Sliding Mode Fractional Order Control for a Single Flexible Link Manipulator

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***Abstract*—This paper presents sliding fractional order control for a single flexible link manipulator. This control strategy takes advantage the robustness of the fractional order control and the sliding mode technique. Fractional calculus is introduced to sliding mode control to design a fractional order sliding mode surface. Lyapunov theory is used to prove the asymptotical stability of the closed-loop system. The proposed controller is compared against the classical PD controller. The simulation results show that the proposed sliding fractional order has smaller error and more robustness comparing to the conventional PD control.**

***Index Terms*— Flexible manipulator, fractional control, sliding mode, stability**

I. Introduction

In recent years, the critical usage of robotic arms in many industrial applications such as welding and painting, and in some scientific experiments such as space discovery, has made controlling these arms a major research area [1-4]. Robotic arms are not just used as an entertainment luxury device as in the past decades. Today, robots have actually replaced humans in operating iterative and dangerous missions which humans do not have the capabilities to do either due to some size limitations, or due to the extreme environments of experiments such as deep depth of the sea, or outer space. Two kinds of manipulators are used: rigid robot and flexible robot. Flexible link manipulators, have the potential for a prosperous future in the fields of modern industry, defense, and space applications. Flexible link manipulators have many advantages compared to heavy and rigid manipulators. Indeed, flexible robots have lower energy consumption, small actuators, higher payload to weight ratio and safe to operate with human due to low inertia [1].

Due to highly coupled nonlinear and time varying dynamic, the flexible link manipulator motion tracking control is one of the most important challenging problems. The Euler–Bernoulli beam theory and the assumed modes method are presented in [2] to develop the equation of motion of flexible link manipulators. Many control strategies have been developed for flexible link manipulators. In [3, 4], the authors raised some challenges encountered for dynamic modeling and control of flexible link manipulators.

A simple linear system approach presented in [5, 6] was developed to control a single-link flexible robot. In [5] LQG was used for the controller design, where in [6], the stable factorization technique was adopted instead. In [7], a composite control system based on sliding mode control, and neural network was proposed. A control system was proposed in [8] which was based on a position-velocity-acceleration feedforward controller and a PID controller for each joint. The independent joint controller recompenses the static and dynamic couplings that exist between the joint while guaranteeing a perfect trajectory tracking.

In [9], a control system based on non-dimensional version of the Euler-Bernoulli beam equation beam equation has been proposed. The authors had also invented a new technique to overcome a time-dependent frequency equation by using a differential version of the frequency equation.

A novel adaptive distributed control system for multiple flexible links manipulators was proposed in [10]. The system was invented to deal with the tracking control problem in the joint space, and to reduce vibrations of the links. The stability of the system was proved using Lyapunov approach. Moreover, the control system was applied on a two-link flexible link manipulator, and showed an improved result over the non-adaptive control versions.

Podlubny had invented the Fractional-order PID controllers for the first time in [11]. From that time, and the fractional-order PID controllers have become a major research area in the literature. The fractional-order PID control system operates by adding the external fractional regulation parameters λ, μ, which can be expressed as $PI_{\lambda}D_{\mu}$. This addition adds more flexibility and robustness to the controlled system, and can assist boosting the overall system's performance at the same time.

In [12], the authors implemented a control system scheme that is a combination of a fuzzy logic controller, and a fractional PID controller, with an automatic parameter tuning method. The idea was to allow all the five parameters of fractional-order PID controller to vary during operation.

A hybrid system that consists of a fractional PID, and the sliding mode control strategy was proposed in [13]. The swarm optimization (PSO) technique was implemented to determine the design parameters. The proposed scheme was implemented on a single flexible

Manuscript received June 14, 2018; revised February 1, 2019.

doi: 10.18178/ijmerr.8.2.228-232

link manipulator, and showed some outstanding results comparing to other similar schemes presented in the literature.

An end-effector position control of a lightweight flexible manipulator using a fractional order controller was proposed in [14]. An interesting characteristic of the design strategy was that the overshoot of the controlled system is independent of the end-effector mass. The proposed control system consists of three nested control loops. After compensating nonlinear effects, the inner loop is responsible for fast motor responses. The middle loop reduces the dynamics complexity of the system, and minimizes its transfer function to a double integrator. Finally, a fractional derivative controller is utilized to shape the outer loop into the form of a fractional order integrator.

This paper presents a sliding fractional control for single flexible link manipulator. This control strategy takes the advantage of the robustness of the fractional order controllers and the sliding mode approach in order to track the desired trajectory of the robot's joint and at the same time reduce the vibration in the flexible link.

