Switching Navigation for a Fleet of Mobile Robots in Multi-Obstacle Regions

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***Abstract*—This paper presents a collision-free navigation control algorithm for a fleet of mobile robots to traverse an unknown obstacle-ridden environment. The proposed leader–follower-based navigation rule guarantees that the robots maintain the minimum-allowed distance from the obstacles while in motion. In this algorithm, the leader plans the safest path based on the information received in each switching period, which results in fewer computations and fast motion. Computer simulation performed confirms the reliability and robustness of the proposed algorithm.**

***Index Terms*—navigation, mobile robot, obstacle avoidance, decision-making algorithm, coverage control**

I. Introduction

Multi-robot networks have been increasingly utilized in fields with complicated situations, such as military, security and surveillance, agriculture, healthcare, and hazardous and toxic environments [1], [2], [3], [4]. A robust and reliable formation control strategy and path planning of a mobile robot network is required for leading a team of mobile robots to find the safest path, while avoiding collisions among the robots as well as between a robot and any obstacles in the area [5], [6].

There are two general categories of path-planning and formation control strategy, namely, global and local formation control strategies. Global formation control strategy is derived from the prior information of the region, whereas local formation control strategy denotes path planning based on real-time information provided by the onboard sensors in the area of interest [7]. Researchers have proposed different formation and path-planning control strategies such as the biological-reaction control method [8], [9], the leader–follower strategy [10], [11], the swarm and flocking algorithm [12], [13], and the behavior-based control strategy [14].

In this work, we propose a leader–follower-based formation control algorithm for a group of mobile robots, wherein the robots are required to form a line for avoiding collisions among teammates as well as with any obstacle in the region.

In this method, the leader collects the information with its onboard sensors from the region and transmits the information to its closest neighbor. It becomes the duty of each teammate to continuously transfer the received information to the closest neighbor in a single-strand chain communication network.

The leader measures the distance between the obstacle and itself in a finite number of directions [15] and updates its heading toward the safest path, which is the one that satisfies the minimum-allowed distance criterion from the obstacle. Every teammate updates its heading based on the information received from its closest neighbor through the single-strand chain communication network. Consequently, the need for continuous measurement and computation of distance, velocity, or repulsive force are not required.

The position estimation switching algorithm is presented in [16]. The validity and robustness of the proposed method was confirmed via mathematical analysis and simulation. The simplicity of the computation involved results in the method being practical; however, it works only when there is a single obstacle in the region of interest. We provide a rigorous mathematical analysis of the proposed algorithm with a proof of states. Furthermore, the computer simulation performed confirms the validity and reliability of the proposed algorithm.

The remaining part of this paper is organized as follows: Section II gives a brief explanation of the related works; in Section III, we present the problem formulation and mathematical analysis; Section IV presents simulation and results; and finally, Section V presents a brief conclusion.

II. Related Works

Virtual structure strategy was applied previously [17] to propose an algorithm that can be used to navigate a multi-robot team in a bounded region. The result confirms the robustness of the proposed algorithm. However, this algorithm requires continuous measurement to establish a robust relationship between the robots.

A behavioral structure was proposed previously [18], [19], [20] for obstacle avoidance. The model assigns a

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control strategy to each team member to maintain its heading in alignment with the orientation of the field's vector. In this algorithm, each agent needs to be prescribed a desired behavior to avoid obstacles. Furthermore, the control algorithm ignores the collision among robots. An artificial potential field algorithm was proposed previously [21] and [22], wherein each obstacle is considered a source of repulsive force whereas the target is considered a source of attractive force. An advantage of this algorithm is that its mathematical model is simple. However, the difficulty in identifying the repulsive force is a disadvantage of this method. A mixed-integer linear-programming-based control algorithm to ensure successful obstacle avoidance was proposed [23]; however, the nondeterministic polynomial-time problem that is used to find a safe path causes the computational complexity to be high.

A mathematically rigorous navigation strategy was proposed by Matveev et al. [24], [25]. Despite the navigation model being feasible, it requires predetermining the distance for each vehicle rather than the entire multi-robot team.

III. Problem Formulation

In this section, we present the dynamic model of the robot and the algorithm for a safe navigation in an unknown area with several multiple convex obstacles.

