Multivariable Control System of Vertical Motion for an Unmanned Underwater Vehicle with Interval Parameters

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***Abstract*—Sophisticated unmanned underwater vehicles require automatic motion control systems. Common approach to synthesis of such systems does not consider parametric uncertainty of underwater vehicles and nonlinearity of some of their elements. Also, the interaction between manipulated variables is not commonly considered. The paper is dedicated to mathematical modeling of a multivariable control system for an unmanned underwater vehicle of a vertical motion, considering its interval parametric uncertainty. The research resulted into a tridimensional mathematical model of the underwater vehicle motion in a vertical plane; a model of a propulsion and steering system of the underwater vehicle and the method of robust controller parametric synthesis for such systems.**

***Index Terms*—underwater vehicle, motion control, vertical motion, robust control, parametric uncertainty, multivariable system**

I. Introduction

Unmanned underwater vehicles (UUVs) of various classes and constructions are applied to solve a vast variety of tasks: research, industrial, defense issues and others. In order to perform its mission, UUVs move in all six degrees of freedom. Manipulating such a motion requires to consider the interaction of UUVs motion parameters; also, UUV, as a control object, is known for its parametric uncertainty and nonlinearity. This poses a highly relevant problem of developing multivariable motion control systems (MCSs) for UUV. To manipulate such objects, some of common approaches can be applied: adaptive control [1-7], fuzzy-logic-based control [8-10], and neural network-based control [11-12]. These approaches have some significant disadvantages, among them are: inability to process rapid parametric variations in wide ranges, complexity of synthesis procedures, a low operating speed compared to simple controllers with the constant parameters. Considering these features of these common approaches, a robust approach to control, which allows to manipulate objects with parametric uncertainty via simple controllers with the constant parameters, is proposed to be used for synthesizing a MCS.

In order to apply this approach, a mathematical model of MCS with the interval parameters must be derived. Existing models have some common disadvantages. Among them are ignoring the interaction between manipulated parameters of UUV motion; assuming angles of attack and drift to be equal to zero; ignoring the uncertainty of hydrodynamic characteristics of UUV; ignoring nonlinearities of a UUV as a control object.

In the paper it is proposed to derive the mathematical model, which will eliminate all these disadvantages and will allow to manipulate a control object with simple linear controllers.

The paper to be considered is dedicated to mathematical modeling of a robust vertical motion control system (VMCS), which would manipulate three parameters: x and y coordinates of UUV and its trim; considering the interval parametric uncertainty and nonlinearity of UUV. To reach the aim, a set of objectives is to be accomplished:

– development of a tridimensional mathematical model of UUV motion in a vertical plane, considering UUV nonlinearities;
– interval linearization of the model;
– introducing the interval parameters;
– development of VMSC structure.

II. Deriving Mathematical Model of UUV Motion in a Vertical Plane

UUV motion in a vertical-longitudinal plane can be considered as the motion in three degrees of freedom: rotation around z axis, which causes UUV trim variation; motion along x axis; motion along y axis – UUV submerging. During derivation of the mathematical model of such a motion it is necessary to consider the interaction of these three degrees of freedom; also, the nonlinearity and the uncertainty of UUV thrusters transfer coefficient and the uncertainty of UUV hydrodynamic characteristics must be considered in the model.

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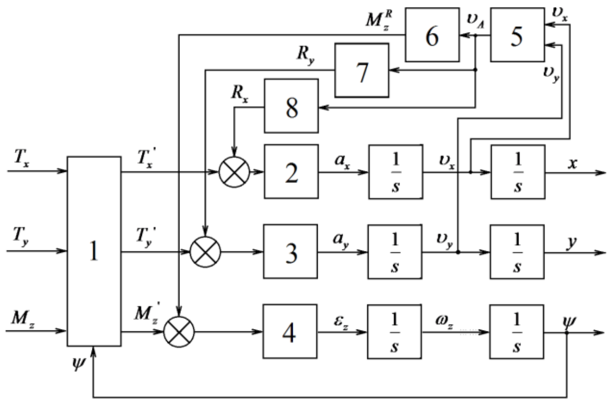

Figure 1. Structure of the proposed model for UUV motion in a vertical plane

The structure of the proposed model, which corresponds to all requirements, listed above, is shown in Fig. 1. The model describes the relations between propulsive forces of horizontal thrusters T_x, vertical thrusters T_y and a propulsive moment of vertical thrusters M_z and UUV coordinates along *x* and *y* axes and UUV trim ψ.