The rest of the paper is organized as follows. Section 2 presents system description and modeling. The sliding fractional order control strategy is presented in section 3. Stability analysis using Lyapunov technique is presented in section 4. Simulation results of the proposed controller on one flexible link manipulator is given in section 5. Finally, the conclusion is presented in section 6.

II. System Description and Modeling

A single flexible link manipulator shown in figure 1 is used in this paper to test the proposed control strategy. The system consists of a motor, a flexible link, and a payload. The angle of the motor $\theta(t)$ is denoted as $q_r(t)$, has an inertia and damping coefficient I_m and b_m, respectively. The motor generates a torque τ. E is defined as the module of Young and I_z as the inertia of z. The single flexible arm, supposed uniform, with length L, linear density ρ and rigidity EI_z. The arm internal friction coefficient is k_e. The flexible link is modeled as an Euler-Bernoulli beam and the deformation is assumed to be small. The payload has a mass M_c and an inertia I_c.

The equation of motion of n flexible-link-manipulator is given using the Lagrangian formulation as follows [4]:

$$M(q)\ddot{q} + C(q,\dot{q}) + Kq = L\tau \tag{1}$$

where M is the mass matrix, K is the rigidity matrix, $C$is the Coriolis vector. τ is the applied torque. q represents the generalized coordinates. Assume that there are totally n rigid modes and m flexible modes, thus:
M and $K \in \Re^{(n+m)\times(n+m)}$, q $and C \in \Re^{(n+m)}$, $\tau \in \Re^n$ and $L = [I_{n\times n} \quad 0_{m\times n}]^T$.

The deformation of the flexible link can be expressed as follows:

$$y(x,t) = \sum_{j=1}^{m} \emptyset_j(x) q_{fj}(t) \tag{2}$$

where q_{fj} is the j-th generalized flexible coordinate of the flexible link, $\emptyset_j(x)$ is its j-th shape function and m is the number of the retained flexible modes.

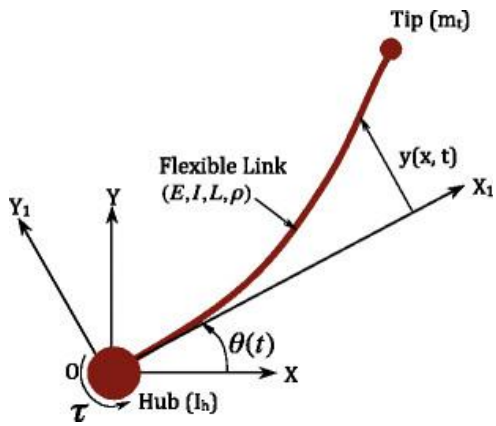

Figure 1. Flexible link robot arm.

Since the proposed system has one flexible link, the number of rigid mode n=1, the dynamical model can be written as:

$$\begin{bmatrix} M_r & M_{rf} \\ M_{fr} & M_f \end{bmatrix}\begin{bmatrix} \ddot{q}_r \\ \ddot{q}_f \end{bmatrix} + \begin{bmatrix} C_r(q,\dot{q}) \\ C_f(q,\dot{q}) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & K_f \end{bmatrix}\begin{bmatrix} q_r \\ q_f \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}\tau \tag{3}$$

where the subscript r denotes the rigid mode and f denotes the flexible mode part.

$q_r \in \Re^n$ are the generalized coordinates associated to the movement of the rigid part, and $q_f \in \Re^m$ are associated to the flexible part. Let define the desired trajectory associated to the rigid part, its first and second-order derivatives be $q_{rd}(t)$, $\dot{q}_{rd}(t)$ and $\ddot{q}_{rd}(t)$ respectively, and $q_{fd}(t)$, $\dot{q}_{fd}(t)$ and $\ddot{q}_{fd}(t)$ associated to the flexible part. In the rest of this paper, two flexible modes for link is considered, i.e. m=2.

The dynamical equation of motion of the flexible manipulators has the following properties:

P1: $M, M_{rr}, M_{ff}$and K_{ff} are symmetric positive definite matrices.

P2: There exists a matrix $H(q,\dot{q}) \in \Re^{(n+m)\times(n+m)}$ such that $C(q,\dot{q}) = H(q,\dot{q})\,\dot{q}$ and $\forall x \in \Re^{n+m}$, $x^T(\dot{M} - 2H)x = 0$.

