Optimum Calculation of Partial Transmission Ratios of Mechanically Driven Systems Using a V-Belt and a Helical Gearbox with First-Step Double Gear Sets

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Abstract—**In this paper, we present a study on the optimum determination of partial transmission ratios of a mechanically driven system using a V-belt and a helical gearbox with second-step double gear sets in order to get the minimum size of the system. The chosen objective function was the cross-sectional dimension of the system. In the optimization problem, the design equation for the pitting resistance of a gear set was investigated. Besides, the equations of the moment equilibrium condition of a mechanical system including a V-belt and a helical gearbox with second-step double gear sets and their regular resistance condition were analyzed. Based on the results of this study, effective formulas for the calculation of the partial ratios of the V-belt and helical gearbox with second-step double gear sets were proposed. Using explicit models, the partial ratios can be easily and accurately determined.**

***Index Terms*—transmission ratio, gearbox design, optimum design, V-belt drive, helical gearbox**

I. Introduction

In the problem of optimum gearbox design, the optimum calculation of partial transmission ratios of the gearbox plays one of the most important roles. This is because partial ratios are the main factors affecting the dimension, the weight, and the cost of the gearbox [1]. Consequently, the optimum determination of the partial ratios of a gearbox has been the subject of much research.

Recently, there have been various studies on the calculation of the partial ratios of gearboxes. Partial ratios were determined for different gearbox types, including helical gearboxes [1–8], bevel-helical gearboxes [1, 3, 9, 10], and worm gearboxes [3, 11]. In addition, different methods have been used for determining the optimum partial ratios. These methods are the graph method [1, 2], the practical method [3], and the modeling method [4–8]. Recently, there have been several studies on the determination of partial ratios for mechanically driven systems using a V-belt and a gearbox [12, 13] as well as a chain drive and a gearbox [14].

From the above mentioned analysis, we concluded that there are no studies on mechanically driven systems using a V-belt and a helical gearbox with first-step double gear sets. Thus, in this paper, we present a study on the optimum calculation of partial ratios for mechanical systems using a V-belt and a helical gearbox with a first-step double gear set to obtain the minimum cross-sectional dimension in the system.

II. Calculation of the Optimum Partial Transmission Ratios

For a helical gearbox with first-step double gear sets (Fig. 1), the cross-sectional dimension becomes minimum when [1]

$$d_{w21} = d_{w22} \tag{1}$$

From Eq. (1) and Fig. 1, for a mechanically driven system using a V-belt and a helical gearbox with first-step double gear sets, the cross-sectional dimension of the system becomes minimum when

$$d_2 = d_{w21} = d_{w22} \tag{2}$$

where d_2 is the driven pulley diameter (mm) and d_{w21} and d_{w22} are the driven diameters of two gear units (mm).

For a helical gearbox with first-step double gear sets, the optimum partial gear ratios u_1 and u_2 are calculated using the following equation [5]:

$$u_2 = 0.8055 \cdot \sqrt[3]{\frac{K_{c2} \cdot \psi_{ba2}}{\psi_{ba1}} \cdot u_g} \tag{3}$$

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where u_g is the transmission ratio of the gearbox, ψ_{ba1} and ψ_{ba2} are the coefficients of the helical gear face width of steps 1 and 2, and K_{C2} is the coefficient, which ranges from 1.1 to 1.3 [5]. For this gearbox, the following values can be chosen: ψ_{ba1} = 0.3 and ψ_{ba2} = 0.35 [10]. Therefore, choosing K_{C2} = 1.2 and substituting ψ_{ba1} and ψ_{ba2} into Eq. (3) yields

$$u_2 = 0.9011 \cdot u_g^{1/3} \quad (4)$$

Equation (4) is used to calculate the partial ratio of the second step (u_2). The partial ratio of the first step (u_1) can be calculated using

$$u_1 = u_g / u_2 . \quad (5)$$

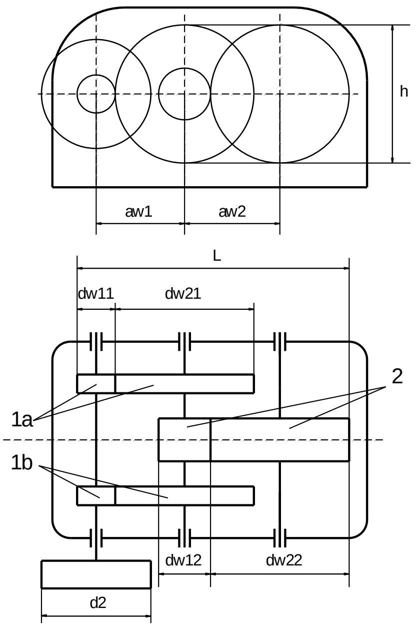

Figure 1: Calculation schema.

