\sigma(z(t)) - \sigma(\hat{z}(t))) \leq e_i^T(t)U_z e_i(t)$$

(11)

By combining the estimates of disturbance with the PI-type control input, the composite controller is inferred as follows

$$u(t) = \hat{d}(t) + Kx(t) = \begin{bmatrix} K_p & K_i \end{bmatrix}$$

(12)

Substituting the composite controller (12) into the augmented system (7), the closed-loop system can be expressed by

$$\dot{\hat{x}}(t) = (\hat{A} + \hat{B}K)x(t) + \hat{B}Ne_i(t) + \hat{B}_y y_d + \hat{B}_d d(t)$$

(13)

Furthermore, integrating the disturbance estimation error model (10) with the closed-loop system (13), we can get

$$\begin{bmatrix} \dot{\tilde{x}}(t) \\
\dot{\tilde{e}}(t) \\
\dot{y}_d \\
\dot{y}_d \\
\dot{W}(t) \end{bmatrix} = 
\begin{bmatrix}
\tilde{A} & \tilde{B}N & \tilde{B} & 0 \\
0 & M + L\hat{B}N & 0 & 0 \\
0 & 0 & 0 & I \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
\end{bmatrix} 
\begin{bmatrix} \tilde{x}(t) \\
\tilde{e}(t) \\
y_d \\
y_d \\
W(t) \end{bmatrix}$$

(14)

IV. THEOREM PROOF

**Theorem:** For the augmented system (14) and the known parameters $\mu_i > 0, i = 1, 2$, if there exist matrices $Q_i = P_i^{-1} > 0$ and $R_i$ satisfying

$$\begin{bmatrix} \text{sym}(\bar{Q} + \bar{R}_i) & \bar{B}_i & \bar{B}_i \\
* & -\mu_i^2 & 0 \\
* & * & -\mu_i^2 I \end{bmatrix} < 0$$

(15)

and $P_2 > 0$, and $R_2$ satisfying

$$\begin{bmatrix} \text{sym}(P_2M + L\bar{B}N) + U_z^T U_z & P_2 \\
* & -\bar{W}^{-1} \end{bmatrix} < 0$$

(16)

Further, the adjustable parameter $W(t)$ is designed as follows

$$\dot{\tilde{W}} = \eta P_2 e_i^T(t)\sigma(\hat{z}(t))$$

(17)

where $\eta$ is a designed positive constant. Then the augmented system (14) under the composite controller (12) is stable and the tracking error satisfies

$$\lim_{t \to \infty} y_d(t) = y_d$$

The gains are given by $K = R_iQ_i^{-1}$ and $L = P_i R_i$.

**Proof:** Select Lyapunov functions as

$$\Phi_1(\tilde{x}(t), t) = \tilde{x}(t)^T P_1 \tilde{x}(t)$$

(18)

and

$$\Phi_2(\langle e_i(t), t \rangle) = e_i(t)^T P_2 e_i(t) + tr\left[\hat{W}\eta^2\tilde{W}\right]$$

(19)

Based on (10), (11) and (16), one has

$$\Phi(e_i(t), t) \leq e_i^T(t)\left(\text{sym}P_2(M + L\hat{B}N)e_i(t) + e_i^T(t)P_2 e_i(t) + e_i^T(t)U_z^T U_z e_i(t)\right)$$

(20)

where $\eta_i$ is a proper constant.

On the other hand, based on the trajectories of (13), we can acquire

$$\Phi_1(\tilde{x}(t), t) = 2\tilde{x}(t)^T P_1 \tilde{x}(t)$$

(21)

$$\Phi(\langle e_i(t), t \rangle) = e_i^T(t)P_2 e_i(t) + tr\left[\hat{W}\eta^2\tilde{W}\right]$$

(22)

Based on Schur complement formula, by pre-multiplying and post-multiplying $\text{diag}\{Q_i^{-1}, I, I\}$ to both sides of (15), we can get

$$\Phi_1(\tilde{x}(t), t) \leq \tilde{x}(t)^T \text{sym}P_2(\tilde{A} + \tilde{B}K + \hat{B}N) \tilde{x}(t) + 2\tilde{x}(t)^T P_1 \hat{B}Ne_i(t)$$

(23)

where $\eta_i > 0$ is a known proper constant.

As a whole, a Lyapunov function for the augmented closed-loop system (14) is chosen as

$$\Phi(\tilde{x}(t), e_i(t), t) = \tilde{x}(t)^T P_1 \tilde{x}(t) + \eta_i e_i^T(t)P_2 e_i(t)$$

(24)

where $\eta_i = (\eta_i^2 + 1)/4\eta_i$. Thus, we can get

$$\Phi(\tilde{x}(t), e_i(t), t) \leq -\eta_0 \tilde{x}(t)^T I + \mu_i^2 y_3^2 + \eta_i \|e_i(t)\|^2$$

(25)

where $\rho(t) = \tilde{x}(t), e_i(t)$.

