Research on Thermodynamic Modeling and Simulation of Hydro-Pneumatic Spring

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***Abstract*—Hydro-pneumatic spring have been widely used in the field of vehicle suspension because of the typical nonlinear characteristics, but the high working temperature will influence the viscosity of oil and the endurance life of rubber seal, therefore, it is particularly necessary to study the thermal dynamics of the hydro-pneumatic spring. In the article, the differential equations of oil and gas medium temperature variation with internal energy are derived, in addition, the random excitation input of hydro-pneumatic spring is determined by dynamic modeling. After that, the thermal equilibrium temperature of hydro-pneumatic spring under different pavement excitation are simulated and analyzed, it can be used to guide the design of suspension.**

***Index Terms*—hydro-pneumatic spring, thermodynamic modeling, vehicle dynamics, heat balance temperature**

I. Introduction

Inert gas is usually used to act as an elastic element for storing vibration energy in oil gas suspension. The vibration energy dissipated by damping device is converted into heat energy, a part of the heat energy is emitted into the environment through the cylinder surface. Another part of the heat energy can change suspension inner energy, thus the oil temperature rises. When the temperature is over the seal temperature limit, the oil-gas leakage phenomenon will appear, so that the whole system can not work normally. System temperature rise is related to many factors, so temperature rise property is researched by establishing oil gas suspension thermal model in this article.

II. Oil Gas Suspension Thermal Physical Model

As shown in Fig. 1, the thermodynamics of physical model of oil gas suspension was established under the forced convection condition. In which D_{hw} is the outer cylinder diameter, D_{hn} is the inner diameter of cylinder, T_y is the oil temperature, T_{w1} is the inner wall temperature of the cylinder, T_{w2} is the outer wall temperature of cylinder, δ_1 is the cylinder wall thickness, T_∞ is the ambient temperature, L_5 is the floating piston thickness, T_q is nitrogen gas temperature, T_{g1} is inner wall temperature of the piston rod, T_{g2} is wall temperature of the piston rod, δ_2 is piston rod wall thickness, L_q is the piston rod length exposed to air, D_{gn} is the inner diameter of piston rod, D_{gw} is the piston rod diameter, V is the velocity of the piston rod, V_c is the outside air flow rate.

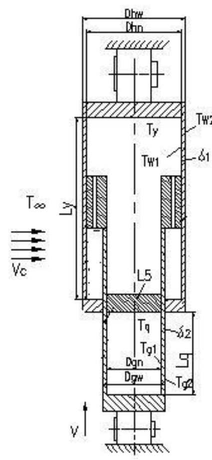

Figure 1. Hydro-pneumatic spring thermal physical model.

III. Hydro-Pneumatic Spring Thermal Mathmatical Model

A. Suspension Input Excitation

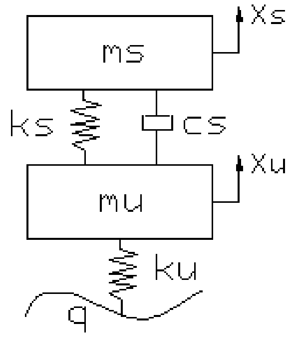

Figure 2. Vehicle dynamics model

According to Newton's laws of motion to establish 1/4 vehicle dynamics model [1], [2]:

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$$\begin{cases} m_s\ddot{x}_s + c_s\left(\dot{x}_s - \dot{x}_u\right) + k_s\left(x_s - x_u\right) = 0 \\ m_u\ddot{x}_u + c_s\left(\dot{x}_u - \dot{x}_s\right) + k_s\left(x_u - x_s\right) + k_t\left(x_u - q\right) = 0 \end{cases} \quad (1)$$

$$M\ddot{X} + C\dot{X} + KX = K_q q \quad (2)$$

In which, $X = [x_s \quad x_u]^T$, $M = \begin{bmatrix} m_s & \\ & m_u \end{bmatrix}$, $C = \begin{bmatrix} c_s & -c_s \\ -c_s & c_s \end{bmatrix}$, $K = \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_t \end{bmatrix}$, $K_q = \begin{bmatrix} 0 \\ k_t \end{bmatrix}$.

Write out the characteristic value and vector equation by using the complex modal method:

$$\left[\lambda_t^2 M + \lambda_t C + K\right]u = 0 \quad (3)$$

In which, λ is characteristic value, u is characteristic vector.

The equation has the non-zero solution necessary and sufficient condition is:

$$\left|\lambda_t^2 M + \lambda_t C + K\right| = 0 \quad (4)$$

In which, n is the number of freedom degree.