From the above analysis, it is obvious that, in order to determine the optimum partial ratios of the systems for getting the minimum system cross section, it is necessary to calculate the diameters d_2 and d_{w22}.

A. Determining the Driven Pulley Diameter (d_2)

For a V-belt set, from the tabulated data for calculating the allowable power [10], the following regression model for determining the driver diameter (d_1) (with the determination coefficient R^2 = 0.9156) has been found:

$$d_1 = 269.7721 \cdot [P_1]^{0.7042} / v^{0.5067} \quad (6)$$

Theoretically, the peripheral velocity of the belt can be calculated as follows:

$$v = \pi \cdot d_1 \cdot n_1 / 60000 . \quad (7)$$

From Eqs. (6) and (7), the diameter of the driver pulley can be determined as follows

$$d_1 = 1093.8 \cdot [P_1]^{0.7923} / n_1^{0.6369} . \quad (8)$$

In addition, the diameter of the driven pulley of a V-belt drive can be determined using the following equation [10]:

$$d_2 = u_b \cdot d_1 \cdot (1 - \varepsilon) . \quad (9)$$

Substituting Eq. (8) into Eq. (9) yields

$$d_2 = 1093.8 \cdot u_b \cdot (1 - \varepsilon) \cdot [p_1]^{0.7923} / n_1^{0.6369} , \quad (10)$$

where ε is the slippage coefficient ($\varepsilon = 0.01 \ldots 0.02$) [10], u_b is the transmission ratio of the V-belt set, and $[P_1]$ is the allowable power of the drive (kW), which is calculated using the following equation:

$$[P_1] = n_1 \cdot [T_1] / \left(9.55 \cdot 10^6\right) \quad (11)$$

Choosing $\varepsilon = 0.015$ and substituting it and (11) into (10) yields

$$d_2 = 0.0032 \cdot u_b \cdot n_1^{0.1554} \cdot [T_1]^{0.7923} \quad (12)$$

B. Determining the Driven Diameter (d_{w22})

For the second step of the gearbox, the driven diameter can be determined as follows [5]:

$$d_{w22} = \left[\frac{4.1571 \cdot [T_{out}] \cdot u_2}{\psi_{ba2} \cdot [K_{02}]} \right]^{1/3} , \quad (13)$$

where ψ_{ba2} = 0.35 (see Section 2), and

$$K_{02} = [\sigma_{H2}]^2 / \left(K_{H2} (Z_{M2} Z_{H2} Z_{\varepsilon 2})^2\right) , \quad (14)$$

where $[\sigma_{H2}]$ is the allowable contact stress of the second-step gear set (MPa). For a helical gear set, $[\sigma_{H2}]$ can be calculated using the following equation [10]:

$$[\sigma_{H2}] = \left([\sigma_H]_1 + [\sigma_H]_2\right) / 2 , \quad (15)$$

where $[\sigma_H]_1$ and $[\sigma_H]_2$ are the allowable contact stresses of the pinion and gear of the second-step gear set (MPa). $[\sigma_H]_1$ is calculated as follows [10]:

$$[\sigma_H]_1 = \sigma^0_{H\lim 1} \cdot K_{HL} / S_H \quad (16)$$

where $\sigma^0_{H\lim 1}$ is the allowable contact stress for the based stress cycle life of the pinion ($\sigma^0_{H\lim 1} = 2 \cdot HB_1 + 70$), HB_1 is the Brinell hardness of the pinion, K_{HL} is the stress cycle life factor, and S_H is the safety factor. With the material of the pinion as ASTM N_0 45, we can have $HB_1 = 250$, $K_{HL} = 1$, and $S_H = 1.1$ [10].

Substituting $\sigma^0_{H\lim 1}$, K_{HL}, and S_H into (16) yields $[\sigma_H]_1 = 518.18$ (MPa). By calculating in the same way for the gear, we get $[\sigma_H]_2 = 490.91$ (MPa). Substituting the values of $[\sigma_H]_1$ and $[\sigma_H]_2$ into Eq. (15) yields $[\sigma_{H2}] = 504.55$ (MPa).