A. Dynamic Model of Sensors

We consider $x_i(t), y_i(t)$ as the Cartesian coordinates and $\theta_i(t)$ as the polar angle of the robot i in the plane. The heading $\theta_i(t) \in (0,2\pi]$ is measured in an anti-clockwise direction along the x axis of a reference frame that is attached to the robot leader. Furthermore, $v_i(t)$ and $\omega_i(t)$ stand for the linear velocity and angular velocity of the robot i, respectively. The following condition must be satisfied:

$$v_{min} < v_i(t) < v_{max} \tag{1}$$

Then, we define a controlled variable U_i that is a function of robots' polar angles and the distances of the robots with the obstacles at time t:

$$U_i = \left[D_{r,t}^{O_j}, \Theta_{r,t}^{O_j}\right] \tag{2}$$

where $D_{r,t}^{O_j}$ denotes a set of all measured Euclidean distances between all agents and the obstacles and $\Theta_{r,t}^{O_j}$ denotes a set of headings concerning the obstacles conveyed by the leader to its closest neighbor, which in turn communicates the same to the closest neighbor in a single-strand chain communication network along the x axis.

B. Navigation Algorithm

The main objective of this study is to develop a robust and reliable algorithm for a team of mobile robots to move smoothly within a region, avoiding static convex obstacles and collision among teammates. The navigation algorithm follows a leader–follower pattern, implying that the team leader finds the safest path based on the proposed navigation law and the rest of the teammates follow the path planned by the leader.

C. Structure of the Navigation Algorithm

The robots are assumed to be initially distributed randomly within the area. The first step is to line up the team members in a queue in order of distance η_i between it and its closest neighbor with respect to the leader.

Each team member is supposed to update their heading and next position at time t with reference to the agent immediately ahead of it. Obviously, the robots' motion has the Markov property as it depends only on the current time.

Each robot detects any object in the region within its sensing range within a radius R. For the sake of simplicity, we considered the sensing range of the robot as a perfect disk shape; however, the hardware and the environmental factors result in an imperfect disk shape in reality [26].

Assumption 1: We suppose that a team of multi-robot x_{ri}, where $i = 1,2,\dots,n$, were randomly deployed in the area of interest. We allocate index 1 to the team leader, x_{r1}, that is located at point p_{r1}. We consider a point $p_{k^*}^{j^*}$ in the outer face of the obstacle $j^* \in j$, where

$$dist\left(p_{r1}, p_{k^*}^{j^*}\right) = \inf\left(dist\left(p_{ri|_{i\neq 1}}, p_k^j\right)\right), k^* \in k \tag{3}$$

In Equation (3), k denotes a set of finite numbers of all visible points of the outer face of the obstacles to the sensors at time t.

We define a set of angles $\Gamma = (\gamma_1, \gamma_2, \dots, \gamma_m)$, each subtended by the maximum visible curve lies between points $\wp_1^j, \wp_2^j$ of each obstacle to the team leader. Then, we define two lines ℓ_1^j and ℓ_2^j from the point p_{r1} of the leader to the points $\wp_1^j, \wp_2^j$ of each obstacle as follows:

$$\begin{cases} \ell_1^j = inf(\| p_{r1} - \wp_1^j \|) \\ \ell_2^j = inf(\| p_{r1} - \wp_2^j \|) \end{cases} \forall\, j = 1,2,\dots,m$$

and

$$\measuredangle \ell_2^{j'} \ell_1^{j''} = \gamma_{m'} \; for \; \gamma_{m'} \in \Gamma, m' = 1,2,\dots,m \tag{5}$$

The points $\wp_1^j, \wp_2^j$ are assumed to be connected by a line $\mathcal{L}_{1j}$, which holds the following condition:

$$\mathcal{L}_{1j} = inf(\| \wp_1^j - \wp_2^j \|) \tag{6}$$

and the set of lines $(\mathcal{L}_{2j}, \mathcal{L}_{3j})$ are perpendicular to $\mathcal{L}_{1j}$ as follows:

$$\begin{cases} \overline{\mathcal{L}_{2j}} = \varepsilon \text{ and } \overline{\mathcal{L}_{3j}} = 2\varepsilon \\ \mathcal{L}_{2j} \perp \mathcal{L}_{1j} \text{ at } \wp_3^j \epsilon \mathcal{L}_{1j} \\ \mathcal{L}_{3j} \perp \mathcal{L}_{1j} \text{ at } \wp_2^j \epsilon \mathcal{L}_{1j} \\ dist(\wp_1^j, \wp_3^j) = dist(\wp_2^j, \wp_3^j) = {}^{\mathcal{L}_{1j}}/_{2} \end{cases} \quad (7)$$

