Dynamics Model of a Differential Drive Mobile Robot Towing an Off-axle Trailer

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***Abstract*—Indoor navigation technology has enabled the exploitation of mobile robots for transportation of goods/materials in industry facilities/warehouses. Nevertheless, the deployment of a number of mobile robots segregated to individual tasks may limit the advantages of such technology. The achievement of a solution where a mobile robot capability is enhanced by aggregating passive mechanisms (i.e. passive trailers) represents a more cost effective and versatile solution. Therefore, the analysis of safety and performance of such configuration is a step forward toward the full deployment of this kind of systems. With this work, we attempt to contribute to this field by studying of the effects of system uncertainties, disturbances and changing operating conditions on the behavior of such systems. Specifically, we model the dynamics of a mobile robot towing an off-axle trailer with two wheels configurations and then incorporate the movement of the caster wheels in the dynamics model. Such a model would enable the synthesis of robust control schemes to achieve a better tracking capability and consistent performance in real operating scenarios.**

***Index Terms*—differential drive mobile robots, dynamics, off-axle trailer**

I. INTRODUCTION

With the development of indoor localization and navigation algorithms, the demand for mobile robots to convey goods/materials in industrial facilities/warehouses has increased steadily in recent years [1]. For example, a significant number of mobile robots have been deployed in several warehouses of major e-commerce stakeholders (e.g. Amazon) to support the fast-paced and dynamic e-commerce operations [2].

In most of these mobile robot applications, each mobile robot is designated to perform a single task, thus increasing the operational cost of the overall solution. The alternative of a mobile robot with hooked passive trailers (i.e. tractor-trailer mobile robots) can be more advantageous in terms of system versatility, multitasking capabilities, and operational costs [3]. However, this configuration entails an under-actuated system with non-holonomic kinematics constraints which translate in a very complex motion planning problem. From the standpoint of motion control, the system complexity is further increased in backward motion (i.e. pushing the trailers) due to the unstable nature of the open-loop system. As such, in the last two decades a significant amount of effort has been devoted to developing path planning and motion control strategies for tractor-trailer mobile robots (TTMRs) in forward (i.e. pulling the trailers) and backward configurations [4][5]. In most of these works, the control methodologies have been designed solely based on the kinematics model of the system, and thus, neglecting the effects of heavy loads, changing operating conditions, and uncertain dynamics in the performance of the TTMRs. The lack of robust control schemes capable of coping with such uncertainties has prevented a wider industrial exploitation of TTMRs with off-axle trailers despite their versatility and operational advantages [6].

With this research, we develop a detailed dynamics model of a differential drive mobile robot (DDMR) towing an off-axle trailer that accounts for the swivel movement of caster wheels. Despite of the amount of literature devoted to investigate the effect of the swivel movement of caster wheels in the motion of DDMRs [7]-[9], to the best our knowledge, none of these works have been focused on studying the dynamic effect on the DDMR of the swivel movement of off-axle trailers' caster wheels. To incorporate such an effect into our models, the kinematic relation of the caster wheels' swivel rate with the velocity of the DDMR is computed considering the pure rolling and non-slipping constraints of the system. Then, the dynamics models are established based on the well-known Lagrange's equation approach [10]. Such models could provide a theoretical insight of the system performance in the presence of friction in the caster wheels' kingpin mechanism and would allow the establishment of operational constraints in the caster wheels' motion for the control scheme design to further improve the operational reliability and safety of the system.

The remainder of this paper is organized as follows. Section II, describes the mechanical configuration of the studied TTMR. Section III, presents the kinematics model of the system under two different trailer's wheels configurations. Section IV, elaborates upon the dynamics model of the TTMR. Section V, presents the simulation result of our model. And finally, section VI, concludes our work.

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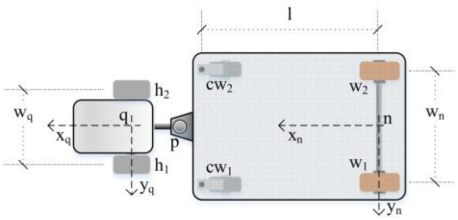
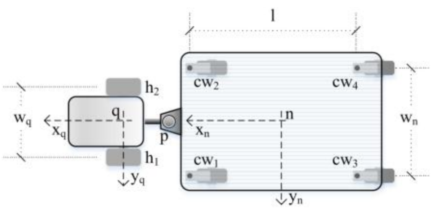
II. System Description



(a)



(b)

Figure 1. DDMR towing an off-axle trailer. (a) Trailer with two fixed wheels and two caster wheels. (b) Trailer with all wheels being caster wheels

For our study, let us consider the DDMR towing an off-axle trailer with two different wheels configurations as depicted in Fig. 1. Let establish a DDMR body coordinate system attached to the wheels axle middle point q with the x-axis (Denoted by x_q) pointing towards the forward movement of the robot and the y-axis (Denoted by y_q) pointing along the robot's wheels axle and outwards the left wheel. The z-axis (Denoted by z_q) completes the orthogonal right-handed coordinate system $x_q y_q z_q$. We assume the robot center of mass, namely cm_q, is located along x_q. The orientation θ_q of the robot is measured from the x-axis of the world reference frame $x_w y_w z_w$ to x_q. The angular rate of the robot's driving wheels h_1 and h_2 is denoted by φ_{h1} and φ_{h2} respectively, and their radius is defined as ρ_h. The distance separation between the wheels is denoted by w_q and $\tau = [\tau_1, \tau_2]^T$ is the torque exerted by the robot's DC motors. Besides, let us consider the trailer body coordinate attached to point n with the x-axis x_n pointing along the forward movement of the trailer and the z-axis z_n being perpendicular to the trailer pointing upwards. The y-axis y_n completes the orthogonal right-handed coordinate system $x_n y_n z_n$. The orientation of the trailer (Denoted by θ_n) is measured from x_w to x_n. Let cm_n be the center of mass of the trailer, and w_1 and w_2 be the fixed wheels of radius ρ_w. Let cw_i denote the caster wheel i, for i =1, 2, 3, 4. The distance from q to the revolute joint p is denoted by r_{qp}, whereas the distance from p to n is denoted by r_{pn}. The length and width of the trailer are represented by l and w_n, respectively. Furthermore, let us define two local coordinate frames to characterize the swivel movement of the caster wheels in terms of the robot's velocity, as depicted in Fig. 2. Let assume the local coordinate system $x_{cwi} y_{cwi} z_{cwi}$ is always aligned with $x'_{cwi} y'_{cwi} z'_{cwi}$, which can rotate freely about the z-axis. The distance from the z_{cwi} to z'_{cwi} is denoted by d_{cw} and corresponds to the swivel lead of the caster wheel.

