Compliance Matrix Based Analysis and Design of Suspension Systems for Chassis Development

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***Abstract*—In the chassis development process, especially for suspension design, simulation has established to reduce both development time and costs. A number of characteristic values are used to characterize and benchmark suspension systems. For front suspension systems, the steering axis plays a vital role. However, two different kinds of steering axes with different meanings exist in literature. This paper presents a methodology for the analysis and design of suspension systems based on the compliance matrix within multi-body simulation. Characteristic values describing both steering feedback and toe behavior are each calculated from the compliance matrix. The characteristic values result from the kinematic and the elastic steering axis. The objective is to provide a comparison of both kinds of steering axes and the resulting characteristic values. The results demonstrate the different meanings of the steering axes and the corresponding characteristic values for suspension characteristics. While the kinematic steering axis defines the lever arms referring to steering feedback, the elastic steering axis is related to the toe behavior. The proposed methodology and the gained insights can be used to improve benchmarking suspension systems and further enhance suspension design.**

***Index Terms*—chassis development, multi-body simulation, suspension analysis, compliance matrix, applied mechanics**

I. Introduction

In the automotive industry, simulation has established as an essential tool in the development process and is gaining more and more importance. The chassis is of particular interest, as it largely defines vehicle dynamics.

A. *Motivation*

The vehicle as well as the chassis development process are based on the V-model, which involves a number of iterations increasing both development time and costs [1]. To avoid these iterations caused by numerous and late modifications in the development process, efforts are being made to gain as much knowledge about the chassis as possible already at the early development stage [2]. Furthermore, the ever-growing competition in the automotive industry has led to shorter development cycles, increasing cost pressure and more model complexity [3].

Therefore, simulation has established in the chassis development process, especially multi-body simulation for the kinematic and compliant design of suspension systems [1], and is becoming more and more important. The benefits include both shorter development cycles and reduced costs, as simulations replace vehicle tests, as well as a comprehensive analysis of the suspension system at an early stage [1]. In simulation based suspension design, characteristic values for particular load cases are extensively used to make a comparison of different suspension systems possible in the early development stage without full vehicle simulations [4]. Characteristic values represent a wide range of customer relevant vehicle characteristics in the fields of safety, ride comfort as well as vehicle handling and relate them to suspension model parameters, such as hardpoints and stiffness characteristics of components [1]. Suspension systems are benchmarked and evaluated based on these characteristic values.

New insights on suspension design as well as more accurate and relevant characteristic values improve the characterization and benchmarking of suspension systems using simulations. Thus, they account for reducing development time and costs by avoiding late modifications after vehicle testing.

B. *Goals*

The approach proposed in this paper improves characterization of suspension systems to optimize their design in the early development stage by means of multi-body simulation. The objective is to develop an approach to determine characteristic values for steering feedback and wheel movement. The characteristic values are then compared to each other and their influence on vehicle behavior is demonstrated.

C. *Structure of the Paper*

The remainder of the paper is structured as follows. Section 2 gives an overview of the state of the art in the field of suspension design. Relevant characteristic values of suspension systems and their meaning for vehicle behavior are described. A methodology for suspension analysis based on the compliance matrix of the suspension system is presented in section 3. The calculation of characteristic values for evaluating steering feedback and toe behavior is outlined as well. Section 4 presents the results of the proposed methodology, which is applied to an exemplary suspension system. The characteristic values for steering feedback and toe behavior are compared to each other and analyzed.

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Section 5 summarizes the results of this work and gives an outlook on further research.

II. Related Work

In the chassis development process, suspension design is done based on characteristic values. They allow an evaluation of suspension characteristics and a comparison of different suspension systems already at an early stage. Numerous characteristic values have been established [1] [4] [5] and much research has been done on studying them. A focus of recent research is on their optimization and on improving the design by taking uncertainties into account [6] [7]. According to [4] [5], the characteristic values can roughly be divided into the values referring to suspension geometry, therefore characterizing wheel movement, and into the values referring to steering geometry, therefore characterizing steering feedback.