Input variables of the model arrive at block 1, which calculates some projections of propulsive forces on x and y axes and adds a trimming moment to a propulsive moment of horizontal thrusters:

$$\begin{aligned} T_x^{'} &= T_x \cdot \cos(\psi) + T_y \cdot \sin(\psi); \\ T_y^{'} &= T_x \cdot \sin(\psi) + T_y \cdot \cos(\psi); \\ M_z^{'} &= M_z + T_x \cdot h_1, \end{aligned} \tag{1}$$

here h_1 – the distance between *x* axis and T_x the line of action.

Blocks 2, 3 and 4 calculate linear and angular accelerations of UUV on the base of the propulsive and hydrodynamic forces and moments:

$$\begin{aligned} a_x &= \frac{T_x^{'} + R_x}{m + \lambda_{11}}; \\ a_y &= \frac{T_y^{'} + R_y}{m + \lambda_{22}}; \\ \varepsilon_y &= \frac{M_z^{'} + M_z^{R}}{J_z + \lambda_{66}}, \end{aligned}$$

here a_x, a_y – UUV acceleration along *x* and *y* axes; ε_z – angular acceleration of UUV around *z* axis; R_x – a drag force; R_y – a lift force; M_z^{R} – a hydrodynamic moment of water resistance to UUV rotation around *z* axis; m – UUV weight; J_z – UUV hull moment of inertia around *z* axis; λ_{ij} – added masses of water. By integrating these accelerations, the projections of UUV velocity on *x* and *y* axes can be calculated. On the base of velocity projections, its absolute value can be found in the block 5:

$$\upsilon_A = \sqrt{\upsilon_x^{2} + \upsilon_y^{2}}. \tag{2}$$

Considering an absolute value of UUV velocity as the absolute value of the water flow around UUV, the hydrodynamic forces and moments can be calculated in blocks 6, 7 and 8:

$$\begin{aligned} R_x &= \frac{1}{2} \cdot c_x \cdot \rho \cdot V^{2/3} \cdot \upsilon_A^{2} \cdot \left[-sign(T_x)\right]; \\ R_y &= \frac{1}{2} \cdot c_y \cdot \rho \cdot V^{2/3} \cdot \upsilon_A^{2} \cdot \left[-sign(T_y)\right]; \\ M_z^{R} &= \frac{1}{2} \cdot m_z \cdot \rho \cdot V \cdot \upsilon_A^{2} \cdot \left[-sign(M_z)\right], \end{aligned} \tag{3}$$

here c_x, c_y, m_z – the hydrodynamic coefficients; ρ – the water density; V – UUV displacement; υ_A – UUV motion velocity. The values of the hydrodynamic forces and moments are used to close feedbacks through the junction points before blocks 2, 3 and 4.

III. Deriving Mathematical Model of Propulsion and Steering System of UUV

In order to derive the mathematical model of UUV VMCS, besides the model of a control object – the model of UUV motion in a vertical plane, it is necessary to derive the model of system's actuator – a propulsion and steering system (PSS). The model of UUV PSS must calculate propulsion forces and moments, provided by thrusters, on the base of the control signals coming from controllers.

The basic elements of UUV PSS – a thruster – will be described with an aperiodic block of the first order. The transfer coefficient of this block will be calculated, considering that the input signal varies from $[-100;100]$. Considering a saturation nonlinearity of the thruster, then, its model can be written as follows:

$$T(t) = \begin{cases} L^{-1}\left[\dfrac{0.01 \cdot T_{\max}}{T_{\partial} \cdot s + 1} \cdot \dfrac{u}{s}\right], |u| \le 100 \\ sign(u) \cdot T_{\max}, |u| > 100 \end{cases},$$

here u – the control signal value; T – the propulsive force, generated by a thruster; $T_{\max}$ – the maximal propulsive force of the thruster; T_t – the time constant of a thruster. Necessary forces can be calculated by multiplying output forces of the thrusters by a number of thrusters, manipulating each motion parameter; moments can be calculated by multiplying output forces of the thrusters by a number of thrusters, manipulating each motion parameter, and the distance between these thrusters and desired axis.