III. Kinematics Model

Let us define the velocity vector of the robot as $\upsilon = [v_q, \omega_q]^T$, where v_q is the forward speed of point q in the robot coordinate system along the axis x_q and ω_q is the angular rate along the axis z_q. Then, in view of Fig. 1, it can be readily checked that:

$$\upsilon = \frac{\rho_h}{2}\begin{bmatrix} 1 & 1 \\ \frac{-1}{w_q} & \frac{1}{w_q} \end{bmatrix}\begin{bmatrix} \dot{\phi}_{h1} \\ \dot{\phi}_{h2} \end{bmatrix}. \tag{1}$$

Let $\vec{r_n} = [x_n, y_n, 0]^T$ denote the position of the trailer in the world reference frame. Hence, the TTMR general coordinates can be expressed as follows:

$$\xi = [x_n, y_n, \theta_n, \theta_q]^T \tag{2}$$

Seeing that tractor-trailer mobile robots are non-holonomic systems undergoing pure rolling and non-slipping constraints [10]. Then, the kinematic constraints of the system results:

$$L_\xi = \begin{bmatrix} \cos(\theta_n) & \sin(\theta_n) & 0 & \sin(\Delta\theta) r_{qp} \\ -\sin(\theta_q) & \cos(\theta_q) & \cos(\Delta\theta) r_{pn} & r_{qp} \end{bmatrix}, \tag{3}$$

With $\Delta\theta = \theta_n - \theta_q$. And the kinematics model becomes:

$$\dot{\xi} = S_\xi \upsilon = \begin{bmatrix} \cos(\theta_n)\cos(\Delta\theta) & -\cos(\theta_n)\sin(\Delta\theta) r_{qp} \\ \sin(\theta_n)\cos(\Delta\theta) & -\sin(\theta_n)\sin(\Delta\theta) r_{qp} \\ -\sin(\Delta\theta)\frac{1}{r_{pn}} & -\cos(\Delta\theta)\frac{r_{qp}}{r_{pn}} \\ 0 & 1 \end{bmatrix} \upsilon. \tag{4}$$

To account for the dynamic effect of the swivel motion of the trailer's caster wheels, the kinematic relation of the caster wheels' swivel rate (Angular speed about z_W) with the velocity of the DDMR, is also computed. For simplicity, we will confine our analysis to the trailer with two caster wheels and two fixed wheels as depicted in Fig. 1a.

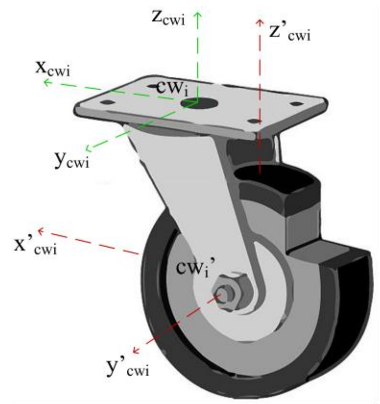


Figure 2. Caster wheels local coordinate systems

Let assume the contact patch of the caster wheel cw_i with the ground is concentrated at a point c_i. In the local coordinate system $x_{cwi} y_{cwi} z_{cwi}$, the position of such a point can be represented as $\vec{r}_{\mathrm{ci}}^{\,\mathrm{cwi}} = [-\mathrm{d_{cw}}, 0, -\mathrm{h_{cw}}]^{\mathrm{T}}$, where $\mathrm{h_{cw}}$ is the height of the caster wheel. In the world reference frame, this position becomes $\vec{r}_{\mathrm{ci|cwi}} = \mathrm{Rot}_z(\psi_{\mathrm{cwi}})\vec{r}_{\mathrm{ci}}^{\,\mathrm{cwi}}$, where Rot_z is the rotation matrix:

$$\mathrm{Rot}_z(\alpha) = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Noting that the position of the caster wheel $\mathrm{cw_i}$ in the trailer coordinate frame can be written as $\vec{r}_{\mathrm{cwi}}^{\,\mathrm{n}} = [\mathrm{l}, -1^{(i+1)}\mathrm{w_n}/2\,, 0]^{\mathrm{T}}$. In the world reference frame, it yields $\vec{r}_{\mathrm{cwi|n}} = \mathrm{Rot}_z(\theta_\mathrm{n})\vec{r}_{\mathrm{cwi}}^{\,\mathrm{n}}$. Hence, the position of the contact point of the caster wheel $\mathrm{cw_i}$ with respect to the world reference frame results:

$$\vec{r}_{\mathrm{ci}} = \vec{r}_{\mathrm{n}} + \vec{r}_{\mathrm{cwi|n}} + \vec{r}_{\mathrm{ci|cwi}} \tag{6}$$

which translates into the velocity vector:

$$\vec{v}_{\mathrm{ci}} = \vec{v}_{\mathrm{n}} + \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_\mathrm{n} \end{bmatrix} \times \vec{r}_{\mathrm{cwi|n}} + \begin{bmatrix} 0 \\ 0 \\ \dot{\psi}_{\mathrm{cwi}} \end{bmatrix} \times \vec{r}_{\mathrm{ci|cwi}}. \tag{7}$$