The characteristic values referring to steering geometry are determined by the kinematic steering axis (Fig. 1), which is the axis of the wheel rotation during steering [1]. The kinematic steering axis yields lever arms for wheel forces resulting in steering torque [4]:

- Scrub radius and caster trail for longitudinal and lateral forces applied at contact patch height
- Kingpin and caster offset for longitudinal and lateral forces applied at wheel center height
- Wheel load arm for vertical forces

For the most part, the focus is on kinematics and their behavior is studied for wheel travel as well as steering. However, due to increasing suspension compliance they are dependent on the load case as well [1] [5] [8]. Although they are regarded as lever arms referring to steering torque, they are also used for studying vehicle behavior, such as straight-line behavior [9] [10] [11].

The characteristic values describing wheel movement focus on instant centers and stiffness characteristics. However, some studies have already suggested an elastic steering axis, which is the steering axis due to wheel forces and yields corresponding characteristic values as the kinematic steering axis.

Zomotor [12] describes an elastokinematic steering axis, where applied longitudinal and lateral forces do not cause any toe change. Therefore, this is the steering axis of the wheel under loads, which differs from the kinematic steering axis. The elastokinematic steering axis then yields characteristic values, such as the elastokinematic scrub radius and caster trail.

Schultz [13] applies Gerrard's [14] concept of the equivalent elastic system with three elastic axes. One of the elastic axes can be seen as the elastic kingpin axis of the suspension system and refers to the toe motion of the wheel. It yields lever arms for tire forces, such as scrub radius and caster trail.

Lee [15] studies the compliance screw axis of suspension systems, which is the axis of the wheel motion under forces caused by the deformation of the bushings. It yields characteristic values, such as kingpin offset and caster trail, and is related to toe and camber change.

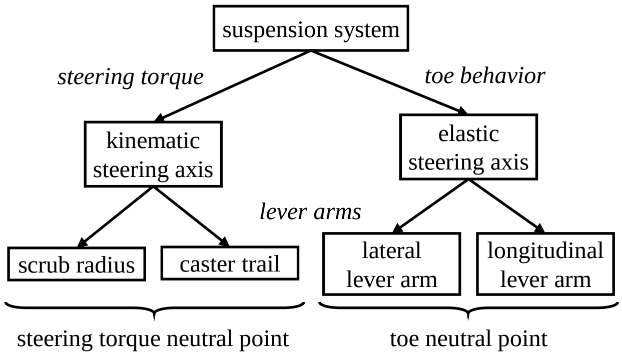

Figure 1. Derived classification of characteristic values resulting from the kinematic and elastic steering axis based on state of the art

Buechner [16] deals with the toe neutral point, which results from the elastic steering axis and yields comparable characteristic values as the kinematic steering axis. The calculation of the toe neutral point from the compliance matrix of the suspension system is described. Influences of the load case, steering system and suspension concept on the toe neutral point are studied.

The impact of these characteristic values resulting from the elastic steering axis on vehicle behavior, such as straight-line driving, has not been investigated yet.

In summary, two different kinds of steering axes exist according to literature, a kinematic and an elastic steering axis (Fig. 1). The kinematic steering axis characterizes steering feedback in the form of steering torque due to wheel forces and yields well-known characteristic values, such as scrub radius and caster trail. In contrast, the elastic steering axis characterizes wheel movement in the form of toe behavior due to wheel forces and yields characteristic values, such as the longitudinal and lateral location of the toe neutral point. While the kinematic steering axis has already been studied extensively, the elastic one has been neglected and has not been analyzed in detail yet. This paper compares both with each other and demonstrates their meaning for suspension behavior. The results specify the state of the art and show the importance to clearly distinguish between these two kinds of steering axes.

III. Method

This paper presents an approach to study the characteristic values of suspension systems describing both steering feedback and wheel movement within a multi-body simulation (Fig. 2). First, a multi-body suspension model is set up and load cases to be studied are defined. During the simulation, the compliance matrix of the suspension system is determined for each simulation step and the characteristic values are calculated from it. After the simulation, the characteristic values for steering feedback and wheel movement are compared to each other and evaluated. Following this, their different meanings for suspension and vehicle characteristics are demonstrated.

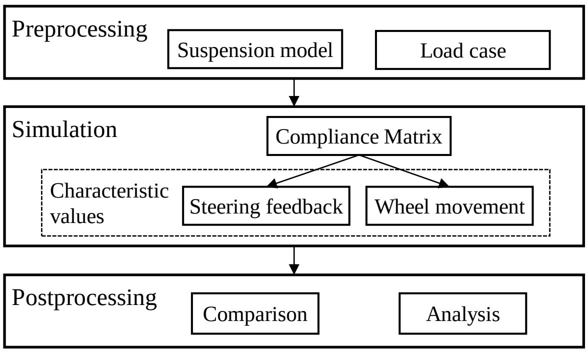

Figure 2. Overall approach proposed in this paper based on [8] [16]

A. Compliance Matrix of the Suspension System

This subsection summarizes the application of the compliance matrix for suspension analysis and design, which is also proposed in [5] [8] [14] [16] [17].