Obviously,

$$T = \begin{bmatrix} \eta_2 & -\eta_1/2 \\
-\eta_1/2 & (-\eta_1^2 + 1)/4\eta_2 \end{bmatrix}$$

(26)

$T$ is a positive definite matrix, so $\lambda_{\text{min}}(T) > 0$. Thus, $\Phi(\tilde{x}(t), e_i(t), t) < 0$, if inequality

$$\|\rho(t)\|^2 > \lambda_{\text{min}}(\mu_i^2 y_3^2 + \mu_i^2 d_i^2(t))$$

holds. I t is noted for any $\tilde{x}(t)$ and $z(t)$, it can be verified that

$$\rho^T(t) \rho(t) \leq \max\left\{\rho_0^T, \lambda_{\text{min}}(\mu_i^2 y_3^2 + \mu_i^2 d_i^2(t))\right\}$$

(27)
assert from (27) that when $t \to \infty$, the variable $\int_{0}^{t} e(\tau) d\tau$ must be bounded. As a result, we can get the dynamical tracking performance that is $\lim_{t \to \infty} y(t) = y_d$.

V. SIMULATION EXAMPLES

In order to illustrate the effectiveness of the designed algorithm for UAV model, the following longitudinal UAV is implemented by using the Matlab simulation software.

In the following, two different kinds of nonlinear kinds of nonlinear exogenous disturbance are described by Neural Network. Furthermore, multi-objective control requirements, including stability, tracking performance, disturbance observation and disturbance rejection.

Case 1: The nonlinear disturbance can be described as

\[
M = \begin{bmatrix} 0 & -4 \\ -4 & 0.3 \end{bmatrix}, \quad N = \begin{bmatrix} 0.5 \\ -0.1 \end{bmatrix}, \quad W^* = \begin{bmatrix} -0.3 & -0.05 \\ 0.4 & 0.45 \end{bmatrix}
\]

\[
\sigma(t) = \begin{bmatrix} \tanh(t) \\ \tanh(t) \end{bmatrix}
\]

Meanwhile, by defining the parameters $\mu_1 = \mu_2 = 1$ and solving LMIs (15), (16), the control gains $K_p = \begin{bmatrix} -0.9754 & -0.2190 & 0.0870 & 0.2304 \end{bmatrix}$,

$K_i = -0.1086$, $L = \begin{bmatrix} 0 & -0.0396 & 0.2010 & 0 & 0 \\ 0 & 0.0811 & -0.0196 & 0 & 0 \end{bmatrix}$

Supposed that the initial values of the state and the disturbance are taken to be $x(0) = [10, 0.3, 0.5, 1]^T$ and $w(0) = [-2, -10]^T$ respectively. The designed tracking objective is selected as $y_d = 10$. Fig. 1 displays the trajectories of harmonic disturbance with attenuation characteristics and its observation value that illustrates the tracking ability of our disturbance observer is satisfactory. When the composite controller is applied, the state responses of UAV systems are shown in Fig. 2. Fig. 3 is the trajectory of system output and the favorable dynamic tracking performance can be embodied.

Figure 1. Disturbance $d_1$ and estimation value.

Figure 2. Responses of UAV system state.

Figure 3. Responses of output.

Case 2: The another type of nonlinear disturbance can be described as

\[
M = \begin{bmatrix} 0 & -5 \\ -1 & 0 \end{bmatrix}, \quad N = \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \quad W^* = \begin{bmatrix} 0 & 0.2 \end{bmatrix}
\]

\[
\sigma(t) = \begin{cases} \frac{2}{e^{3t} + 1} & if \ t > 0 \\ -3, & if \ t \leq 0 \end{cases}
\]

The control gains $K_p$, $K_i$ and observer gain $L$ are found to be

$K_p = \begin{bmatrix} -0.9831 & -0.2207 & 0.0331 & 0.2076 \end{bmatrix}$

$K_i = -0.1283$, $L = \begin{bmatrix} 0 & 0.0190 & 0.1322 & 0 & 0 \\ 0 & -0.0074 & -0.0329 & 0 & 0 \end{bmatrix}$

The situation of the Case 2 is shown in from Fig. 4 to Fig. 6.

Figure 4. Disturbance $d_2$ and estimation value.
In this paper, a novel Neural Network modeling-based anti-disturbance tracking control framework is proposed for UAVs system. Based on the Neural Network disturbance models, the composite controller is designed by combining the disturbance observer with PI-type control input. As a result, two LMI-based convex optimization design schemes are adopted to ensure the augmented closed-loop systems stable and convergence of the tracking error to zero.

VI. CONCLUSION

In this paper, a novel Neural Network modeling-based anti-disturbance tracking control framework is proposed for UAVs system. Based on the Neural Network disturbance models, the composite controller is designed by combining the disturbance observer with PI-type control input. As a result, two LMI-based convex optimization design schemes are adopted to ensure the augmented closed-loop systems stable and convergence of the tracking error to zero.

REFERENCES


Bei Liu received the B.Sc. degree from the College of Information Engineering, Yangzhou University, Yangzhou, China, in 2017. Now, he is a postgraduate student in Yangzhou University. His current research interests include robust control and anti-disturbance control.

Yang Yi received the M.Sc degree from the School of Information Engineering, Yangzhou University, Yangzhou, China, in 2005, and Ph.D. degree from the School of Automation, Southeast University, Nanjing, China, in 2009. Now, he is an associate professor in Yangzhou University. From July 2012 to July 2013, he was a visiting scientist in the School of Computing, Engineering and Mathematics, University of Western Sydney, Australia. His current research interests include stochastic systems, intelligent systems and anti-disturbance control.