For a trailer with two caster wheels and two fixed wheels (See Fig. 1a), such velocity vectors become:

$$\vec{v}_{\mathrm{c1}} = \begin{bmatrix} d_{\mathrm{cw}}\sin(\psi_{\mathrm{cw1}})\,\dot{\psi}_{\mathrm{cw1}} - \left(\mathrm{l}\sin(\theta_n) + \frac{\mathrm{w_n}}{2}\cos(\theta_n)\right)\dot{\theta}_n + \dot{x}_n \\ -d_{\mathrm{cw}}\cos(\psi_{\mathrm{cw1}})\,\dot{\psi}_{\mathrm{cw1}} - \left(\frac{\mathrm{w_n}}{2}\sin(\theta_n) - \mathrm{l}\cos(\theta_n)\right)\dot{\theta}_n + \dot{y}_n \\ 0 \end{bmatrix}$$
$$\vec{v}_{\mathrm{c2}} = \begin{bmatrix} d_{\mathrm{cw}}\sin(\psi_{\mathrm{cw2}})\,\dot{\psi}_{\mathrm{cw2}} - \left(\mathrm{l}\sin(\theta_n) - \frac{\mathrm{w_n}}{2}\cos(\theta_n)\right)\dot{\theta}_n + \dot{x}_n \\ -d_{\mathrm{cw}}\cos(\psi_{\mathrm{cw2}})\,\dot{\psi}_{\mathrm{cw2}} + \left(\frac{\mathrm{w_n}}{2}\sin(\theta_n) + \mathrm{l}\cos(\theta_n)\right)\dot{\theta}_n + \dot{y}_n \\ 0 \end{bmatrix} \tag{8}$$

Furthermore, for a trailer with wheels configuration as depicted in Fig. 1b, the caster wheels' wheel axle velocity results as indicated in (9)-(10). Note that in this configuration, we assume the center of mass of the trailer is located at the middle of the platform and coincides with the local reference frame $\mathrm{x_n y_n z_n}$.

$$\vec{v}_{\mathrm{cw1}} = \begin{bmatrix} d_{Cw}\sin(\psi_{\mathrm{cw1}})\,\dot{\psi}_{\mathrm{cw1}} - \left(\frac{1}{2}\sin(\theta_\mathrm{n}) + \frac{\mathrm{w_n}}{2}\cos(\theta_\mathrm{n})\right)\dot{\theta}_1 + \dot{x}_\mathrm{n} \\ -d_{\mathrm{cw}}\cos(\psi_{\mathrm{cw1}})\,\dot{\psi}_{\mathrm{cw1}} - \left(\frac{\mathrm{w_n}}{2}\sin(\theta_\mathrm{n}) - \frac{1}{2}\cos(\theta_\mathrm{n})\right)\dot{\theta}_\mathrm{n} + \dot{y}_\mathrm{n} \\ 0 \end{bmatrix}$$
$$\vec{v}_{\mathrm{cw2}} = \begin{bmatrix} d_{\mathrm{cw}}\sin(\psi_{\mathrm{cw2}})\,\dot{\psi}_{\mathrm{cw2}} - \left(\frac{1}{2}\sin(\theta_\mathrm{n}) - \frac{\mathrm{w_n}}{2}\cos(\theta_\mathrm{n})\right)\dot{\theta}_\mathrm{n} + \dot{x}_\mathrm{n} \\ -d_{\mathrm{cw}}\cos(\psi_{\mathrm{cw2}})\,\dot{\psi}_{\mathrm{cw2}} + \left(\frac{\mathrm{w_n}}{2}\sin(\theta_\mathrm{n}) + \frac{1}{2}\cos(\theta_\mathrm{n})\right)\dot{\theta}_\mathrm{n} + \dot{y}_\mathrm{n} \\ 0 \end{bmatrix} \tag{9}$$

$$\vec{v}_{\mathrm{cw3}} = \begin{bmatrix} d_{\mathrm{cw}}\sin(\psi_{\mathrm{cw3}})\,\dot{\psi}_{\mathrm{cw3}} + \left(\frac{1}{2}\sin(\theta_\mathrm{n}) - \frac{\mathrm{w_n}}{2}\cos(\theta_\mathrm{n})\right)\dot{\theta}_\mathrm{n} + \dot{x}_\mathrm{n} \\ -d_{\mathrm{cw}}\cos(\psi_{\mathrm{cw3}})\,\dot{\psi}_{\mathrm{cw3}} - \left(\frac{\mathrm{w_n}}{2}\sin(\theta_\mathrm{n}) + \frac{1}{2}\cos(\theta_\mathrm{n})\right)\dot{\theta}_\mathrm{n} + \dot{y}_\mathrm{n} \\ 0 \end{bmatrix}$$
$$\vec{v}_{\mathrm{cw4}} = \begin{bmatrix} d_{\mathrm{cw}}\sin(\psi_{\mathrm{cw4}})\,\dot{\psi}_{\mathrm{cw4}} + \left(\frac{1}{2}\sin(\theta_\mathrm{n}) + \frac{\mathrm{w_n}}{2}\cos(\theta_\mathrm{n})\right)\dot{\theta}_\mathrm{n} + \dot{x}_\mathrm{n} \\ -d_{\mathrm{cw}}\cos(\psi_{\mathrm{cw4}})\,\dot{\psi}_{\mathrm{cw4}} + \left(\frac{\mathrm{w_n}}{2}\sin(\theta_\mathrm{n}) - \frac{1}{2}\cos(\theta_\mathrm{n})\right)\dot{\theta}_\mathrm{n} + \dot{y}_\mathrm{n} \\ 0 \end{bmatrix} \tag{10}$$

In view of the kinematic constraints of the caster wheel, the velocity in (7) can be expressed in the local coordinate system $\mathrm{x_{cwi}y_{cwi}z_{cwi}}$ as follows:

$$\vec{v}_{\mathrm{ci}}^{\,\mathrm{cwi}} = \mathrm{Rot}_z(-\psi_{\mathrm{cwi}})\vec{v}_{\mathrm{ci}} = \begin{bmatrix} \|\vec{v}_{\mathrm{ci}}^{\,\mathrm{cwi}}\| \\ 0 \\ 0 \end{bmatrix} \tag{11}$$

where $\|\vec{v}_{\mathrm{ci}}^{\,\mathrm{cwi}}\|$ is the norm of the vector $\vec{v}_{\mathrm{ci}}^{\,\mathrm{cwi}}$, i.e., the speed of $\mathrm{c_i}$ along the axis $\mathrm{x_{cwi}}$. Noting that the velocity of the origin of the local coordinate system $\mathrm{x'_{cwi}y'_{cwi}z'_{cwi}}$ is equal to $\vec{v}_{\mathrm{ci}}$, thus, the swivel rate of the caster wheel $\dot{\psi}_{\mathrm{cwi}}$ can be computed in terms of υ by using (8) and (11), as indicated in (12).