III. Sliding Fractional Order Control Strategy

In this section, a sliding fractional order controller is designed for flexible link manipulator. First, the dynamical model (3) can be written in terms of two equations as follows:

$$M_r\ddot{q}_r + M_{rf}\ddot{q}_f + C_r = \tau \tag{4}$$

$$M_{fr}\ddot{q}_r + M_f\ddot{q}_f + C_f + K_{ff}q_f = 0 \tag{5}$$

From equation (5), we can write:

$$\ddot{q}_f = -M_f^{-1}\left[M_{fr}\ddot{q}_r + C_f + K_f q_f\right] \tag{6}$$

Inserting (6) in (4), we obtain

$$M_r^* \ddot{q}_r + C_r^* + K_f^* q_f = \tau \qquad (7)$$

where $M_r^* = M_r - M_{rf} M_f^{-1} M_{fr}$; $C_r^* = C_r - M_{rf} M_f^{-1} C_f$; $K_f^* = -M_{rf} M_f^{-1} K_f$.

Define the sliding surface as follows:

$$s_r = K_1 e_r + K_2 D^{-\mu} e_r + K_3 D^{\alpha} e_r + \dot{e}_r \qquad (8)$$

where $e_r = q_r - q_{rd}$, and $e_f = q_f - q_{fd}$ are the error signals for joint angle and flexible coordinates respectively. $D^{-\mu} e_r$ is the integration rate to a fractional order integrator and $D^{\alpha} e_r$ is the fractional order derivative term. μ and α are additional free parameters. K_1, K_3 and K_3 are positive gain parameters.

Note that in the proposed design $q_{rd}(t), \dot{q}_{rd}(t), \ddot{q}_{rd}(t)$, $q_{fd}(t), \dot{q}_{fd}(t)$, and $\ddot{q}_{fd}(t)$ must be carefully chosen to satisfy the control objective $(q_r, q_f) = (q_{rd}, 0)$.

From the sliding surface (8), we can write:

$$\dot{s}_r = K_1 \dot{e}_r + K_2 D^{1-\mu} e_r + K_3 \frac{d}{dt} D^{\alpha} e_r + \ddot{e}_r \qquad (9)$$

Let us propose the following control law:

$$\tau = C_r^* + K_f^* q_f + M_r^* \left[\ddot{q}_{rd} + K_1 \dot{e}_r + K_2 D^{1-\mu} e_r + K_3 \frac{d}{dt} D^{\alpha} e_r + K_r sgn(s_r) \right] \qquad (10)$$

where K_r is a positive gain parameter.

The error dynamics are obtained by inserting the control law (10) in the new dynamical model (7) as follows:

$$\ddot{e}_r + K_1 \dot{e}_r + K_2 D^{1-\mu} e_r + K_3 \frac{d}{dt} D^{\alpha} e_r + K_r sgn(s_r) = 0 \qquad (11)$$

IV. Stability Analysis

The goal is to drive the tracking error asymptotically to zero for any arbitrary initial conditions and uncertainties. To achieve this objective, the Lyapunov theory is used.

The second derivative of the tracking error can be expressed as follows:

$$\ddot{e}_r = \ddot{q}_{rd} - \ddot{q}_r = \dot{s}_r - K_1 \dot{e}_r - K_2 D^{1-\mu} e_r - K_3 \frac{d}{dt} D^{\alpha} e_r \qquad (12)$$

The acceleration term of the rigid part $\ddot{q}_r$ can be obtained from (12) as

$$\ddot{q}_r = -\dot{s}_r + \ddot{q}_{rd} + K_1 \dot{e}_r + K_2 D^{1-\mu} e_r + K_3 \frac{d}{dt} D^{\alpha} e_r \qquad (13)$$

Using equation (13), the equation (7) can be written as:

$$M_r^* \left[-\dot{s}_r + \ddot{q}_{rd} + K_1 \dot{e}_r + K_2 D^{1-\mu} e_r + K_3 \frac{d}{dt} D^{\alpha} e_r \right] + C_r^* + K_f^* q_f = \tau \qquad (14)$$

From equation (14), the time derivative of the sliding surface can be written as

$$\dot{s}_r = -M_r^{*-1} \left[\tau - C_r^* - K_f^* q_f \right] + \ddot{q}_{rd} + K_1 \dot{e}_r + K_2 D^{1-\mu} e_r + K_3 \frac{d}{dt} D^{\alpha} e_r \qquad (15)$$

Now, let consider the following positive Lyapunov function

$$V(t) = \frac{1}{2} {s_r}^2 \qquad (16)$$

Take the time derivation of $V(t)$ to get:

$$\dot{V}(t) = s_r \dot{s}_r$$
$$= s_r \left(-M_r^{*-1} \left[\tau - C_r^* - K_f^* q_f \right] + \ddot{q}_{rd} + K_1 \dot{e}_r + K_2 D^{1-\mu} e_r + K_3 \tfrac{d}{dt} D^{\alpha} e_r \right)$$
$$= s_r \left(-M_r^{*-1} \left[\tau - C_r^* - K_f^* q_f \right] + \ddot{q}_{rd} + K_1 \dot{e}_r + K_2 D^{1-\mu} e_r + K_3 \frac{d}{dt} D^{\alpha} e_r \right)$$

