Comparison of Friction Estimation Models for Rotary Triple Inverted Pendulum

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***Abstract*—Most of mechanical robotic systems include non-negligible nonlinearities due to the complex dynamic behaviors of frictions. In this study, joint frictions of Rotary Triple Inverted Pendulums (RTIP) are examined based on its experimental and simulation dynamic responses. RTIP might be considered as the most appropriate mechanical setup to investigate friction phenomena and understand the frictions' influence in the dynamics of any mechanical system. In this paper, three different friction estimation models such as Non-Conservative, Linear and Non-linear friction models are compared to estimate the joint frictions of the RTIP developed in our laboratory. Non-Conservative friction estimation model considers only viscous frictions. Linear friction model is dependent on Coulomb and viscous frictions. The Non-Linear friction model is the sum of five types of frictions: the zero drift error of friction, the Coulomb friction, the viscous friction, and the experimental friction. Based on comparative experimental friction analysis, the joint frictions of the RTIP are estimated more effectively using a Non-Linear friction model.**

***Index Terms*—Rotary Triple Inverted Pendulums (RTIP), Linear Friction Estimation, Non-linear Friction Estimation, Non- Conservative Friction Estimation, Gradient Descent (GD) algorithm, Pattern Search (PS) algorithm**

I. Introduction

Rotary inverted pendulum system (RIPS) is an underactuated mechanical system, naturally unstable, open-loop with nonlinear dynamics [1]. It is a practical laboratory setup to understand human dynamics and test the different nonlinear controllers [2].

Frictions are very important in control engineering systems, such as in pneumatic and hydraulic systems, anti-lock brakes for cars and robotic systems [3]. Frictions are highly nonlinear and they can result in steady state errors, limit cycles, and poor performance in different systems [4]. It is, therefore, important for control engineers to understand friction phenomena and to estimate the ideal frictions for each system. Today, using the computational power available, it is possible to deal effectively with frictions. Frictions estimation has the potential to ameliorate the quality, economy, and safety of any system [5].

In this paper, a rotary triple inverted pendulum (RTIP) is used to estimate the friction coefficients in the joints of the pendulum as seen in "Fig. 1". This work presents three different models applied to estimate the joint friction of an RTIP: Non-Conservative, Linear and Non-linear friction estimation models.

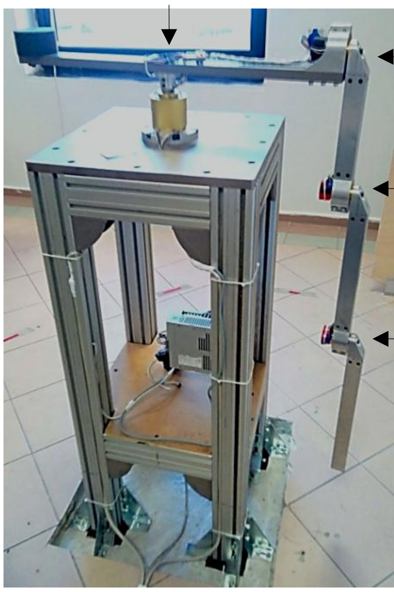

Figure 1. Experimental setup: RTIP

A dynamic model simulation of the RTIP is explained in Section II. The friction estimation models applied to the RTIP will be explained in Section III. In section IV the pendulum friction estimation models are validated with experimental results. The last section is devoted to the conclusion.

II. Dynamic Model Simulation

In this part of the paper, a dynamic model simulation of the RTIP will be explained.

- **Rotary Triple Inverted Pendulum**

RTIP takes the classic rotary single and double rotary pendulum problems to the next level of complexity [6]. The proposed RTIP is composed of a rotary arm that attaches to a servomotor which provides a torque to the base arm to control the whole system. Furthermore, the balance mass is added to the system in the control phase. Three pendulums are mounted respectively at the arm. It is an underactuated and extremely nonlinear unstable system, because of the gravitational force and the

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coupling arising from the Coriolis and Centripetal forces [7]. The orientation of the horizontal arm is represented by the angle θ_0, and the positions of the three pendulums by the angles θ_1, θ_2 and θ_3 respectively. θ_1, θ_2 and θ_3 are equal to zero when the three pendulums are pointed down in the upright position. By using the Newton-Euler method, the motion equations of the RTIP are determined. [8]. The coordinate systems attached to the joints are shown in "Fig. 2".