$$\begin{aligned} \dot{\psi}_{\mathrm{cw1}} = -\frac{v_\mathrm{q}}{\mathrm{d_{cw}r_{pn}}}\Big(&\mathrm{l}\sin(\Delta\theta)\cos(\psi_{\mathrm{cw1}} - \theta_\mathrm{n}) \\ &+ \left(\frac{\mathrm{w_q}}{2}\sin(\Delta\theta) + \mathrm{r_{pn}}\cos(\Delta\theta)\right)\sin(\psi_{\mathrm{cw1}} - \theta_\mathrm{n})\Big) \\ &+ \frac{\omega_q \mathrm{r_{qp}}}{\mathrm{d_{cw}r_{pn}}}\Big(\left(\mathrm{r_{pn}}\sin(\Delta\theta) - \frac{\mathrm{w_q}}{2}\cos(\Delta\theta)\right)\sin(\psi_{\mathrm{cw1}} - \theta_\mathrm{n}) \\ &- \mathrm{l}\cos(\Delta\theta)\cos(\psi_{\mathrm{cw1}} - \theta_\mathrm{n})\Big) \\ \dot{\psi}_{\mathrm{cw2}} = -\frac{v_\mathrm{q}}{d_{\mathrm{cw}}\mathrm{r_{pn}}}\Big(&\mathrm{l}\sin(\Delta\theta)\cos(\psi_{\mathrm{cw2}} - \theta_\mathrm{n}) \\ &- \left(\frac{\mathrm{w_n}}{2}\sin(\Delta\theta) - \mathrm{r_{pn}}\cos(\Delta\theta)\right)\sin(\psi_{\mathrm{cw2}} - \theta_\mathrm{n})\Big) \\ &+ \frac{\omega_q \mathrm{r_{qp}}}{d_{\mathrm{cw}}\mathrm{r_{pn}}}\Big(\left(\mathrm{r_{pn}}\sin(\Delta\theta) + \frac{\mathrm{w_n}}{2}\cos(\Delta\theta)\right)\sin(\psi_{\mathrm{cw2}} - \theta_\mathrm{n}) \\ &- \mathrm{l}\cos(\Delta\theta)cos(\psi_{\mathrm{cw2}} - \theta_\mathrm{n})\Big) \end{aligned} \tag{12}$$

For a trailer with all wheels being caster wheels (See Fig. 1b), the kinematics relations result similar to (12) for $\dot{\psi}_{\mathrm{cw1}}$ and $\dot{\psi}_{\mathrm{cw2}}$, but substituting 1 for ½ in each case; and:

$$\begin{aligned} \dot{\psi}_{\mathrm{cw3}} = -\frac{v_\mathrm{q}}{d_{\mathrm{cw}}\mathrm{r_{pn}}}\Big(&-\frac{\mathrm{l}}{2}\sin(\Delta\theta)\cos(\psi_{\mathrm{cw3}} - \theta_n) \\ &+ \left(\frac{\mathrm{w_n}}{2}\sin(\Delta\theta) + \mathrm{r_{pn}}\cos(\Delta\theta)\right)\sin(\psi_{\mathrm{cw3}} - \theta_n)\Big) \\ &+ \frac{\omega_q \mathrm{r_{qp}}}{d_{\mathrm{cw}}\mathrm{r_{pn}}}\Big(\left(\mathrm{r_{pn}}\sin(\Delta\theta) - \frac{\mathrm{w_n}}{2}\cos(\Delta\theta)\right)\sin(\psi_{\mathrm{cw3}} - \theta_n) \\ &+ \frac{\mathrm{l}}{2}\cos(\Delta\theta)\cos(\psi_{\mathrm{cw3}} - \theta_n)\Big) \\ \dot{\psi}_{\mathrm{cw4}} = -\frac{v_\mathrm{q}}{d_{\mathrm{cw}}\mathrm{r_{pn}}}\Big(&-\frac{\mathrm{l}}{2}\sin(\Delta\theta)\cos(\psi_{\mathrm{cw4}} - \theta_\mathrm{n}) \\ &- \left(\frac{\mathrm{w_n}}{2}\sin(\Delta\theta) - \mathrm{r_{pn}}\cos(\Delta\theta)\right)\sin(\psi_{\mathrm{cw4}} - \theta_\mathrm{n})\Big) \\ &+ \frac{\omega_q \mathrm{r_{qp}}}{d_{\mathrm{cw}}\mathrm{r_{pn}}}\Big(\left(\mathrm{r_{pn}}\sin(\Delta\theta) + \frac{\mathrm{w_n}}{2}\cos(\Delta\theta)\right)\sin(\psi_{\mathrm{cw4}} - \theta_\mathrm{n}) \\ &+ \frac{\mathrm{l}}{2}cos(\Delta\theta)cos(\psi_{\mathrm{cw4}} - \theta_\mathrm{n})\Big) \end{aligned} \tag{13}$$

The caster wheel swivel movement expressed in terms of the mobile robot velocity vector allows us to develop detailed dynamics models which can be utilized to synthesize more robust control schemes. In the next section, we derived a model which incorporates the kinematics equations (8) and (12) for more realistic results. To the best of our knowledge, no other work in the current literature has addressed this issue before.

IV. Dynamics Model

Following [10], the dynamics model of the TTMR can be represented as:

$$M(\bar{\xi})\ddot{\bar{\xi}} + n(\bar{\xi})\dot{\bar{\xi}} + \tau_d = B(\bar{\xi})\tau - \bar{L}_{\bar{\xi}}\lambda \tag{14}$$

where $M(\bar{\xi})$is a square positive definite inertia matrix, $n(\bar{\xi})$is the centripetal and Coriolis matrix, $\tau_d$accounts for the unknown disturbances and unmodeled dynamics, $B(\bar{\xi})$is the input transformation matrix, $\bar{L}_{\bar{\xi}}$represents the kinematics constraints of the system, λ is the Lagrange multiplier vector, and $\bar{\xi}$ is the augmented coordinates of the system expressed as:

$$\bar{\xi} = [\xi^{\mathrm{T}}, \dot{\phi}_{h1}, \dot{\phi}_{h2}]^{\mathrm{T}} \tag{15}$$