In this paper, a multi-link front suspension is studied. Bushings, springs as well as bump and rebound stops are modeled with the corresponding stiffness characteristics. Steering compliance resulting from the hardy disc and torsion bar of the electromechanical steering system is considered as well. The steering assist characteristic for highway driving is used. The suspension system with its kinematic and compliant properties is regarded as an elastic system connecting the wheel to the chassis (Fig. 3) [5] [14] [18]. Therefore, it can be described by its stiffness matrix or the compliance matrix respectively.

The compliance matrix C summarizes all compliant properties for all degrees of freedom. In this paper, an 18×18 matrix is used to consider the coupling between three parts, which include the left and right wheel (with reference to the wheel center) as well as the rack.

$$C = \begin{bmatrix} C_{\text{wheel,left}} & \cdots & C_{\text{wheel,left,rack}} \\ \vdots & C_{\text{wheel,right}} & \vdots \\ C_{\text{rack,wheel,left}} & \cdots & C_{\text{rack}} \end{bmatrix} \quad (1)$$

As the compliance matrix is determined for each simulation step, its compliant properties are equivalent to the ones of the suspension system for the respective operating point. The suspension system's motion $\vec{d}$, that is its translation and its rotation, due to external forces and moments $\vec{F}$ can be calculated using the compliance matrix.

$$\vec{d} = C\vec{F}, \quad (2)$$

where

$$\vec{F} = \begin{pmatrix} \vec{F}_{\text{wheel,left}} \\ \vec{F}_{\text{wheel,right}} \\ \vec{F}_{\text{rack}} \end{pmatrix} \quad (3)$$

$$\vec{d} = \begin{pmatrix} \vec{d}_{\text{wheel,left}} \\ \vec{d}_{\text{wheel,right}} \\ \vec{d}_{\text{rack}} \end{pmatrix} \quad (4)$$

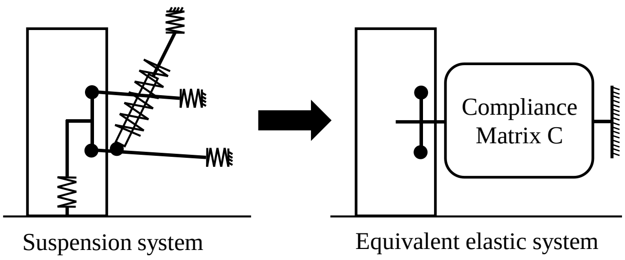

Figure 3. Suspension system as an elastic system represented by its compliance matrix based on [14]

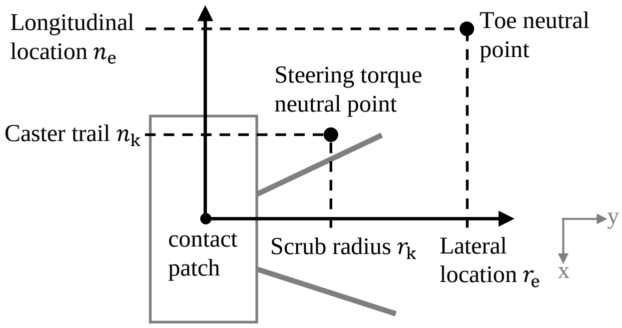

Figure 4. Comparison of steering torque neutral point and toe neutral point in the plane (top view, left wheel, contact patch height)

The calculation of characteristic values for steering feedback and wheel movement, such as the steering torque neutral and toe neutral point (Fig. 4), from the compliance matrix is described in detail in the following subsections.

B. Analysis and Evaluation of Kinematic Steering Axis

This subsection deals with the kinematic steering axis and resulting characteristic values. As the state of the art in the previous section showed, these are regarded as lever arms referring to steering torque. The calculation of these characteristic values from the compliance matrix and their relation with the resulting rack force, which is equivalent to the steering torque, is outlined below.

