On the Synthesis of Cylindrical Linear Rack Mechanisms

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Abstract— The science of the gear mechanisms, which is an independent direction of “Applied Mechanics” covers two main scientific directions: theory of gearing and geometric theory of gear transmissions. The main tasks, which are solved by this direction of the science of the gear mechanisms are: development of adequate approaches to the synthesis and related to it generation of the active tooth surfaces of different types of gear mechanisms; choice of rational form of active tooth surfaces; creation of calculating methods for design and elaboration of innovative drives and etc. The current study is dedicated to the synthesis of one less studied class of spatial gear transmissions – cylindrical rack drives. This type of drives serves to transform the rotation of a cylindrical worm into translational displacement of a gear rack. An analytical dependencies, defining the basic geometric characteristics of this type three-link mechanisms, for the cases when the rotating link is a cylindrical worm, which active tooth surfaces are asymmetric cylindrical convolute, Archimedean and involute helicoids, are presented in the study. This motions transformer should be treated as a special case of spatial three-link rack drive, which rotating link is a convolute, Archimedean and involute worm with a constant axial pitch.

Index Terms— rack drive, mathematical modeling, synthesis, cylindrical linear helicoid, action surface, region of mesh

I. INTRODUCTION

The main purpose of each transmission mechanisms is to realize motions transformation upon a preliminary defined law with a necessary exactness. In the group of gear mechanisms, certain significance for the technique has spatial rack mechanisms, transforming motions of the type (R ↔ T) [1].

The reason for this is that, as a mechanical system, this type of gear sets does not have an alternative for cases of motions transformation at a big power, as an alternative for small power and displacements are the electric drives with electronic control. Hydraulic drives represent a definite alternative, but for the achievement of high positioning accuracy - high-tech and expensive servo systems are applied.

The non-orthogonal crossed placement of the rotation axis of one movable link towards the directrix of the rectilinear translation of the second movable link is a premise for existence of a greater number of free parameters. The search of suitable combinations among them, enables the possibility when the rack mechanisms are synthesized, to control the desired exploitation characteristics: increased reliability and durability; low vibration activity and noiseless and etc. Here, it will be mentioned that even a slight deviation from the orthogonal placement of the crossing of the specific motions axes of the movable links leads to increment of the overlap coefficient and hence of the increment of the gear loading capacity. As a rule, the positive technological and exploitation qualities of the gear mechanism are result of achieving an optimal correlation between their specific kinematic characteristics. This determines the prevailing kinematic character of the chosen approach to the synthesis and to the studied models also.

The successful implementation of spatial rack transmissions with new kinematic and strength characteristics into techniques is obstructed from the insufficient knowledge of the common principles of this transformation, due to the lack of offered specific approaches to the mathematical modeling oriented towards their synthesis.

The global structure of every mathematical model of the studied type gear mechanism is determined from [2]:

- The purpose of the three-link mechanism in terms of the defined law of motions transformation;
- Placement of the specific axes of the movable links and the type of conjugation of the Σi, elements of high kinematic joints (point or linear tangent contact);
- The technological devices related to the choice of the geometry of the generating (instrumental) surfaces and the chosen principle of generation by T. Olivier.

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Two approaches for generation of the tooth surfaces $\Sigma_2$ of the gear rack (link $i = 2$) of the studied mechanism will be shown below:

- $\Sigma_2$ is generated through an enveloping in accordance with the second principle of T. Olivier; $\Sigma_2$ is generated (cut) by instrument $J$, which generating (enveloping) surfaces $\Sigma_j$ are a geometrically identical with the tooth surfaces $\Sigma_1$ of the rotating link $i = 1$, i.e. $\Sigma_j \equiv \Sigma_1$; the tool $J$ occupies the same relative position towards the blank of the link $i = 2$; the relative motion of the instrument $J$ towards the blank of the link $i = 2$ is the same as it is of the $\Sigma_j$ towards $\Sigma_2$ in condition of work meshing. Hence, the generated surfaces $\Sigma_j$ have a linear contact with $\Sigma_1$.

- $\Sigma_2$ is a cylindrical surface, for which it is easy to determine the analytical type of its normal section and that is the normal profile of the tooth of the gear rack. If the direction of the longitudinal line of the tooth and the geometry of its normal profile are known, then $\Sigma_2$ can be generated by copying. For this case too, $\Sigma_1$ and $\Sigma_2$ have a linear contact.

II. STUDY OF THE GEOMETRY OF THE CYLINDRICAL RACK DRIVES

A. Cylindrical Convolute Rack Drive

The study of this class rack drives is realized on the basis of the shown in Fig. 1 geometric scheme for generation of cylindrical convolute helicoids [1, 2] and on Fig. 2- in which a spatial rack mechanism with rotating cylindrical helicoids (cylindrical worm) is shown.