The dynamics model (14) can be obtained by computing the kinetic energy of each element of the TTMR and using the Lagrange's equation:

$$\frac{d}{dt}\left(\frac{\partial E}{\partial \dot{\bar{\xi}}}\right) + \frac{\partial E}{\partial \bar{\xi}} = F - \bar{L}_{\bar{\xi}}\lambda \tag{16}$$

where $E = U - V$ is the Lagrangian function, U is the kinetic energy of the system, V is the potential energy, and F is the force vector acting on the system. In general, the kinetic energy of a rigid body can be represented as:

$$U_j = \frac{1}{2}m_j\|v_j\|^2 + \frac{1}{2}\vec{\omega}_j^{\mathrm{T}} I_j \vec{\omega_j} \tag{17}$$

where $U_j$denotes the kinetic energy of the object j, $m_j$its mass, $v_j$the velocity with respect to the world reference frame, $I_j$the moment of inertia about the local axis of rotation, and $\vec{\omega_j}$ the angular velocity with respect to the world reference frame expressed in the local frame.

In our analysis, we neglect the kinetic energy of the trailer's fixed wheels, and thus, the models only account for the kinetic energy of the trailer's caster wheels and body, and the kinetic energy of the DDMR's body and driving wheels. It should be noted that due to the caster wheel's complex mechanism, the computation of the kinetic energy may result somewhat involved [11]. Hence, we only consider the mass of the caster wheels' wheel so that (17) becomes:

$$U_{\mathrm{cwi}} = \frac{1}{2}m_{\mathrm{cwi}}\|\vec{v}_{\mathrm{cwi'}}\|^2 + \frac{1}{2}\vec{\omega}_{\mathrm{cwi}}^{\mathrm{T}}\bar{I}_{\mathrm{cwi}}\vec{\omega}_{\mathrm{cwi}} \tag{18}$$

with m_{cwi} being the mass of the caster wheel i's wheel. The norm of the velocity $\|\vec{v}_{\mathrm{cwi'}}\|$ can be calculated from the velocity vectors (8).

For consistency with our kinematics model (4), let us simplify and express (14) in terms of the mobile robot velocity vector υ as follows:

$$\bar{M}(\bar{\xi})\dot{\upsilon} + \bar{n}(\bar{\xi})\upsilon = \bar{B}(\bar{\xi})\tau - \bar{\tau}_d \tag{19}$$

where $\bar{\tau}_d$ accounts for the dynamic effect of the swivel movement of the caster wheels, and:

$$\begin{aligned} \bar{M} &= \bar{S}_{\bar{\xi}}^{\mathrm{T}} M \bar{S}_{\bar{\xi}}, \bar{n} = \bar{S}_{\bar{\xi}}^{\mathrm{T}}(M\dot{\bar{S}}_{\bar{\xi}} + n\bar{S}_{\bar{\xi}}) \\ \bar{B} &= \bar{S}_{\bar{\xi}}^{\mathrm{T}} B = \frac{1}{\rho_h}\begin{bmatrix} 1 & 1 \\ -\mathrm{w_q}/2 & \mathrm{w_q}/2 \end{bmatrix} \end{aligned} \tag{20}$$

The components of the matrices in (20), relates to the mass and inertia of every constituent of the TTMR. For more details, on the derivation of (19) refer to appendix A.

For the computation of the dynamic effect of the swivel motion of the caster wheels $\bar{\tau}_d$, we neglect the higher order derivative terms of ψ_{cwi} (i.e. $\ddot{\psi}_{\mathrm{cwi}}$) in the dynamics model. Therefore, the swivel motion effect of two caster wheels yields:

$$\bar{\tau}_d = \frac{d_{\mathrm{cw}} m_{\mathrm{cw}}}{2\mathrm{r_{pn}}}\begin{bmatrix}\bar{\tau}_d[1] \\ \bar{\tau}_d[2]\end{bmatrix} \tag{21}$$

$$\begin{aligned} \bar{\tau}_d[1] &= \dot{\psi}_{\mathrm{cw1}}^2\left(\left(\frac{\mathrm{W_n}}{2}\mathrm{s}_{\Delta\theta} + \mathrm{r_{pn}}\mathrm{c}_{\Delta\theta}\right)\mathrm{c}_{\psi_{\mathrm{cw1}}-\theta_{\mathrm{n}}} + \mathrm{l}\mathrm{s}_{\Delta\theta}\mathrm{s}_{\psi_{\mathrm{cw1}}-\theta_{\mathrm{n}}}\right) \\ &\quad + \dot{\psi}_{\mathrm{cw2}}^2\left(-\left(\frac{\mathrm{W_n}}{2}\mathrm{s}_{\Delta\theta} - \mathrm{r_{pn}}\mathrm{c}_{\Delta\theta}\right)\mathrm{c}_{\psi_{\mathrm{cw2}}-\theta_{\mathrm{n}}} + \mathrm{l}\mathrm{s}_{\Delta\theta}\mathrm{s}_{\psi_{\mathrm{cw2}}-\theta_{\mathrm{n}}}\right) \\ \bar{\tau}_d[2] &= \mathrm{r_{qp}}\dot{\psi}_{\mathrm{cw1}}^2\left(\left(\frac{\mathrm{W_n}}{2}\mathrm{c}_{\Delta\theta} - \mathrm{r_{pn}}\mathrm{s}_{\Delta\theta}\right)\mathrm{c}_{\psi_{\mathrm{cw1}}-\theta_{\mathrm{n}}} + \mathrm{l}\mathrm{c}_{\Delta\theta}\mathrm{s}_{\psi_{\mathrm{cw1}}-\theta_{\mathrm{n}}}\right) \\ &\quad + \mathrm{r_{qp}}\dot{\psi}_{\mathrm{cw2}}^2\left(-\left(\frac{\mathrm{W_n}}{2}\mathrm{c}_{\Delta\theta} + \mathrm{r_{pn}}\mathrm{s}_{\Delta\theta}\right)\mathrm{c}_{\psi_{\mathrm{cw2}}-\theta_{\mathrm{n}}} + \mathrm{l}\mathrm{c}_{\Delta\theta}\mathrm{s}_{\psi_{\mathrm{cw2}}-\theta_{\mathrm{n}}}\right) \end{aligned} \tag{22}$$

where c_* denotes $\cos(*)$, s_* denotes $\sin(*)$, and $\dot{\psi}_{\mathrm{cw1}}$ and $\dot{\psi}_{\mathrm{cw2}}$ can be computed using (12). The dynamics model of the TTMR with a trailer in configuration as shown in Fig. 1b, can be computed following a similar approach. For the sake of brevity, we only present the dynamic effect of the four caster wheels in (23).