As shown in Fig. 1, $\mathcal{L}_{1j}$, $\mathcal{L}_{3j}$,and $\mathcal{L}_{4j}$ represent the catheti and the hypotenuse of the right-angle triangle $\varDelta \wp_1^j \wp_2^j \wp_5^j$, where

$$\alpha_j = tan^{-1} \frac{\mathcal{L}_{3j}}{\mathcal{L}_{1j}} \quad (8)$$

Definition 1: We consider $\mathfrak{D} = (d^{12}, d^{23}, \ldots, d^{(m-1)\,m})$ as a set of all minimum distances between the $\wp_1^{j'}$ and $\wp_3^{j''}$ of any two obstacles j' and j'' in anti-clockwise direction. The robots can traverse between any two obstacles within a distance $d^{(k-1)\,k} \in \mathfrak{D}_e \subseteq \mathfrak{D}$ if and only if

$$\frac{d^{(k-1)k}}{2r} \gg 1 + \varepsilon \quad (9)$$

Equation (10) shows that the leader has different options to choose from when $m > 2$, which leads to more computation, more energy consumption, and bewilderment unless the leader chooses a $d^{(\tilde{k}-1)\,\tilde{k}}$, which satisfies the following condition:

$$d^{(\tilde{k}-1)\,\tilde{k}} = sup\big(d^{(k-1)k} \in \mathfrak{D}_e\big), k = 1,2, \ldots, m \quad (10)$$

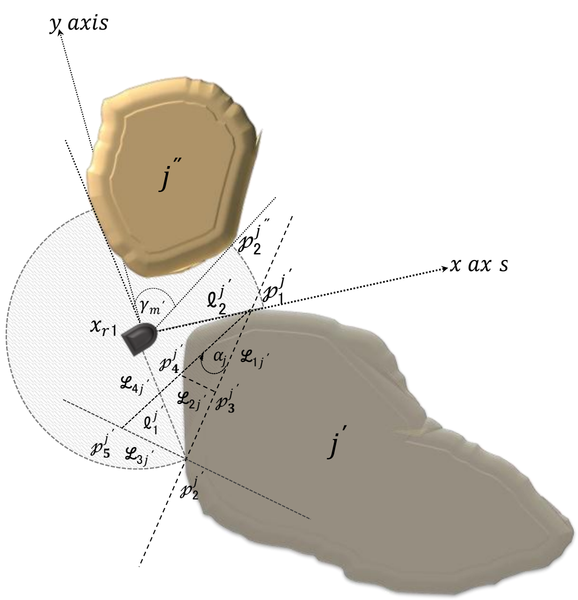

Figure 1. Measuring distances by the leader.

Proposition 1: If the robot x_{r1} starts moving from the location $\mathcal{O}_1$ toward the location $\mathcal{O}_2$ with two obstacles j' and j'' in between, where $\left(p_{r1}, p_{k^*}^{j'}\right)|_{\mathcal{O}_1} < dist\left(p_{r1}, p_{k^{**}}^{j''}\right)|_{\mathcal{O}_1}$ in the direction of the vector $\overrightarrow{\tilde{\mathcal{L}}_\iota}$as in

$$\tilde{\mathcal{L}}_i = \frac{1}{2}\sqrt{\mathcal{L}_{1j'}{}^2 + \mathcal{L}_{1j''}{}^2 + 2\mathcal{L}_{1j'}\mathcal{L}_{1j''}cos(\alpha_1 + \alpha_2)} \quad (11)$$

and the length of $\tilde{\ell}_\iota$ as follows:

$$\tilde{\ell}_\iota = inf\left(\frac{\mathcal{L}_{1j'}}{2}, \frac{\mathcal{L}_{1j''}}{2}\right) \quad (12)$$

Based on the $\ell_1^{j'}$, which is considered as the *x* axis of the attached frame to the leader robot, $inf\left(dist\left(p_{r1}, p_{k^*}^{j'}\right)|_{\mathcal{O}_2}, dist\left(p_{r1}, p_{k^{**}}^{j''}\right)|_{\mathcal{O}_2}\right) \geq \varepsilon - \mu_0$. (13)

In Equation (14), $\mu_0$represents the tolerance of allowed distance result from the measurement error.