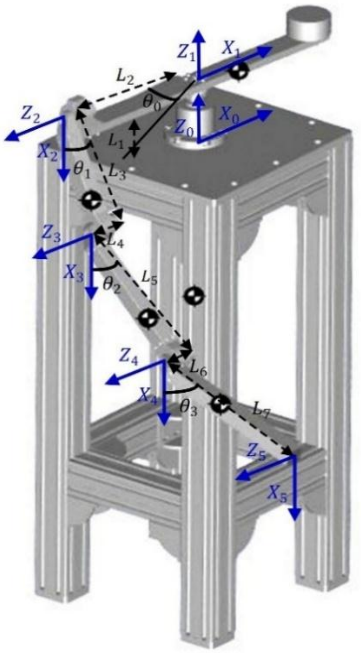

Figure 2. The coordinate system of the RTIP

The dynamic torque equations of the RTIP may be written in a matrix form, as follows:

$$\mathrm{D}(\theta)\ddot{\theta} + \mathrm{C}(\theta,\dot{\theta})\dot{\theta} + \tau_f(\theta,\dot{\theta}) + \mathrm{G}(\theta) = \tau_i \qquad (1)$$

where $\theta, \dot{\theta}$ and $\ddot{\theta}$ are the vectors of joint angles, the angular velocities, and the angular accelerations respectively. $\mathrm{D}(\theta)$ is the mass matrix, $\mathrm{C}(\theta,\dot{\theta})$ is the Coriolis and centrifugal force vector, $\tau_f(\theta,\dot{\theta})$ is the friction torque vector , $\mathrm{G}(\theta)$ is the gravity vector and τ_i is the command torque vector [9]. "Equation (1)" is derived and simulated in Matlab/Simulink. The joints' position angles θ_0 , θ_1 , θ_2 and θ_3 are obtained through experiments. For example, the initial conditions are chosen as follows, $\theta_0 = 0°$, $\theta_1 = 20°$, $\theta_2 = 30°$ and $\theta_3 = 40°$. For this case, position angles of the four joints of the RTIP are shown in "Fig. 3". In order to verify the simulation results obtained from equation 1, a Mechanical-Sim model of the RTIP is developed on Matlab. The output results from the Mechanical-Sim model match exactly those obtained from the Simulink simulation. In the dynamic model, some parameters like body masses, inertia, and lengths of the pendulums can be directly measured. However, the friction coefficients should be determined experimentally to have the most accurate dynamic model of the RTIP.

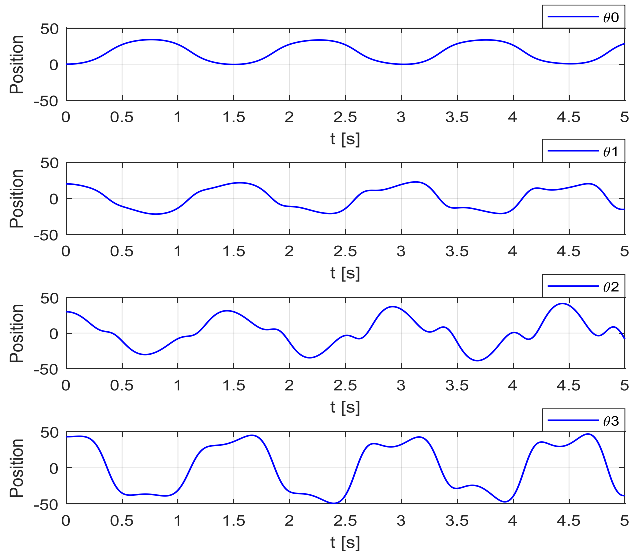

Figure 3. Positions of 4 joints of the TRIP

III. FRICTION ESTIMATION MODELS

Most of existing scientific papers in the field of friction estimation for inverted pendulums use an equivalent viscous friction as the overall friction [10]. In this section, three models of friction estimation will be studied. The first model is a non-conservative friction model that is used to estimate the natural damping friction coefficient (viscous friction) in the joint of each pendulum. The second one is a linear model, used to estimate Coulomb and viscous friction coefficients. The last model is a non-linear model including five parameters; the zero-drift error of the friction torque, Coulomb friction coefficient, viscous friction coefficient, and the experimental friction coefficients.

A. Non-Conservative Friction Estimation Model

This model is based on non-conservative torques estimation due to friction in the Rotary Inverted Pendulum. The friction in the joints of the pendulums is well modeled by a damping constant (viscous friction) [11]. The non-conservative friction torque in the pendulum joints is given as follows:

$$\tau_d = \frac{d\,Đ(\theta_i)}{d\theta_i} = \frac{d}{d\theta_i}\left(\frac{1}{2}\overline{C_p}\dot{\theta}_i^{\,2}\right) = \overline{C_p}\dot{\theta}_i \qquad (2)$$

where $Đ(\theta_i)$ is the Rayleigh's dissipation function and $\overline{C_p}$ is the damping constant [12].