From (12) and (21)-(23), it should be noted that the distance from the fixed wheels axle to the revolute joint p influences significantly the swivel motion dynamic effects. This is, the disturbance torque acting on the DDMR due to the caster wheels' swivel is directly proportional to the distance between q and p, and the distance from p to n.

On the other hand, the disturbance force can be reduced by increasing the distance from the revolute joint p to the trailers fixed wheels axle. However, these mechanical parameters also dictate the capability of the TTMR to ensure a proper tracking performance as evidenced in [12]. Therefore, a trade-off on the mechanical configuration of the system should be made to diminish such a dynamic disturbance while guaranteeing low steady-state tracking errors.

$$
\begin{aligned}
\bar{\tau}_d[1] = {} & \dot{\psi}_{cw1}^2 \left(2\left(\frac{w_n}{2} s_{\Delta\theta} + r_{pn} c_{\Delta\theta}\right) c_{\psi_{cw1}-\theta_n} + l s_{\Delta\theta} s_{\psi_{cw1}-\theta_n}\right) \\
& + \dot{\psi}_{cw2}^2 \left(-2\left(\frac{w_n}{2} s_{\Delta\theta} - r_{pn} c_{\Delta\theta}\right) c_{\psi_{cw2}-\theta_n} + l s_{\Delta\theta} s_{\psi_{cw2}-\theta_n}\right) \\
& + \dot{\psi}_{cw3}^2 \left(2\left(\frac{w_n}{2} s_{\Delta\theta} + r_{pn} c_{\Delta\theta}\right) c_{\psi_{cw3}-\theta_n} - l s_{\Delta\theta} s_{\psi_{cw3}-\theta_n}\right) \\
& + \dot{\psi}_{cw4}^2 \left(-2\left(\frac{w_n}{2} s_{\Delta\theta} - r_{pn} c_{\Delta\theta}\right) c_{\psi_{cw4}-\theta_n} - l s_{\Delta\theta} s_{\psi_{cw4}-\theta_n}\right) \\
\bar{\tau}_d[2] = {} & r_{qp} \dot{\psi}_{cw1}^2 \left(2\left(\frac{w_n}{2} c_{\Delta\theta} - r_{pn} s_{\Delta\theta}\right) c_{\psi_{cw1}-\theta_n} + l c_{\Delta\theta} s_{\psi_{cw1}-\theta_n}\right) \\
& + r_{qp} \dot{\psi}_{cw2}^2 \left(-2\left(\frac{w_n}{2} c_{\Delta\theta} + r_{pn} s_{\Delta\theta}\right) c_{\psi_{cw2}-\theta_n} + l c_{\Delta\theta} s_{\psi_{cw2}-\theta_n}\right) \\
& + r_{qp} \dot{\psi}_{cw3}^2 \left(-2\left(\frac{w_n}{2} c_{\Delta\theta} - r_{pn} s_{\Delta\theta}\right) c_{\psi_{cw3}-\theta_n} - l c_{\Delta\theta} s_{\psi_{cw3}-\theta_n}\right) \\
& + r_{qp} \dot{\psi}_{cw4}^2 \left(2\left(\frac{w_n}{2} c_{\Delta\theta} + r_{pn} s_{\Delta\theta}\right) c_{\psi_{cw4}-\theta_n} - l c_{\Delta\theta} s_{\psi_{cw4}-\theta_n}\right)
\end{aligned}
\tag{23}
$$

V. Numerical Simulation

In this section, we present the simulation results of a TTMR with trailer wheels configuration as in Fig. 1b, following two prescribed trajectories under the kinematic controller introduced in [13]. The first trajectory is a circular path of radius 23.5 m, whereas the second trajectory is an irregular path with sharp turns to highlight the swivel motion effect of the caster wheels. The TTMR parameters were chosen as: $m_B = 50$ kg, $m_h = 2$ kg, $m_n = 150$ kg, $m_{cw} = 2.5$ kg, $w_q = 0.45$ m, $l = 0.985$ m, $w_n = 0.8$ m, and $r_{qcm} = 0.125$ m, $r_{qp} = 0.5$ m, $r_{pn} = 0.75$ m.

The results for the circular path are presented in Fig. 3 – Fig. 5. The swivel motion effect is negligible in such a smooth path. Fig. 5 illustrates the force affecting the DDMR which is clearly related to the force exerted by the caster wheel's wheel due to the continuous change of its the angular momentum. On the other hand, for the second trajectory (See Fig. 6), the effect of the caster wheels on the performance of the TTMR is significantly higher as shown in Fig. 7 and Fig. 8. The kinematic controller fails to guarantee a proper tracking performance in sharp turns, allowing oscillations and an excessive heading offset between the DDMR and the trailer. Such a performance degradation relates to not only the rapid change in controller's reference signals but also to the system dynamics and disturbances, such as the swivel motion dynamic effect. This outcome evidences that it is necessary to carry out a more comprehensive study on unmodeled dynamics and disturbances that arise from the swivel mechanism of caster wheels.