Using the control law (10), the final version of $\dot{V}$ becomes:

$$\dot{V}(t) = -K_r s_r sgn(s_r) \qquad (17)$$

K_r is positive definite, in which clearly $\dot{V}(t) < 0$. Using Barbalat Lemma [15], the error dynamics resulting from the above control law (10) are asymptotically stable in the sense of Lyapunov.

V. Simulation Results

This section presents the simulation results of the proposed sliding fraction controller to single flexible link manipulator. To show the contribution of the proposed control, a comparison with a classical PD type control is presented.

The nominal parameters of the proposed system is given in Table 1.

TABLE I. Nominal Parameters

Parameter name	*Nominal value*
Motor inertia *(I_m)*	0.02 kg m^2
Beam length *(L)*	1 m
Beam linear density *(ρ)*	0.62 kg/m
Beam rigidity *(EI_z)*	12.85 N m^2
Payload mass *(M_c)*	0.3 kg

The PD control law is given by the following expression:

$$\tau = K_p (q - q_d) + K_v (\dot{q}_d - \dot{q}) \qquad (18)$$

where $K_p$and K_d are positive gain parameters.

The following polynomial function is used to generate the desired trajectory of the rigid coordinate.

$$q_{rd}(t) = \frac{\pi}{2} \left(5 \left(\frac{t}{T_f} \right)^3 - \frac{15}{2} \left(\frac{t}{T_f} \right)^4 + 3 \left(\frac{t}{T_f} \right)^5 \right) \qquad (19)$$

for $0 \leq t \leq T_f = 7s$, and $q_{rd}(t) = \frac{\pi}{4}$ for $t \geq T_f$.

As mentioned in the previous section that the goal is to reduce the link vibration. For this reason, the desired positions of the flexible modes are set to zero.

Figures (2)-(5) show the simulation of the sliding fractional controller.

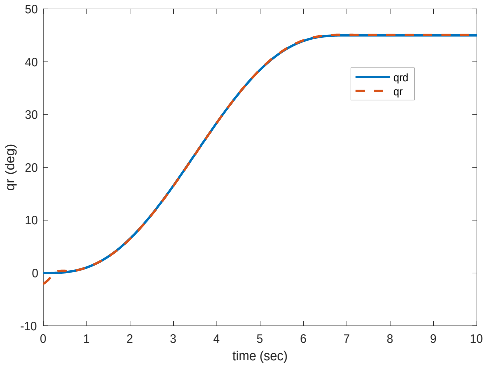

Figure 2. Tracking trajectory in the joint space.

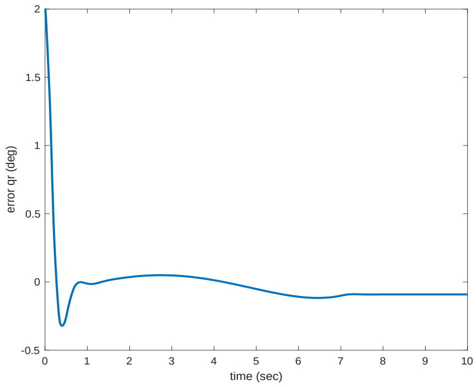

Figure 3. Joint tracking error.

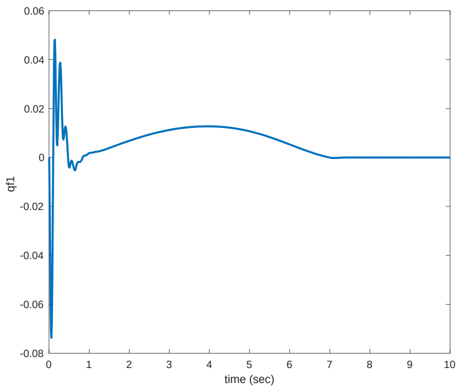

Figure 4. First vibration mode of the link.

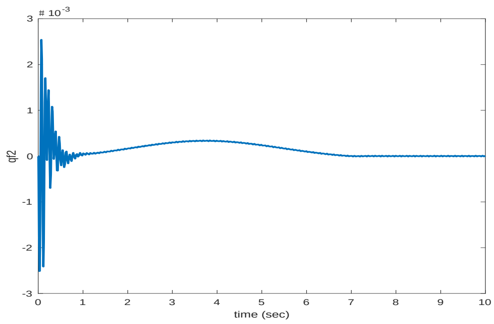

Figure 5. Second vibration mode of the link.