Corresponding to any characteristic value λ_{tr}, and the corresponding characteristic vector is u_r, put it into the formula (3):

$$\left[\lambda_{tr}^2 M + \lambda_{tr} C + K\right]u_r = 0 \quad r = 1, \cdots, 2n \quad (5)$$

Put the state variable y into the formula (2) [3] ,

$$M'\dot{y} + K'y = F(t) \quad (6)$$

In which, $y = \begin{bmatrix} \dot{Z} \\ Z \end{bmatrix}$, $F(t) = \begin{bmatrix} 0 \\ K_q q \end{bmatrix}$, $M' = \begin{bmatrix} 0 & M \\ M & C \end{bmatrix}$, $K' = \begin{bmatrix} -M & 0 \\ 0 & K \end{bmatrix}$.

$$m_r = u_r^T\left[2\lambda_{tr} m + c\right]u_r \quad (7)$$

The complex modal response array is：

$$z = \left[z_1 \cdots z_{2n}\right]^T \quad (8)$$

The formula (5) is decoupled and written the scalars form[4]：

$$\dot{z}_r - \lambda_{tr} z_r = m_r^{-1} u^T k_u q \quad (9)$$

Using complex modal transformation:

$$X_n = u z_r, \ \dot{X}_n = \lambda_{tr} u \dot{z}_r \quad (10)$$

The oil gas suspension relative displacement is：$\Delta x = x_s - x_u$.

And suspension relative velocity is： $\Delta\dot{x} = \dot{x}_s - \dot{x}_u$

The oil gas spring relative displacement is：

$$\Delta x_t = \left(x_s - x_u\right)/i$$

And spring relative velocity is：

$$\Delta\dot{x}_t = \left(\dot{x}_s - \dot{x}_u\right)/i$$

The 4-Runge Kutta method is used to solve the thermal mathematical model:

$$\begin{cases} z_{r(i+1)} = z_{ri} + \dfrac{h}{6}\left(K_1 + 2K_2 + 2K_3 + K_4\right) \\ K_1 = m_r^{-1}u^T k_u q(t) + \lambda_{tr} z_{ri} \\ K_2 = m_r^{-1}u^T k_u q(t + h/2) + \lambda_{tr}\left(z_{ri} + \dfrac{h}{2}K_1\right) \\ K_2 = m_r^{-1}u^T k_u q(t + h/2) + \lambda_{tr}\left(z_{ri} + \dfrac{h}{2}K_2\right) \\ K_4 = m_r^{-1}u^T k_u q(t + h) + \lambda_{tr}\left(z_{ri} + hK_3\right) \end{cases} \quad (11)$$

B. Thermal Mathematical Model

Firstly, the oil internal energy could be written as:

$$\frac{d\left(U_y\right)}{dt} = m_y c_{py}\frac{dT_y}{dt} \quad (12)$$

In which m_y is oil mass, c_{py} is specific heat capacity at constant pressure for oil.

As well as the external work done on oil is:

$$\frac{dW_y}{dt} = \Delta p A_h \frac{dS}{dt} = \Delta p A_h V_d \quad (13)$$

The heat transfer between the cylinder and the outside air is convective heat transfer. According to the Newton's law of cooling, the general expression of heat transfer is determined as follows:

$$\frac{dQ_{wo}}{dt} = h_{wo}A_{wo}\left(T_{w2} - T_\infty\right) \quad (14)$$

In which, h_{wo} is convective heat transfer coefficient for the outer surface of the cylinder, in addition, the area of air contact is:

$$A_{wo} = \pi D_{hw} L_y \quad (15)$$

The heat transfer caused by the temperature difference between the inner and outer walls of the cylinder is calculated according to the heat conduction equation in the column coordinate system:

$$\frac{dQ_w}{dt} = -2\pi L_y \lambda_r \frac{T_{w1} - T_{w2}}{\ln\left(D_{hn}/D_{hw}\right)} \quad (16)$$

In which, λ_r is thermal conductivity of cylinder material.