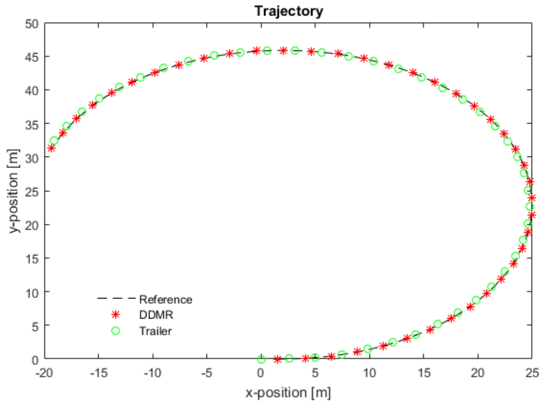


Figure 3. TTMR prescribed trajectory #1

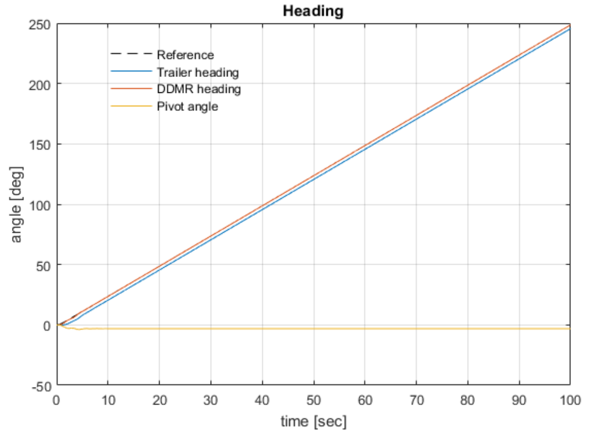


Figure 4. TTMR and pivot angle (tractor-trailer angle in p) for trajectory # 1

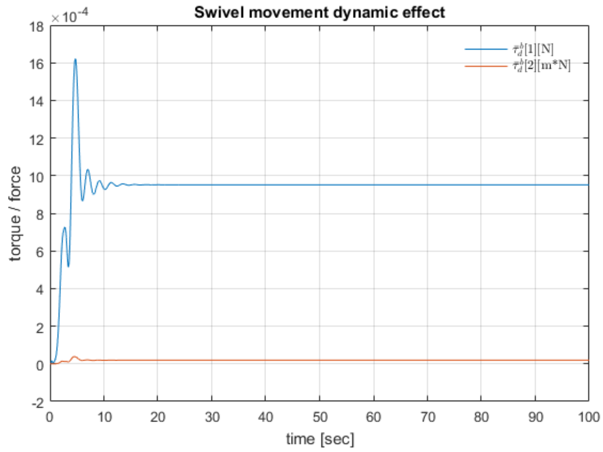


Figure 5. Caster wheels' swivel movement dynamic effect for trajectory #1

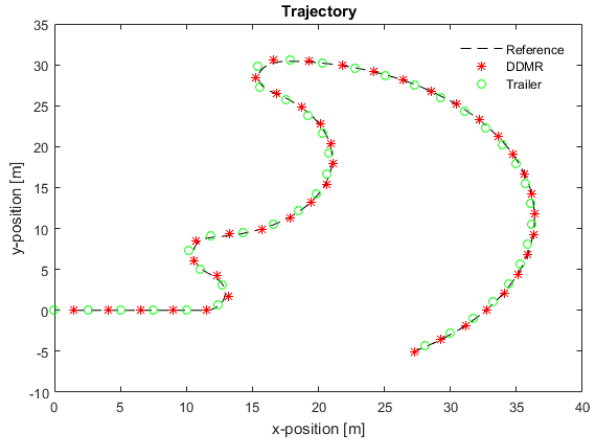


Figure 6. TTMR prescribed trajectory # 2

VI. Conclusion

In this work, we modeled the dynamic effect of the swivel motion of caster wheels in tractor-trailer mobile robots. Such an effect is characterized in terms of the kinematics relation with the mobile robot velocity to facilitate the integration with the system dynamics model.

Our analysis covered a trailer with two wheels configurations commonly implemented in such a mechanical setup. The simulation outcome shows how the unmodeled dynamics may diminish the controller performance when they are neglected in the control designing stage. Therefore, the development of more accurate dynamics models for mobile robots with off-axle trailers is required to develop robust controllers capable of guaranteeing safe and proper operational performance to enable a wider exploitation of TTMRs in highly demanding applications.

The moment of inertia tensor in the world reference frame is defined as:

$$\bar{I}_{\mathrm{cwi}} = \begin{bmatrix} \bar{I}^{xx}_{\mathrm{cwi}} & \bar{I}^{xy}_{\mathrm{cwi}} & 0 \\ \bar{I}^{xy}_{\mathrm{cwi}} & \bar{I}^{yy}_{\mathrm{cwi}} & 0 \\ 0 & 0 & \bar{I}^{zz}_{\mathrm{cwi}} \end{bmatrix} \tag{24}$$

With:

$$\begin{aligned} \bar{I}^{xx}_{\mathrm{cwi}} &= I^{y}_{\mathrm{cwi}}\sin(\psi_{\mathrm{cwi}})^2 + I^{z}_{\mathrm{cwi}}\cos(\psi_{\mathrm{cwi}})^2 \\ \bar{I}^{xy}_{\mathrm{cwi}} &= I^{z}_{\mathrm{cwi}}\cos(\psi_{\mathrm{cwi}})\sin(\psi_{\mathrm{cwi}}) - I^{y}_{\mathrm{cwi}}\cos(\psi_{\mathrm{cwi}})\sin(\psi_{\mathrm{cwi}}) \\ \bar{I}^{yy}_{\mathrm{cwi}} &= I^{z}_{\mathrm{cwi}}\sin(\psi_{\mathrm{cwi}})^2 + I^{y}_{\mathrm{cwi}}\cos(\psi_{\mathrm{cwi}})^2, \bar{I}^{zz}_{\mathrm{cwi}} = I^{z}_{\mathrm{cwi}} \end{aligned} \tag{25}$$

where $I^{x}_{\mathrm{cwi}} = I^{z}_{\mathrm{cwi}}$ and $I^{y}_{\mathrm{cwi}}$are the moment of inertia about the axes of the local coordinate system $x'_{cw}y'_{cw}z'_{cw}$ attached to cw_i' (See Fig. 2) and initially aligned with $x_n y_n z_n$. Moreover, the angular velocity in the world reference frame $\vec{\omega}_{\mathrm{cwi}}$ is:

$$\vec{\omega}_{\mathrm{cwi}} = \begin{bmatrix} -\dot{\phi}_{\mathrm{cwi}}\sin(\psi_{\mathrm{cwi}}) \\ \dot{\phi}_{\mathrm{cwi}}\cos(\psi_{\mathrm{cwi}}) \\ \dot{\psi}_{\mathrm{cwi}} \end{bmatrix} \tag{26}$$

where $\dot{\phi}_{\mathrm{cwi}}$ is the angular rate of the caster wheel's wheel about the axis y'_{cwi}.