Proof: As shown Fig. 2, if the robot moves from location $\mathcal{O}_1$ to the location $\mathcal{O}_3$, with the heading α_1 based on $\ell_1^{j'}$and the length of $\frac{\mathcal{L}_{1j'}}{2}$, the distance between the robot and the obstacle j' is more than ε. In the same way, if the leader moves to the location $\mathcal{O}_4$ with the heading $\delta_{j'j''} - \alpha_2$ based on $\ell_1^{j'}$and the length $\frac{\mathcal{L}_{1j''}}{2}$, the minimum-allowed distance between the robot and the obstacle j"would be satisfied.

The vector $\overrightarrow{\tilde{\mathcal{L}}_\iota}$ denotes the resultant vector of $\vec{l}_i^{j'}$ and $\vec{l}_i^{j''}$ (see Fig. 2).

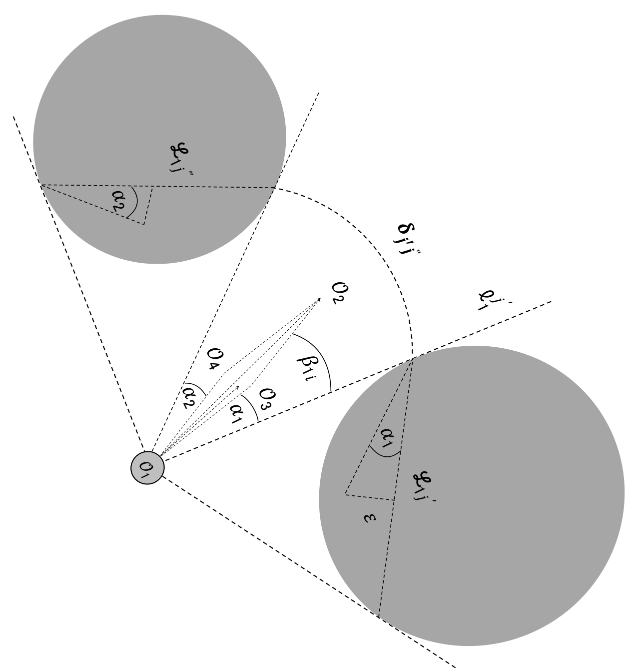

Figure 2. Computation of the switching position.

Consequently, we find the angle β_{1i} while Equation (10) holds:

$$\beta_{1i} = tan^{-1}\left(\frac{\mathcal{L}_{1j'} \sin\alpha_1 + \mathcal{L}_{1j''} \sin\alpha_2}{\mathcal{L}_{1j'} \cos\alpha_1 + \mathcal{L}_{1j''} \cos\alpha_2}\right); \qquad (14)$$

therefore,

$$\alpha_1 \leq \beta_{1i} \leq \delta_{j'j''} - \alpha_2 \qquad (15)$$

and Equation (12) guarantees that the leader would settle at location $\mathcal{O}_2$ while satisfying Equation (13).

Since the safest path is planned by the leader, each teammate updates its own heading and distance based on the information received from the robot preceding it to avoid any consequent collision. The algorithm for performing the updates for each follower is defined as follows:

$$\beta_{ki} = tan^{-1}\left(\frac{y_{(k-1)i} - y_{ki}}{x_{(k-1)i} - x_{ki}}\right) \qquad (16)$$

Furthermore, each robot updates the maximum specified distance between its current position and the position of the robot ahead using

$$dist_{ki}\left(x_{kr}, x_{(k+1)r}\right) = \| \mathcal{O}_{(k-1)i} - \mathcal{O}_{ki} \| \qquad (17)$$

where the index k denotes the number of robots while the index i denotes the permutation of each robot at each step.

IV. Simulation and Discussion

Fig. 3 shows how the multi-robot team moves in the region while avoiding the obstacles. We consider a team of 5 pointwise robots and the first robot assigned is regarded as the leader of the group.

The tolerance is considered as $\mu_0 = 0.1\varepsilon$. where $\varepsilon = 0.1$ meters. Obviously, the leader can choose the safest path in the area by applying the navigation rules (11) and (12); other teammates can as well do same by following the leader.