K_{H2} is the contact load factor for the pitting resistance. As $K_{H2} = 1.1–1.3$ [10], we can choose $K_{H2} = 1.2$. Z_{M2} is the material factor. As the pinion and gear are made of steel, Z_{M2} is 274 (MPa$^{1/3}$) [10]. Z_{H2} is the surface condition factor. As the pinion and gear are standard and the helix angles are $8^0 \div 20^0$, $Z_{H2} =$ $1.74 \div 1.67$ [10], and we can choose $Z_{H2} = 1.71$. $Z_{\varepsilon 2}$ is the load sharing factor: $Z_{\varepsilon 2} = (1/\varepsilon_\alpha)^{1/2}$ [10], with ε_α being the contact ratio, which can be calculated using the following equation [10]:

$$\varepsilon_\alpha = \left[1.88 - 3.2\left(1/z_1 + 1/z_2\right)\right] \cdot \cos\beta \tag{17}$$

Practically, the helix angles are $8^0 \div 20^0$, and the number of teeth of the pinion and gear ranges from 15 to 90. From these, the value of the transverse contact ratio is $Z_{\varepsilon 2} = 0.7628 \div 0.8344$. Consequently, the value of $Z_{\varepsilon 2}$ can be chosen as the average of these values, that is, $Z_{\varepsilon 2} = 0.7986$.

Substituting the values of $[\sigma_{H2}]$, K_{H2}, Z_{M2}, Z_{H2}, and $Z_{\varepsilon 2}$ into Eq. (14) yields

$$K_{02} = 504.55^2 / \left(1.2 \cdot (274 \cdot 1.71 \cdot 0.7986)^2\right) = 1.5152\,. \tag{18}$$

Substituting $\psi_{ba2} = 0.35$ and $K_{02} = 1.5152$ into Eq. (13) yields

$$d_{w22} = 1.9865 \cdot \left[T_{out}\right]^{1/3} \cdot u_2^{1/3} \tag{19}$$

C. Determining the Partial Ratios

From Eqs. (2), (12), and (19), we have

$$0.0032 \cdot u_b \cdot n_1^{0.1554} \cdot [T_1]^{0.7923} = 1.9865 \cdot \left[T_{out}\right]^{1/3} \cdot u_2^{1/3} \tag{20}$$

Theoretically, the permissible torque on the drive shaft, that is, $[T_1]$, can be determined from the permissible torque on the output shaft, $[T_r]$, using the following equation:

$$[T_1] = [T_r] / \left(u_t \cdot \eta_t\right) \tag{21}$$

where u_t is the total transmission ratio of the system and η_t is the total efficiency of the system and is calculated as follows:

$$\eta_t = \eta_d \cdot \eta_{br}^2 \eta_o^3 \tag{22}$$

where η_d is the V-belt efficiency, which ranges from 0.95 to 0.96 [2]; η_{br} is the helical gear transmission efficiency, which ranges from 0.96 to 0.98 [2]; and η_o is the transmission efficiency of a pair of rolling bearings, which ranges from 0.99 to 0.995 [2]. Choosing $\eta_d = 0.955$, $\eta_{br} = 0.97$, and $\eta_o = 0.992$ [10] and substituting Eqs. (4), (21), and (22) into Eq. (23), noting that $u_g = u_t / u_b$, yields

$$u_b = 43.6183 \cdot \left[T_{out}\right]^{-0.6267} \cdot n_1^{0.326} \cdot u_t^{1.2544} \tag{23}$$

Equation (23) is used to determine the speed ratio of the V-belt driver. After obtaining u_b, the ratio of the gearbox is calculated using $u_g = u_t / u_b$, and the partial speed ratios of the gearbox, that is, u_1 and u_2, can be found using Eqs. (5) and (4), respectively.

III. Conclusion

The minimum cross-sectional dimension of a mechanically driven system using a V-belt and a helical gearbox with first-step double gear sets can be found by optimally splitting the total transmission ratio of the system. The equations for the calculation of the partial ratios of the V-belt and the helical gearbox for obtaining the minimum cross-sectional dimension of the system were derived. Using explicit models, the partial ratios of the V-belt driver and gear steps of the gearbox can be easily and accurately calculated.

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