The lever arms referring to steering torque due to forces applied to the wheels are calculated from (2) subject to the condition

$$d_{\text{rack,ty}} = 0. \quad (5)$$

The force vector $\vec{F}$ in (3) consists of the respective forces and the corresponding moments of the considered load case, which is relevant for each lever arm. A more detailed derivation of the characteristic values for steering feedback using the compliance matrix can be found in [8].

The scrub radius r_{k} as lever arm for longitudinal wheel forces at contact patch height R [4] is then expressed as

$$r_{\text{k}} = \frac{c_{14,1} - c_{14,5}R}{c_{14,6}}. \quad (6)$$

The caster trail n_{k} as lever arm for lateral wheel forces at contact patch height R [4] is given by

$$n_{\text{k}} = \frac{c_{14,2} + c_{14,4}R}{c_{14,6}}. \quad (7)$$

The respective lever arms at wheel center height, that is kingpin and caster offset, can be derived analogously. The wheel load arm p_{k} as lever arm for vertical wheel forces [4] is derived using the same approach and is given by

$$p_{\mathrm{k}} = -\frac{c_{14,3}}{c_{14,6}}. \tag{8}$$

The steering linkage ratio i_{steer} [4], which is defined as the ratio between toe angle and rack travel, is also derived using the compliance matrix. Furthermore, it also describes the ratio between a moment around the z-axis applied to the wheel and the resulting rack force.

$$i_{\mathrm{steer}} = \frac{c_{14,14}}{c_{6,14}} \tag{9}$$

Referring to [4], the resulting rack force change ΔF_{rack} is calculated with the lever arms and the respective forces as well as the steering linkage ratio. The rack force change resulting from one-sided braking forces F_{br} is given by

$$\Delta F_{\mathrm{rack}}\,(F_{\mathrm{br}}) = \int \left(\frac{r_{\mathrm{k}}\,(F_{\mathrm{br}})}{i_{\mathrm{steer}}\,(F_{\mathrm{br}})}\right) dF_{\mathrm{br}}. \tag{10}$$

The resulting rack force change caused by other force components, such as lateral or vertical forces, is calculated analogously with the corresponding lever arms.

By using the total steer ratio [4], that is the ratio between toe angle and steering wheel angle, instead of the steering linkage ratio in (10), the steering torque change resulting from wheel forces can be calculated.

C. Analysis and Evaluation of the Elastic Steering Axis

In this subsection, the calculation of the toe neutral point and corresponding characteristic values resulting from the elastic steering axis is presented. According to [5] [12] [13] [14] [16], the toe neutral point is defined as the location, where longitudinal and lateral forces applied to the wheel do not cause any toe change. The location of the toe neutral point is given by its longitudinal and its lateral location. Additionally, an analogous lever arm for vertical forces, comparable to the wheel load arm, is proposed, which has not been considered in literature before. Furthermore, the calculation of the toe change due to wheel forces is addressed.

The toe neutral point of the left wheel is calculated from (2) subject to the condition

$$d_{\mathrm{wheel,left,rz}} = 0. \tag{11}$$

The force vector $\vec{F}$ in (3) consists of the respective forces and the corresponding moments of the considered load case, which is relevant for each lever arm. A more detailed derivation of the characteristic values for wheel movement using the compliance matrix can be found in [16].

The lateral location of the toe neutral point r_{e} at contact patch height R is the equivalent of an elastokinematic scrub radius and is expressed as

$$r_{\mathrm{e}} = \frac{c_{6,1} - c_{6,5}R}{c_{6,6}}. \tag{12}$$

Its lateral location n_{e} at contact patch height R is the equivalent of an elastokinematic caster trail and is given by

$$n_{\mathrm{e}} = -\frac{c_{6,2} + c_{6,4}R}{c_{6,6}}. \tag{13}$$

The toe neutral wheel load arm p_{e}, a new characteristic value, is defined as the lever arm for vertical forces not causing any toe change and is a measure for the sensitivity of the toe behavior to vertical forces.

$$p_{\mathrm{e}} = -\frac{c_{6,3}}{c_{6,6}} \tag{14}$$

The resulting toe change Δr_{z} is calculated with the lever arms derived from the toe neutral point and the respective forces as well as the rotational compliance around the z-axis $c_{6,6}$. The toe change Δr_{z} resulting from one-sided braking forces F_{br} is given by

$$\Delta r_{\mathrm{z}}\,(F_{\mathrm{br}}) = \int \left(r_e\,(F_{\mathrm{br}}) \cdot c_{6,6}(F_{\mathrm{br}})\right) dF_{\mathrm{br}}. \tag{15}$$

The toe change resulting from other force components is calculated analogously with the corresponding lever arms.