B. Linear Friction Estimation Model

The linear friction F_L in the inverted pendulum joints are considered as a combination of viscous F_V and Coulomb frictions F_C [13].

$$F_L = F_C + F_V \qquad (3)$$

The viscous friction is proportional to the angular velocity $\dot{\theta}_i$ and is given as follows:

$$F_V = B_i\,\dot{\theta}_i \qquad (4)$$

where B_i is the constant coefficient. The Coulomb friction is related to the normal force N_f [14] that is derived as follows:

$$N_f = m\omega^2 l + mg\cos(\theta) \tag{5}$$

where ω is the angular velocity, m is the pendulum mass and l is the distance from the pendulum rotation center to the mass center.

$$F_C = C_i \,\mathrm{sgn}(\dot{\theta}_i).(\mathrm{ml}\,\dot{\theta}_i^{\,2} + \mathrm{mg}\cos(\theta_i)) \tag{6}$$

where C_i are the dynamic friction coefficients and $\mathrm{sgn}(.)$is the signum function. Thus, the linear friction F_L expression is:

$$F_L = B_i\,\dot{\theta}_i + C_i\,\mathrm{sgn}(\dot{\theta}_i).(\mathrm{ml}\,\dot{\theta}_i^{\,2} + \mathrm{mg}\cos(\theta_i)) \tag{7}$$

The friction resisting moment due to the rotation may be calculated by using “equation (8)”

$$f = l\left(B_i\,\dot{\theta}_i + C_i\,sgn(\dot{\theta}_i).(ml\,\dot{\theta}_i^{\,2} + mg\cos(\theta_i))\right) \tag{8}$$

C. Non-Linear Friction Estimation Model

The non-linear friction estimation model is a more advantageous description of the joints’ friction because it contains five types of different friction coefficients [15]. It can be defined by a nonlinear equation:

$$\tau_f = f_o + f_c\,\mathrm{sgn}(\dot{\theta}_i) + f_v\,(\dot{\theta}_i) + f_a\mathrm{atan}(f_b\,\dot{\theta}_i) \tag{9}$$

where f_o is the zero-drift error of the friction torque, f_c is the Coulomb friction coefficient, f_v is the viscous friction coefficient, f_a and f_b are the experimental friction coefficients. Also, $\dot{\theta}_i$ is the angular velocity, $\mathrm{sgn}(.)$is the signum function and atan is the arctangent function.

IV. Experiments

A. Data Collection

The RTIP developed in our laboratory is shown in “Fig. 1”. The horizontal arm of the pendulum is driven by an AC servomotor with a 10-ratio gearbox (quasi-direct drive). The arm angle θ_0 is measured with the motor encoder which has a resolution of 2048 pulses per revolution. The pendulums angles θ_1, θ_2 and θ_3 are measured with three encoders that have also the same resolution. The encoder signals are passed through the slip ring mounted in the joints. To receive the angles’ signals from the encoders, a dSPACE controller is used. The resultant data is collected through the angle of the horizontal arm. $\theta_0$is fixed at 0 degrees, then at t = 0 seconds, the initial positions of the pendulums θ_1, θ_2 and θ_3 are equal to 45 degrees. All of them have the same sampling interval 1 ms. The experimental simulation time of θ_1, θ_2 and θ_3 is taken at t=80 seconds. The experimental hardware configuration is shown in “Fig. 4”.

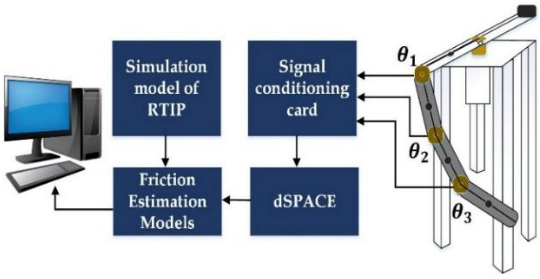

Figure 4. The experimental hardware configuration

B. Estimation results

The estimated results of the friction coefficients for the Non-Conservative Friction Model (NCFM) are given in Table I. The Gradient Descent (GD) method is selected for the current optimization case. This method is based on a sequential quadratic programming (SQP) algorithm to estimate the viscous friction coefficients $C_{p\,(i)}$ [16], which minimizes the value of the function $e = \|\theta_i(t) - \hat{\theta}_i(t)\|$ where $\theta_i(t)$ is the position value of the angles obtained experimentally and $\hat{\theta}_i(t)$ is the position value of the angles obtained from the mathematical model of The RTIP [17]. The experiments are carried out during 80s, however, for the graphs’ clarity, only the [0, 10s] intervals are shown.