The convection heat transfer between the inner wall of the cylinder and the oil is as follows:

$$\frac{dQ_{wi}}{dt} = h_{wi}A_{wi}\left(T_y - T_{w1}\right) \quad (17)$$

In which, h_{wi} is convective heat transfer coefficient on the inner surface of cylinder. Contact area with oil liquid is:

$$A_{wi} = \pi D_{hn} L_y \tag{18}$$

The floating piston is calculated by the heat conduction mode, and the heat transfer is as follows:

$$\frac{dQ_f}{dt} = \lambda_r A_x \cdot \frac{T_y - T_q}{L_5} \tag{19}$$

According to the principle of heat transfer:

$$T_{w2} = T_y - \frac{dQ_w}{dt}\left[\frac{1}{h_{wi}\pi D_{hn} L_y} - \frac{\ln\left(D_{hn}/D_{hw}\right)}{2\pi L_y \lambda_r}\right] \tag{20}$$

Further, the expression of integrated heat transfer in oil liquid is derived:

$$\frac{dQ_w}{dt} = \frac{\left(T_y - T_\infty\right)}{\frac{1}{h_{wo}\pi D_{hw} L_y} + \frac{1}{h_{wi}\pi D_{hn} L_y} - \frac{\ln\left(D_{hn}/D_{hw}\right)}{2\pi L_y \lambda}} \tag{21}$$

The total heat emitted from the liquid to the outside is as follows:

$$\frac{dQ_y}{dt} = \lambda_r \cdot \pi D_x^2 \cdot \frac{T_y - T_q}{4L_5} + \frac{\left(T_y - T_\infty\right)}{\frac{1}{h_{wo}\pi D_{hw} L_y} + \frac{1}{h_{wi}\pi D_{hn} L_y} - \frac{\ln\left(D_{hn}/D_{hw}\right)}{2\pi L_y \lambda_r}} \tag{22}$$

The expression of oil temperature rise is obtained:

$$m_y c_{py} \frac{dT_y}{dt} + \left(C_y + B_y\right) T_y = \Delta p A_u V_d + C_y T_q + B_y T_\infty \tag{23}$$

In which,

$$B_y = \frac{1}{\frac{1}{h_{wo}\pi D_{hw} L_y} + \frac{1}{h_{wi}\pi D_{hn} L_y} - \frac{\ln\left(D_{hn}/D_{hw}\right)}{2\pi L_y \lambda}}$$

$$C_y = \frac{\lambda \pi D_{gn}^2}{4L_5}$$

Thermodynamic energy expression of nitrogen could be deduced according to the Vander Waals equations of the real gas state:

$$\frac{dU_q}{dt} = m_n c_{q\upsilon} \frac{dT_q}{dt} + \frac{m_n{}^2 a}{V_q{}^2} \frac{dV_q}{dt} \tag{24}$$

In which $\frac{dV_q}{dt} = A_x V_d$.

As well as the external work done on gas is:

$$\frac{dW_q}{dt} = pA_x V_d = pV_d \pi D_x^2 / 4 \tag{25}$$

The convective heat transfer between the outer wall of the piston rod and the air can be determined according to the Newton cooling formula:

$$\frac{dQ_{go}}{dt} = h_{go} A_{go} \left(T_{g2} - T_\infty\right) \tag{26}$$

In which, h_{go} is convective heat transfer coefficient on the external surface of accumulator, the area of contact with the air is:

$$A_{go} = \pi D_{xw} L_q \tag{27}$$

And the unit time heat transfer of the inner wall and nitrogen is:

$$\frac{dQ_{gi}}{dt} = h_{gi} A_{gi} \left(T_q - T_{g1}\right) \tag{28}$$

In which, h_{gi} is convective heat transfer coefficient on the inner surface of accumulator, the contact area with nitrogen is similar to that of:

$$A_{gi} = \pi D_x L_q \tag{29}$$

The unit time heat conduction equation caused by the temperature difference between the inner and outer walls of the piston rod is as follows:

$$\frac{dQ_g}{dt} = -2\pi L_q \lambda_r \frac{T_{g1} - T_{g2}}{\ln\left(D_x / D_{xw}\right)} \tag{30}$$

It is known from the heat transfer relationship that the work temperature formula is:

$$T_{g2} = T_q - \frac{dQ_g}{dt}\left[\frac{1}{h_{gi} A_{gi}} - \frac{\ln\left(D_x / D_{xw}\right)}{2\pi L_q \lambda_r}\right] \tag{31}$$

The expression of comprehensive heat transfer of nitrogen is obtained:

$$\frac{dQ_g}{dt} = \frac{\left(T_q - T_\infty\right)}{\frac{1}{h_{go}\pi D_{xw} L_q} + \frac{1}{h_{gi}\pi D_x L_q} - \frac{\ln\left(D_x / D_{xw}\right)}{2\pi L_q \lambda_r}} \tag{32}$$