The simulation results of the PD control are presented in Figures (6)-(9).

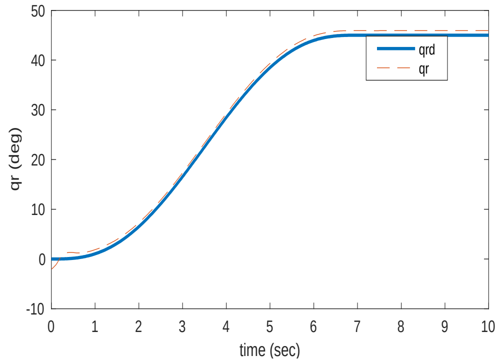

Figure 6. PD control: joint trajectory response.

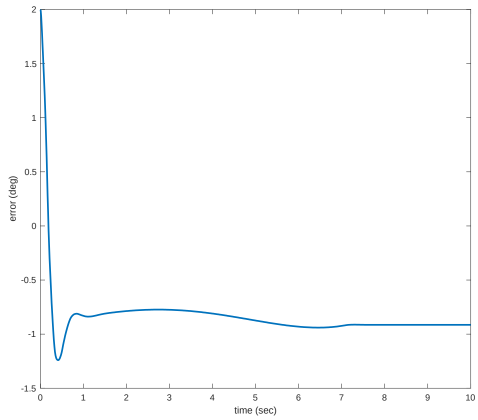

Figure 7. PD control: joint tracking error.

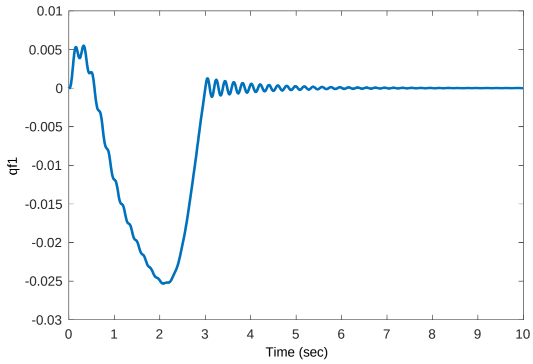

Figure 8. PD control: first vibration mode of the link.

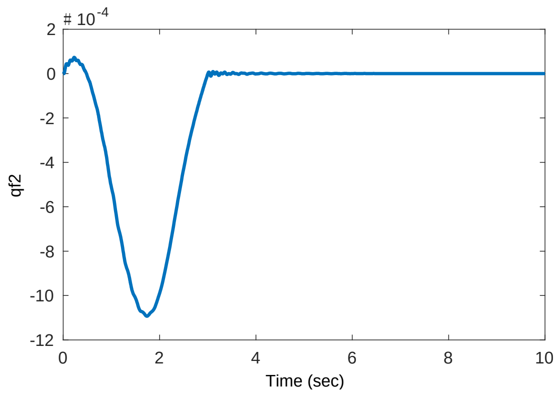

Figure 9. PD control: second vibration mode of the link.

In the simulation results, Figure 2 presents tracking trajectory in the joint space. This good tracking is confirmed by Figure 3 which shows the tracking error. Despite the different initial conditions, the proposed controller track perfectly the desired trajectory. For the flexible part, Figures 4 and 5 show the first and second vibration mode of the flexible link. According to these two figures, we can conclude that the proposed controller capable to eliminate the vibration of the flexible link. For the PD control, Figures 6 and 7 show the tracking trajectory and the tracking error respectively while figures 8 and 9 show the first and second vibration mode of the link.

According to the simulation results we can conclude that, with the proposed controller, the tracking error of the joint is smaller than the one resulting from the PD control. These results show the effectiveness of the proposed technique, which is based on fractional calculus and the sliding mode technique. The proposed technique provides improved robustness and more extra parameters (α, μ) in the search space than the classical PD. For the future work, this proposed controller can be extended to the adaptive version.

VI. CONCLUSION

In this paper, a sliding fractional order control law was presented. The main advantage of this proposed controller is the combination of the robustness of the fraction controller and sliding mode control. This proposed controller was applied to non-minimum phase system such as the flexible link manipulator. Compared to the classical PD control, the theoretical analysis and simulation results show that the proposed sliding fractional control achieves better tracking performance and capable to eliminate the vibration in the flexible link. Asymptotical stability of the closed-loop system has been guaranteed by using Lyapunov theory.

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