TABLE I. Estimation Results For NCFM

friction coefficients	The joints of pendulums		
	Joint (1)	Joint (2)	Joint (3)
C_p [Nm.s/rad]	5.6178e-04	2.9319e-10	9.0673e-04

“Fig. 5” presents the experiments’ position signals obtained from the dSPACE controller, and the simulation signals, with the coefficients of the non-conservative friction estimation for the joints of the RTIP.

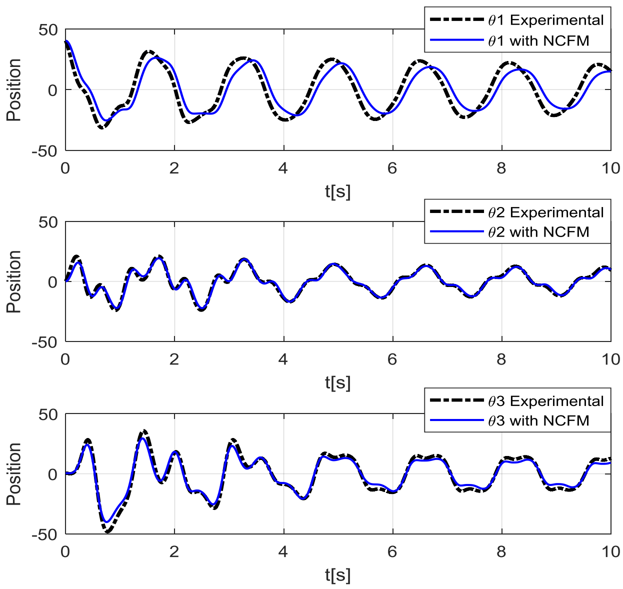

Figure 5. Experimental position signals and the NCFM simulation results

To estimate the friction coefficients of the Linear Friction Model (LFM) and the Non Linear Friction Model (NLFM), the Pattern Search (PS) method is selected for optimization. The PS algorithm starts by calculating a sequence of points that may or may not reach the optimal value. The PS proceeds by creating a group of points around the given initial point, called mesh. If a point in the mesh is found to improve the estimation of the experiment's output at that current point, the algorithm sets the new point as the current point at the next iteration [18]. The estimation's results of the linear friction model and the non-linear friction model are presented in Table II and III.

TABLE II. ESTIMATION RESULTS FOR LFM

Friction coefficients	The joints of pendulums		
	Joint (1)	Joint (2)	Joint (3)
B_i [Nm.s/rad]	6.1865e-04	3.1009e-07	2.2292e-04
C_i [Nm]	2.7550e-05	4.9864e-09	0.0168

"Fig. 6" shows the experiments' position signals obtained from the dSPACE controller and the signals with the coefficients of the linear friction estimation for the joints of the RTIP.

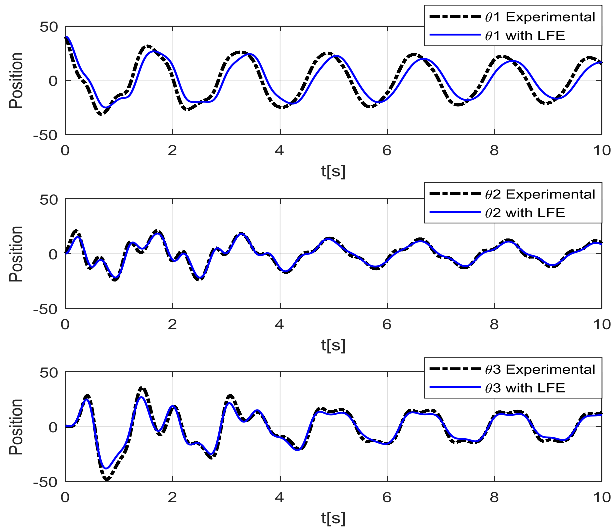

Figure 6. Experimental position signals and the NLFM simulation results

TABLE III. ESTIMATION RESULTS FOR NLFM

friction coefficients	The joints of pendulums		
	Joint (1)	Joint (2)	Joint (3)
f_o [Nm]	0.0038	1.5280e-06	0.001
f_c [Nm]	9.5940e-04	8.8846e-04	0.0165
f_v [Nm.s/rad]	0.0011	0.0315	0.0577
f_a [Nm]	0.0869	0.1876	7.2715e-04
f_b [Nm]	0.0159	0.1876	0.0456

"Fig. 7" illustrates the experiment's position signals obtained from the dSPACE controller and the signals with the coefficients of the non-linear friction estimation for the joints of the RTIP.