And the gas total calories which transfer to the outside is:

$$\frac{dQ_q}{dt} = \frac{\left(T_q - T_\infty\right)}{\frac{1}{h_{go}\pi D_{xw} L_q} + \frac{1}{h_{gi}\pi D_x L_q} - \frac{\ln\left(D_x / D_{xw}\right)}{2\pi L_q \lambda_r}} - \lambda_r \pi D_x^2 \frac{T_y - T_q}{4L_5} \tag{33}$$

The expression of temperature rise of nitrogen is obtained:

$$m_q c_{q\upsilon} \frac{dT_q}{dt} + \left(B_q + C_y\right) T_q = \left(\frac{p\pi D_{gw}^2}{4} - \frac{m_q{}^2 a}{V_q{}^2} A_{gw}\right) V_d + B_q T_\infty + C_y T_y \tag{34}$$

In which,

$$B_q = \frac{1}{\frac{1}{h_{go}\pi D_{gw} L_q} + \frac{1}{h_{gi}\pi D_{gn} L_q} - \frac{\ln\left(D_{gn} / D_{gw}\right)}{2\pi L_q \lambda}}$$

The system temperature rise could be analyzed through the first law of thermodynamics:

$$\frac{dW}{dt}-\frac{dQ}{dt}=\frac{dU}{dt} \tag{35}$$

In which, $\frac{dW}{dt}$ is the system get the power from outside, $\frac{dQ}{dt}$ is the system heat transfer to the outside in a unit time, $\frac{dU}{dt}$ is the internal energy increment of the system in a unit time.

IV. Heat Balance Calculation of Oil Gas Suspension

The thermodynamic model of hydro-pneumatic spring is calculated through the mathematical software Matlab. Parameters are set as follows: $D_{hw}=0.145m$, $D_{hn}=0.125m$, $D_{gn}=0.08m$, $D_{gw}=0.095m$, $L_y=0.37m$, $L_q=0.3m$.

The initial temperature and the ambient temperature are equal. Setting the vehicle speed is 10m/s, the calculation results as the following diagrams:

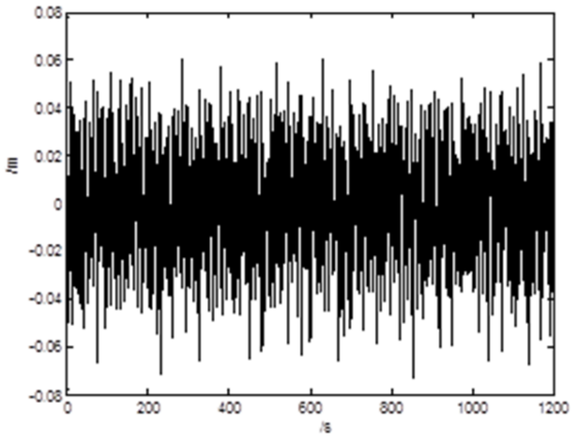

Figure 3. Road spectrum of D grade.

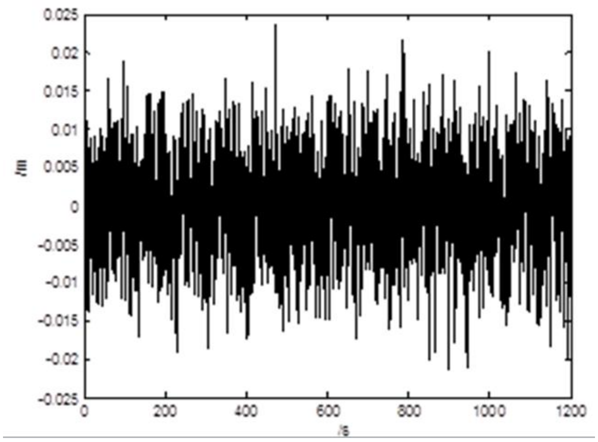

Figure 4. Pebble pavement spectrum.

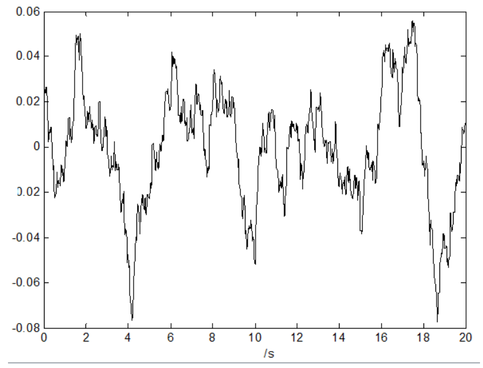

Figure 5. Unevenness of sand and gravel pavement.