IV. Deriving Model of UUV VMSC with Interval Parametric Uncertainty

A. Interval Linearization and Introduction of Interval Parameters

It is well known that the thrusters can be described with “saturation” nonlinearity. In order to consider this nonlinearity during the synthesis procedure, it is proposed to use an interval linearization method. The method consists of the substitution of a non-linear function with the constant coefficients with a linear function with interval coefficients. The interval coefficients must be chosen to provide inclusion of a nonlinear function domain of values into a domain of values of a function with the interval coefficients. An example of the interval linearization of a square-law function is shown in Fig. 2.

$$\upsilon_x^{\,2} \approx [0;2] \cdot |\upsilon_x|$$

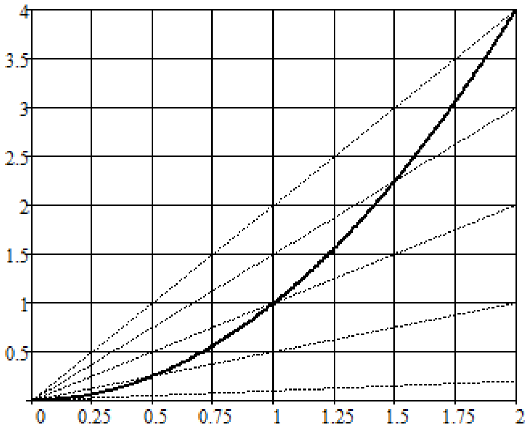

Figure 2. Structure of UUV VMCS

In Fig. 2, a domain of values of the function with the interval coefficient includes a nonlinear function domain of values. Also, considering features of a robust approach to analysis and synthesis of control systems, a controller will be synthesized to provide desired control quality in the worst combinations of the interval parameters values, including a linearization coefficient.

According to the interval linearization method, let us replace the constant transient coefficient with the interval one. The linearized model of a thruster can be written as follows:

$$\mathrm{W}_{\partial}(t) = \frac{0.01 \cdot [0; T_{\max}]}{T_{\partial} \cdot s + 1}.$$

The tridimensional model of UUV motion in a vertical plane, which is shown in fig. 1, includes (2) and (3), which must be linearized. To simplify the model, let us assume, that in (2) υ_A is equal to υ_x. Expression (3) includes the hydrodynamic coefficients, which depend on UUV angles of attack and drift. Considering the uncertainty of these parameters, linearized (3) can be written as follows:

$$R_x = \frac{1}{2} \cdot [c_x] \cdot \rho \cdot V^{2/3} \cdot \upsilon_x \cdot [k_l];$$

$$R_y = \frac{1}{2} \cdot [c_y] \cdot \rho \cdot V^{2/3} \cdot \upsilon_x \cdot [k_l];$$

$$M_z^{\,R} = \frac{1}{2} \cdot [m_z] \cdot \rho \cdot V \cdot \upsilon_x \cdot [k_l],$$

here $k_l = [0; \upsilon_x^{\max}]$ – the interval linearization coefficient, the right border of which is determined by the maximal velocity of UUV motion along *x* axis.

B. Development of System Structure

The mathematical model and the structure of UUV VMCS can be developed on the base of the mathematical model of the control object, the actuator and transfer coefficients of sensors.

The proposed structure, shown in Fig. 2, is the common structure of multivariable control system, built on the base of compensation control principle. Control quality in such systems is determined by controllers in direct connections; the independence of manipulated variables is reached while synthesizing controllers in cross-connections.

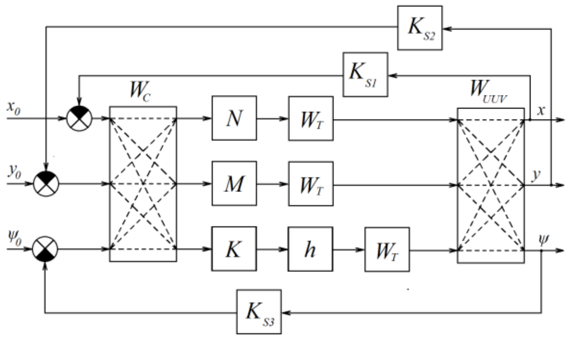

Figure 3. Structure of UUV VMCS

In Fig. 3 the following designations were used: x_0 and x – setpoint and the actual value of x coordinate; y_0 and y – setpoint and the actual value of y coordinate; ψ_0 and ψ – setpoint the actual value of UUV trim; W_C – a tridimensional compensator-controller; W_T – the thruster transfer function; W_{UUV} – the tridimensional mathematical model of a manipulated process – UUV motion in a vertical plane (see fig. 1); N, M, K – a number of the thrusters, manipulating each parameter; h – the thruster moment shoulder; K_{S1}, K_{S2}, K_{S3} – the transfer coefficients of sensors.

In order to provide the desired control quality, despite the interval parametric uncertainty of UUV, it is proposed to synthesize controllers in direct connections with the help of the method, described in [13-14]. This method is based on the root approach to controller synthesis and the system poles domination principle. This allows to determine the control quality of the system by allocating only a small group of dominant poles according to the desired characteristics of a transient process and allocating all other poles far enough from the dominant ones in order to minimize their influence on the system.