IV. RESUTLS

In this section, the proposed methodology is applied to an exemplary front suspension system. The characteristic values from the kinematic steering axis, which describe steering feedback, and from the elastic steering axis, which characterize toe behavior, are calculated as explained in the previous section. The first subsection compares the characteristic values resulting from both steering axes to each other for the first time. The following two subsections demonstrate their meanings for suspension and vehicle characteristics.

A. Comparison of Kinematic and Elastic Steering Axis

In this subsection, the respective characteristic values resulting from the kinematic and elastic steering axis, which include lateral (Fig. 5), longitudinal (Fig. 6) and vertical (Fig. 7) lever arms, are compared to each other for parallel wheel travel.

The comparison of the lateral lever arms, this is scrub radius and the lateral location of the toe neutral point, is depicted in Fig. 5. While the scrub radius is nearly constant during wheel travel, the lateral location of the toe neutral point varies widely. For bump and the most part of rebound, the scrub radius is smaller than the lateral location of the toe neutral point. The longitudinal lever arms (Fig. 6), that is caster trail and the longitudinal location of the toe neutral point, show a similar behavior. They increase during bump and decrease during rebound. The variations, however, are smaller than the ones of the lateral location of the toe neutral point.

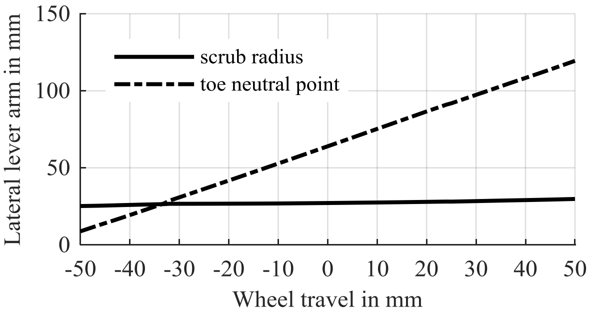

Figure 5. Comparison of scrub radius and lateral location of the toe neutral point for wheel travel

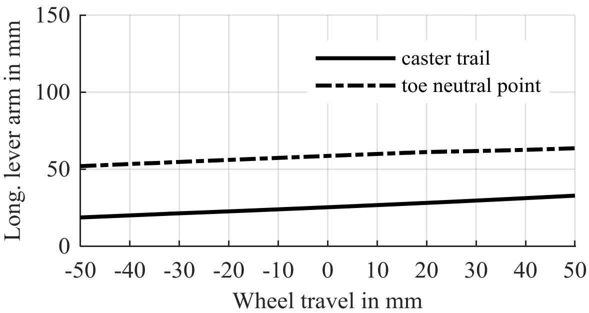

Figure 6. Comparison of caster trail and longitudinal location of the toe neutral point for wheel travel

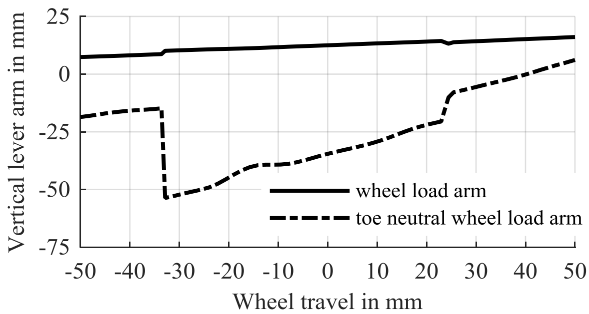

Figure 7. Comparison of wheel load arm and toe neutral wheel load arm for wheel travel

According to Fig. 4, both the steering torque neutral point and the toe neutral point are located on the inner side of the wheel and in front of it. The vertical lever arms, that is the wheel load arm and the toe neutral wheel load arm, are shown in Fig. 7. Both increase during bump and decrease during rebound. The variation of the wheel load arm is smaller than the one of the toe neutral wheel load arm. The stiffness characteristics of bump and rebound stops cause the steps in the curve of the toe neutral wheel load arm.