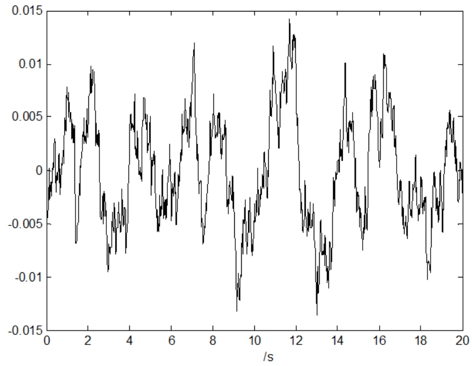

Figure 6. Asphalt pavement unevenness.

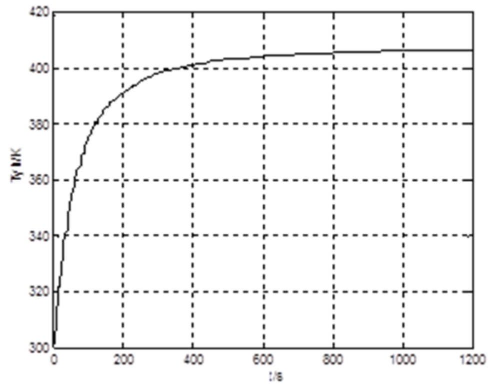

Figure 7. Temperature rise of suspension on the D grade road.

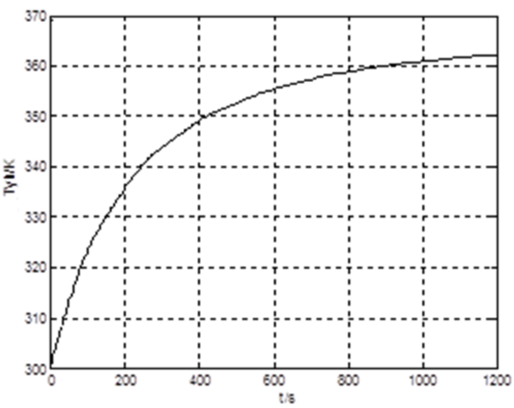

Figure 8. .temperature rise of suspension on the pebble pavement.

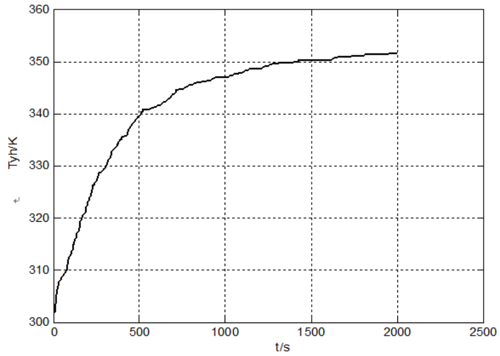

Figure 9. Temperature rise of suspension on sand and gravel pavement.

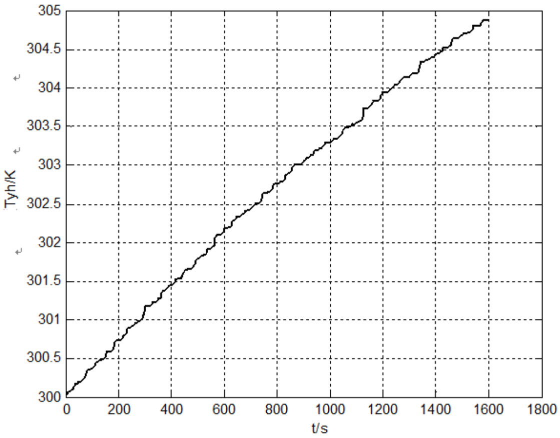

Figure 10. temperature rise of suspension on asphalt pavement.

As the Fig. 3-6 shows, the road surface roughness of standard D grade is significantly greater than the other pavement, and it can be seen from Fig.7-10, the vehicle which install the oil gas suspension travel on the four road surface at the same speed and same time range, the suspension oil temperature reached to 405K when the vehicle on the D grade road , and it reached to 362K when vehicle travel on the pebble road, so we can get the result that following the increase of the pavement spectrum density, vehicle suspension need to attenuate more impact energy from the ground. At the same time, the suspension thermal model also could lay the foundation for the further development of the power consumption of the system.

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