The synthesis of controllers in cross-connections is proposed to perform as follows:

1. Derive the transfer functions describing cross-connections between inputs and outputs of the system:

$$\mathrm{W}_{ij}(s,\vec{k},\vec{q}) = \frac{A(s,\vec{k},\vec{q})}{B(s,\vec{k},\vec{q})},$$

here $\vec{k}$ – a vector of controller parameters, $\vec{q}$ – a vector of interval parameters.

2. Derive a relation between a steady state of their step responses and controller parameters:

$$h(\infty,\vec{k},\vec{q}) = \mathrm{W}_{ij}(0,\vec{k},\vec{q}) = \frac{A(0,\vec{k},\vec{q})}{B(0,\vec{k},\vec{q})},$$

here $h(\infty,\vec{k},\vec{q})$ – is a steady state of step response of the cross-connections in the system.

3. Define the maximal value of these steady states and develop the system of inequalities on the base of results obtained due to the previous step.

$$\begin{cases} \mathrm{W}_{0,1}(0,\vec{k},\vec{q}) < h_{0,1}{}^{\max}; \\ \dots \\ \mathrm{W}_{i,j}(0,\vec{k},\vec{q}) < h_{i,j}{}^{\max}; \\ \dots \\ \mathrm{W}_{n,n-1}(0,\vec{k},\vec{q}) < h_{\mathrm{n,n-1}}{}^{\max}; \end{cases},$$
$$i, j \in [1;n], i \neq j$$

here $h_{i,j}{}^{\max}$ – the maximal values of steady states of cross-connection transfer functions step responses; n – a number of the direct control channels of a multivariable system.

4. If quantity of inequalities in the system is not enough to calculate all the parameters of a controller, consider the velocity error and acceleration error and add more inequalities to the system.

$$\begin{cases} \mathrm{W}_{0,1}(0,\vec{k},\vec{q}) < h_{0,1}{}^{\max}; \\ \dots \\ \mathrm{W}_{i,j}(0,\vec{k},\vec{q}) < h_{i,j}{}^{\max}; \\ \dots \\ \mathrm{W}_{n,n-1}(0,\vec{k},\vec{q}) < h_{\mathrm{n,n-1}}{}^{\max}; \\ \left.\dfrac{\partial^k}{\partial s^k}\mathrm{W}_{0,1}(s,\vec{k},\vec{q})\right|_{s=0} < \partial^k h_{0,1}{}^{\max}; \\ \dots \\ \left.\dfrac{\partial^k}{\partial s^k}\mathrm{W}_{i,j}(s,\vec{k},\vec{q})\right|_{s=0} < \partial^k h_{i,j}{}^{\max}; \\ \dots \\ \left.\dfrac{\partial^k}{\partial s^k}\mathrm{W}_{n,n-1}(s,\vec{k},\vec{q})\right|_{s=0} < \partial^k h_{n,n-1}{}^{\max}; \end{cases}$$
$$i, j \in [1;n], i \neq j$$

here $\partial^k h_{i,j}{}^{\max}$ – is the maximal value of steady states of k-th derivatives of cross-connection transfer functions.

5. Solve the system of inequalities.

A robust system, synthesized with the help of described method, will provide the desired quality of UUV motion control in conditions of its parametric uncertainty.

V. Conclusion

The research resulted in the mathematical model of UUV VMCS, considering nonlinearity of UUV elements. The model can be used for VMCS simulation modeling, but it is inapplicable to solving a problem of synthesis. To make the model applicable to UUV VMCS synthesis, the model was linearized with the help of the interval linearization method. Several interval parameters were introduced to the model in order to consider their uncertainty: the angles of attack and drift, the coefficients of the hydrodynamic forces and moments, the transient coefficients of the thrusters. The set of the interval parameters can be easily expanded, according to features of UUV. For example, for UUV with manipulators the mass of UUV and coordinates of the center of masses can be considered as the interval parameter; for UUV with adjustable geometrics added masses of water it can be considered as the interval parameters.

Through the analysis of UUV VMCS features and known approaches to manipulating objects with parametric uncertainty, a synthesis method for robust controllers of the considered system was formulated. The method allows to provide the desired control quality in each control channel of the system and reduce any disturbances, caused by other control channels in conditions of the interval parametric uncertainty.

Future development of VMCS will be dedicated to controller synthesis and simulation modeling of the synthesized system. It is proposed to derive the mathematical model and synthesize MCS, capable to manipulate all six degrees of freedom despite the parametric uncertainty after that the approach under consideration is tested on VMSC..

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