Furthermore, it is not possible to experience collision when the agents update their heading and distances based on the updating rules (16) and (17).

In Fig. 4, each graph represents the distance between the leader and each obstacle in the region. As Fig. 4 confirms, the distance between the leader and the obstacles never exceeded the minimum-allowed distance except in step 11, wherein a difference of 0.08 meters is an acceptable tolerance value for μ_0.

According to Table I, the robots update their headings and distances in just 11 switching steps in a non-continuous manner that results in a fast motion. Furthermore, the leader has the minimum distance from the obstacle 2 in switching step 11, which is $0.82\, m \times 10^{-1}$; it satisfies rule (14) and confirms the validity and reliability of the developed algorithm for avoiding multiple obstacles in a region by a team of multiple robots.

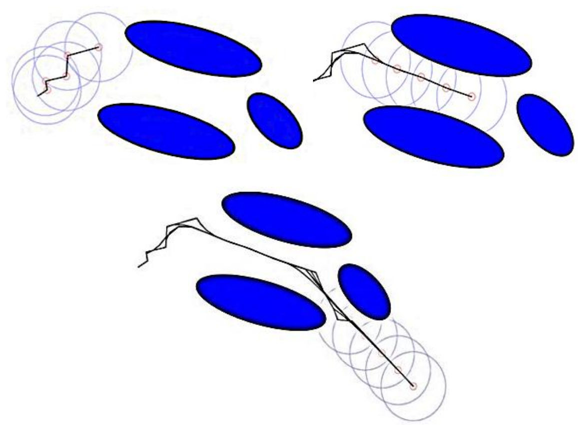

Figure 3. Obstacle avoidance by the team of mobile robots.

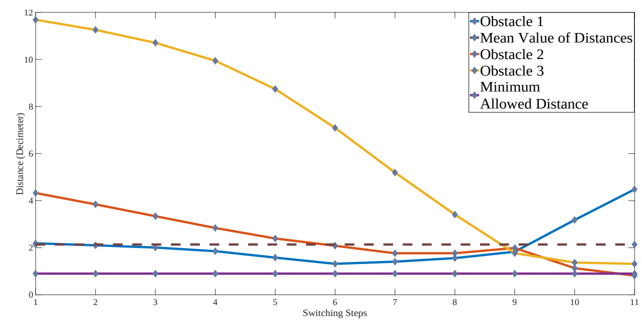

Figure 4. Distance of the leader and the obstacles $(m \times 10^{-1})$.

TABLE I. Distances from the Obstacles in Each Switching Steps $(m \times 10^{-1})$

Switching steps	Distance from Obstacle 1	Distance from Obstacle 2	Distance from Obstacle 3
1	11.68	4.33	2.19
2	11.25	3.84	2.1
3	10.71	3.34	2.01
4	9.94	2.84	1.85
5	8.74	2.39	1.58
6	7.09	2.08	1.32
7	5.19	1.76	1.41
8	3.41	1.77	1.56
9	1.77	1.99	1.83
10	1.37	1.13	3.18
11	1.32	0.82	4.48

V. Conclusion

In this study, we considered a group of robots in a region occupied by convex obstacles. The obstacles were assumed to be smooth and static. The mission of the team of mobile robots was to traverse the area while avoiding the obstacles. We developed a novel algorithm based on the leader–follower concept, wherein the leader is the only agent that plans the safest path and navigates the other team members around the area of interest in just a few switching steps. Each follower needs to update its heading and distance based on the information received from its nearest neighbour.

Both mathematical analysis and simulation results confirm the robustness and validity of the proposed algorithm. As discussed in Section IV, the navigation rules (12) and (13) guarantee that the multi-robot team moved through the path that was planned by the leader while avoiding static obstacles in the region with minimum computation. Furthermore, updating rules (17) and (18) ensure a collision-free motion by the team members.

Future work would focus on performing this study in a dynamic non-convex environment. Furthermore, according to the structure of the proposed algorithm, information gathering, data processing, and path planning take place at distinct time intervals by the leader, which communicates relevant information to other team members. As a result, sensors and the processors do not need to perform data capturing and processing continuously, thus resulting in energy saving by the multi-robot team: this can also be considered potential future work.

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