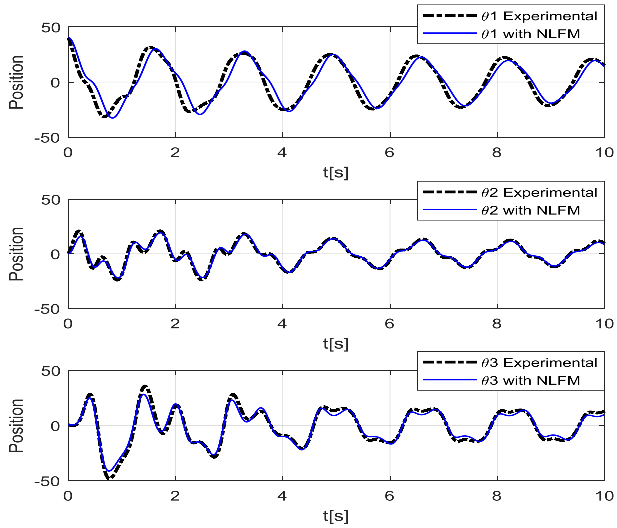

Figure 7. Experimental position signals and the NLFM simulation results

To evaluate the performance of the LFM, NLFM and NCFM, the Root Mean Squared Error (RMSE) between the modeled signals θ_i and the measured signal $\hat{\theta}_i$ is calculated based on the following equation (10). The RMSEs are given in table IV. [19]

$$\mathrm{RMSE} = \sqrt{\frac{1}{\mathrm{N}}\sum_{\mathrm{i=1}}^{\mathrm{N}}(\theta_{\mathrm{i}} - \hat{\theta}_i)^2} \tag{10}$$

TABLE IV. RMSE BETWEEN θ_i AND $\hat{\theta}_i$ FOR LFM, NLFM AND NCFM

Joints	RMSE (°)		
	NCFM	LFM	NLFM
Joint (1)	0.0052	0.0049	0.0025
Joint (2)	0.0071	0.0065	0.0047
Joint (3)	0.0085	0.0079	0.0035

In order to understand the dynamic friction behaviors in the RTIP, the friction forces and velocities in each joint are given in "Fig. 8". The nonlinear relationship between the calculated friction forces and the joint velocities may be observed in this figure. This relationship should be explained with more complex models for an accurate friction estimation.

V. CONCLUSION

In this paper, the performances of three different friction estimation models (Non-conservative, Linear and Non- Linear) are compared in terms of RMSEs of joints of the RTIP. Based on the performance comparison, the NLFM produces the least RMSE in the results for all joints of the RTIP. The RMSE of LFM becomes less than that of the NCFM. In future work, a better friction estimation model needs to be enhanced in the control of the complex robotic systems such as an adaptive friction estimation model, which is developed using the joint velocities and accelerations of the RTIP.

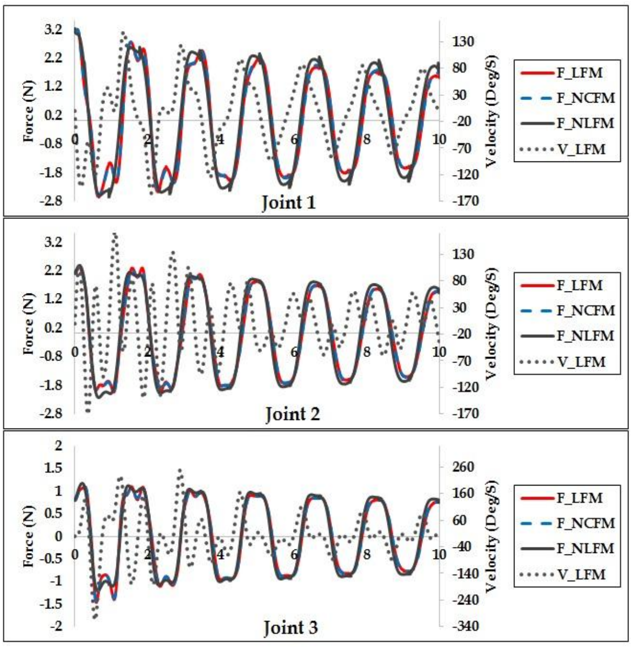

Figure 8. Friction forces and the velocity in each joint of the RTIP

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