The results show that the characteristic values of the kinematic and elastic steering axis, and therefore the lever arms for steering feedback and toe behavior, differ from each other significantly. Future suspension design needs to consider both steering axes and not only the kinematic one.

B. Kinematic Steering Axis and Steering Feedback

According to the state of the art, the kinematic steering axis defines lever arms for steering feedback. Therefore, the relationship between the characteristic values derived from the kinematic steering axis and the steering feedback in the form of the resulting rack force due to wheel forces is studied in this subsection.

The scrub radius as the lever arm for braking forces is further investigated here. Referring to mu-split braking, a one-sided braking force is applied to the left wheel. The scrub radius (Fig. 8) decreases under braking forces, as the stiffness of the trailing arm bushing increases due to its nonlinear stiffness characteristic. The steering linkage ratio (Fig. 9) is nearly constant during the load case. The one-sided braking force in combination with the scrub radius and the steering linkage ratio results in a rack force (Fig. 10). The calculated rack force according to (10) matches the simulated rack force for the load case.

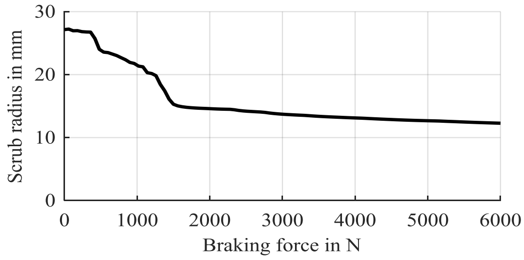

Figure 8. Scrub radius for one-sided braking force

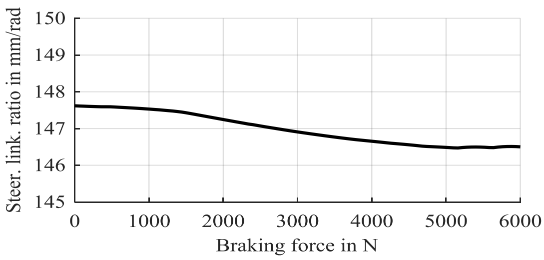

Figure 9. Steering linkage ratio for one-sided braking force

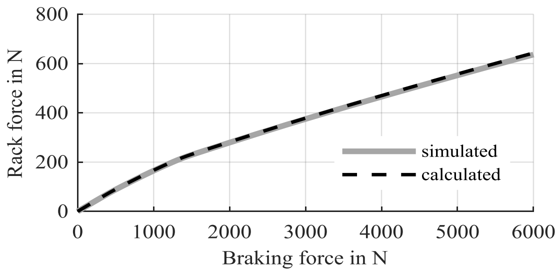

Figure 10. Resulting rack forces for one-sided braking force

Thus, the scrub radius calculated from the compliance matrix is actually the lever arm for braking forces. The same correlation can also be demonstrated for the caster trail and lateral forces as well as for the wheel load arm and vertical forces.

The results clearly illustrate the relationship between the kinematic steering axis with its respective characteristic values and steering feedback.

C. Elastic Steering Axis and Wheel Movement

In this subsection, the elastic steering axis and its meaning for suspension characteristics is studied in detail. Referring to literature, the elastic steering axis, which is the steering axis under loads, is related to the toe behavior of the suspension system. Therefore, the relationship between the characteristic values derived from the elastic steering axis, that is the toe neutral point, and the resulting toe angle due to wheel forces is analyzed.

The lateral location of the toe neutral point is considered as the lever arm for longitudinal forces referring to the toe behavior. Thus, a one-sided braking force is applied to the left wheel again. Under braking forces, the lateral lever arm increases and the toe neutral point moves away from the contact patch (Fig. 11). The increase is caused by changes in the stiffness characteristics of the suspension and steering system. The braking force and the lateral location of the toe neutral point as lever arm create a moment around the z-axis. Therefore, the rotational compliance (Fig. 12) defines the resulting toe angle due to this moment. The comparison of the simulated and the calculated toe angle is depicted in Fig. 13. The calculated toe change from the compliance matrix according to (15) matches the simulated one. The only difference is the static toe angle, because this is not considered in (15). Furthermore, the rotational compliance refers to the wheel center, whereas the braking force is applied at contact patch height. The same results can also be obtained for the longitudinal location of the toe neutral point and lateral forces. Thus, the toe neutral point calculated from the compliance matrix defines the lever arms referring to the toe angle caused by wheel forces.