Furthermore, in view of (1) and (4), the kinematics model for the augmented coordinates (15) becomes, where:

$$\bar{S}_{\bar{\xi}} = \begin{bmatrix} S_{\bar{\xi}} \\ T_{\upsilon\phi} \end{bmatrix}, T_{\upsilon\phi} = \frac{1}{\rho_h}\begin{bmatrix} 1 & -\frac{w_q}{2} \\ 1 & \frac{w_q}{2} \end{bmatrix} \tag{27}$$

Hence, $\ddot{\bar{\xi}} = \dot{\bar{S}}_{\bar{\xi}}\upsilon + \bar{S}_{\bar{\xi}}\dot{\upsilon}$. Noting that $\bar{S}_{\bar{\xi}}^{T}\bar{L}_{\bar{\xi}} = 0$, the dynamics model (14) can simplified as indicated in (19).In the case of a trailer with two fixed wheels and two caster wheels, the elements of the matrices in (20) are:

$$\begin{aligned} \bar{M}_{11} &= 2\frac{I^{y}_{h}}{\rho_h^2} + \frac{I^{z}_{tr}}{r_{pn}^2}s^2_{\Delta\theta} + m_T c^2_{\Delta\theta} \\ \bar{M}_{12} &= \bar{M}_{21} = \left(I^{z}_{tr}/r_{pn}^2 - m_T\right)s_{\Delta\theta}c_{\Delta\theta}r_{qp} \\ \bar{M}_{22} &= \frac{I^{y}_{h}{w_q}^2}{2\rho_h^2} + \left(\frac{I^{z}_{tr}}{r_{pn}^2}c^2_{\Delta\theta} + m_T s^2_{\Delta\theta}\right)r^2_{qp} - 2e_2 r_{qp} + I^{z}_{T} \end{aligned} \tag{28}$$

where I^{y}_{h} is the moment of inertia of the driving wheels along the rotation axle, the total mass of the system is given by $m_T = m_B + 2m_h + m_n + 2m_w + 2m_{cw}$, with m_B and m_h being the mass of the DDMR's body and driving wheels, respectively. m_n is the mass of the trailer's body, m_w and m_{cw} are the mass of the fixed and caster wheels, respectively. Moreover,

$$\begin{aligned} I^{z}_{tr} = (2m_h + m_B)r^2_{pn} + I^{z}_{n} + \frac{l^2}{4}m_n + m_{cw}\left(\frac{{w_n}^2}{2} + 2l^2\right) \\ + 2I^{z}_{w} + \frac{{w_n}^2}{2}m_W \end{aligned} \tag{29}$$

where and denote the moment of inertia of the trailer's body and fixed wheels about the z-axis. Besides,

$$\begin{aligned} \bar{n} = \begin{bmatrix} \bar{n}_{11} & \bar{n}_{12} \\ -\bar{n}_{12} & -\bar{n}_{11} \end{bmatrix}, \bar{n}_{11} = c_{\Delta\theta}s_{\Delta\theta}\left(\frac{I^{z}_{tr}}{r^2_{pn}} - m_T\right)\Delta\theta \\ \bar{n}_{12} = -e_2\dot{\theta}_q + e_1\dot{\theta}_n\frac{r_{qp}}{r_{pn}} - \Delta\theta\frac{r_{qp}}{r^2_{pn}}\left(m_T c^2_{\Delta\theta}r^2_{pn} + I^{z}_{B}s^2_{\Delta\theta}\right) \end{aligned} \tag{30}$$

with $e_2 = (m_B+2m_h)r_{qp}+r_{qcm}m_B$ where is the distance from q to the DDMR's center of mass. In addition, $e_1 = m_B+2m_h r_{pn}+lm_n/2+2lm_{cw}$, and

$$I^{z}_{B} = m_B(r_{qp} + r_{qcm})^2 + 2m_h\left(r^2_{qp} + \frac{{w_q}^2}{4}\right) + I^{z}_{q} + 2I^{z}_{h} \tag{31}$$

where I^{z}_{q} and I^{z}_{h} represent the moment of inertia of the DDMR's body and driving wheels about the z-axis of the local reference frame $x_q y_q z_q$, accordingly.

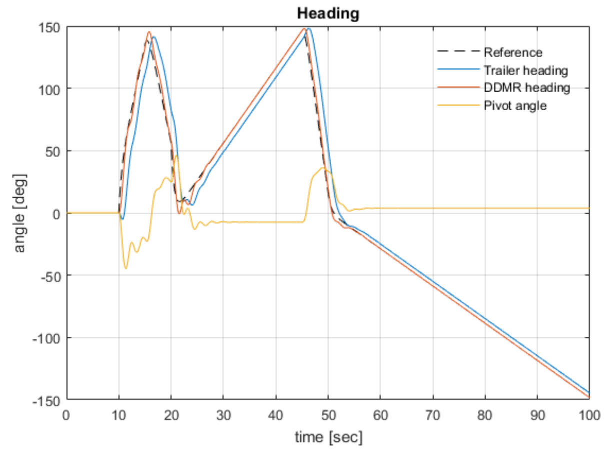


Figure 7. TTMR and pivot angle (tractor-trailer angle in p) for trajectory # 2

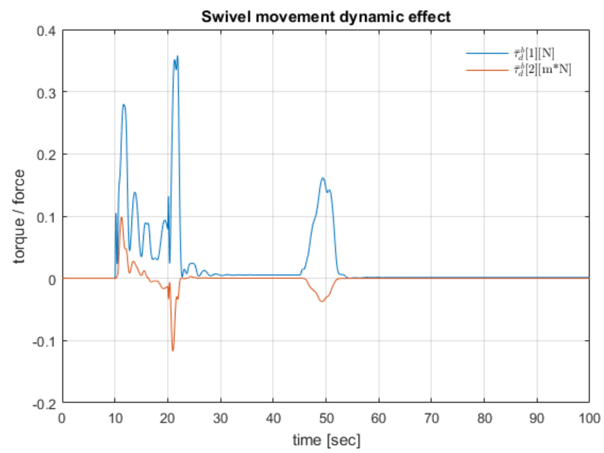


Figure 8. Caster wheels' swivel movement dynamic effect for trajectory # 2

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