The results of this subsection demonstrate the proposed relationship between the elastic steering axis with its respective characteristic values and the toe behavior of the suspension system.

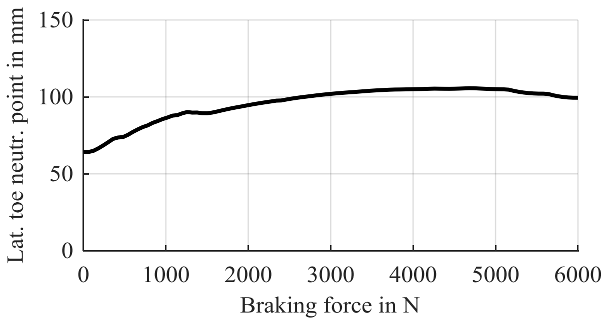

Figure 11. Lateral location of toe neutral point for one-sided braking force

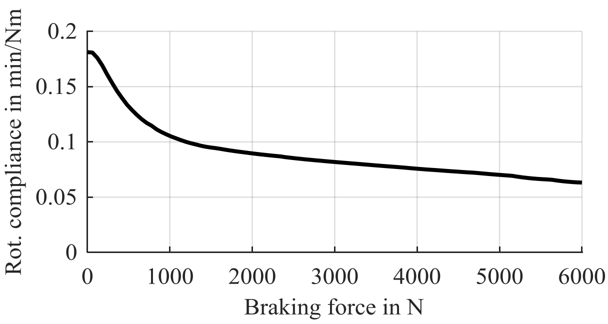

Figure 12. Rotational compliance around the z-axis for one-sided braking force

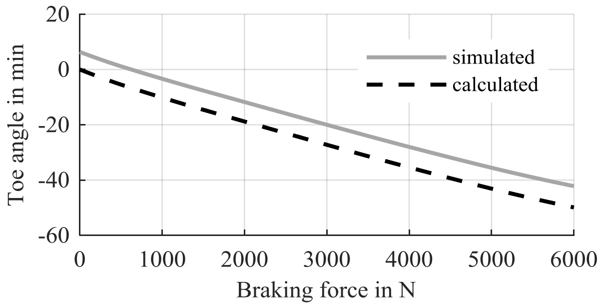

Figure 13. Resulting toe angle for one-sided braking force

V. Conclusion and Outlook

This paper presents a methodology for the analysis and design of suspension systems based on their compliance matrix. The steering axis of front suspension systems is in the center of interest. According to the state of the art, two different kinds of steering axes exist. The kinematic steering axis defines characteristic values, such as scrub radius and caster trail, and the elastic steering axis yields characteristic values, such as the longitudinal and lateral location of the toe neutral point. In contrast to the state of the art, they are now compared to each other and their meaning for suspension characteristics is studied in detail.

The obtained results of this study show for the first time that the elastic steering axis differs considerably from the kinematic steering axis. While the kinematic steering axis defines the lever arms for wheel forces, which characterize steering feedback in the form of rack force, the elastic steering axis yields the lever arms for wheel forces regarding toe behavior. In summary, the results in this paper conform to the new classification proposed in Fig. 1 and specify the state of the art. It is important to clearly distinguish between these two kinds of steering axes, because they have completely different meanings. The gained insights can be used to improve the characterization of suspension systems and thus chassis development with the newly defined characteristic values and further enhance suspension design. Additionally, this paper suggests the use of the compliance matrix method for the comprehensive analysis and design of suspension systems.

Future work needs to be done on the evaluation of differences between the kinematic and the elastic steering axis regarding suspension characteristics. With this in mind, the differences should be analyzed systematically for various load cases and different suspension concepts. Following this, target areas for the characteristic values can be derived. Furthermore, it is strongly recommended to investigate the influences of the characteristic values from both kinds of steering axes on vehicle characteristics in full vehicle simulations.

VI. Contributions

Stefan Buechner (as the first author) initiated and implemented this paper. He drew up the overall concept of the proposed methodology. Patrick Streubel and Robert Buchmann made contributions to the implementation of the proposed methodology within the scope of their master theses. Markus Lienkamp made an essential contribution to the conception of the research project. He revised the paper critically for important intellectual content. Markus Lienkamp gave final approval of the version to be published and agrees to all aspects of the work. As a guarantor, he accepts responsibility for the overall integrity of the paper.

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