The process of these helical surfaces generation is considered in fixed co-ordinate system $S_p^{(j)}(O_p^{(j)}, x_p^{(j)}, y_p^{(j)}, z_p^{(j)})$ and it is as follows. The generatrix $L^{(j)}$ does not cross the axis $O_p^{(j)}z_p^{(j)}$ which coincides with the geometric axis of the gear. $L^{(j)}$ and $O_p^{(j)}z_p^{(j)}$ conclude an angle $0,5\pi < \xi^{(j)} < \pi$. At the same time, line $L^{(j)}$ belongs to plane $T^{(j)}$, which is tangential to directed circle cylinder $C^{(j)}$. The generation of the cylindrical convolute helicoid $\Sigma_j^{(j)}$ by the line $L^{(j)}$ is realized through axial helical motion with longitudinal axis $O_p^{(j)}z_p^{(j)}$ with parameter $p_s^{(j)} = constant$. It can be shown easily, that this type of helicoids should be treated as special cases of conic linear helicoids [1, 3, 4], when the cross parameter (tangent parameter) $p_s = 0$. Hence, they are linear helicoids with axial parameter $p_t^{(j)} = p_s^{(j)} = constant$. Then, the cylindrical convolute helicoid is described by the vector equation:

$$\overrightarrow{P_1^{(j)}} = \overrightarrow{r_0^{(j)}} + \overrightarrow{\xi^{(j)}} + \overrightarrow{u^{(j)}}.$$  \hspace{1cm} (1)

where $\overrightarrow{P_1^{(j)}}$ is a radius-vector of point $N^{(j)}$ from the cylindrical convolute helicoid $\Sigma_j^{(j)}$; $\overrightarrow{r_0^{(j)}}$ - radius-vector of the directed cylinder $C^{(j)}$; $u^{(j)}$, $\overrightarrow{\xi^{(j)}}$ - coordinates of the helical surface $\Sigma_j^{(j)}$; $\overrightarrow{\xi^{(j)}} = p_s^{(j)}\overrightarrow{\theta^{(j)}}$ - axial displacement of the generatrix $L^{(j)}$.

When (1) is written in the co-ordinate system, $S_p^{(j)}(O_p^{(j)}, x_p^{(j)}, y_p^{(j)}, z_p^{(j)})$, then it is obtained:

$$x_p^{(j)} = r_0^{(j)} \cos\theta^{(j)} \pm u^{(j)} \sin\xi^{(j)} \sin\theta^{(j)},$$

$$y_p^{(j)} = r_0^{(j)} \sin\theta^{(j)} \mp u^{(j)} \sin\xi^{(j)} \cos\theta^{(j)},$$

$$z_p^{(j)} = p_s^{(j)}\theta^{(j)} \mp u^{(j)} \cos\xi^{(j)}.$$  \hspace{1cm} (2)

When $U^{(j)} = u^{(j)} \sin\xi^{(j)} \neq 0$ is substituted in (2), then the system (2) has the following type:

$$x_p^{(j)} = r_0^{(j)} \cos\theta^{(j)} \pm U^{(j)} \sin\theta^{(j)},$$

$$y_p^{(j)} = r_0^{(j)} \sin\theta^{(j)} \mp U^{(j)} \cos\theta^{(j)},$$

$$z_p^{(j)} = p_s^{(j)} \theta^{(j)} \mp U^{(j)} \cot\xi^{(j)}.$$  \hspace{1cm} (3)

The systems of equations (3), as well as (2), describe the pair of cylindrical convolute helicoids, which parameter of distribution is

$$h^{(j)} = p_s^{(j)} + u_0^{(j)} \cot \xi^{(j)}.$$  \hspace{1cm} (4)
The normal vectors to the cylindrical helical surfaces $\Sigma_i^{(j)} (j = I, 2)$ at their arbitrary point are illustrated by their corresponding scalar components:

$$n_{i,s}^{(j)} = \mp h^{(j)} \sin \vartheta^{(j)} + U^{(i)} \cot \xi^{(j)} \cos \vartheta^{(j)},$$

It is obvious from (5), that $\overline{n}_{i,s}^{(j)} \neq \overline{0}$ for each point from $\Sigma_i^{(j)} (j = I, 2)$. Hence, the cylindrical convolute helicoid has only ordinary points.

The most general type of the action surface/mesh region of the rack mechanisms can be presented by the following equations system:

$$\begin{align*}
\rho_{s}^{(j)} &= M_{m_s} \rho_{s}^{(j)}, \\
\rho_{l}^{(j)} &= \rho_{s}^{(j)}(U^{(i)}, \vartheta^{(j)}), \\
f(U^{(i)}, \vartheta^{(j)}, \varphi_i) &= \overline{n}_{i,s} \overrightarrow{V}_{12} = 0, \\
n_{i,s}^{(j)} &= L_{m_s} n_{i,s}^{(j)},
\end{align*}$$

where $\rho_{s}^{(j)}$ is a column-matrix of the radius-vector $\overline{p}_{s}^{(j)}$ of an arbitrary point from cylindrical linear helicoids $\Sigma_i^{(j)}$ in the fixed co-ordinate system $S$; $\rho_{l}^{(j)}$ - column-matrix of the vector $\overline{p}_{l}^{(j)}$ of an arbitrary point from $\Sigma_i^{(j)}$ in its fixed co-ordinate system $S_p$ of the helicoid; $M_{m_s}$ - 4x4 transition matrix from $S_{p}$ into $S$; $L_{m_s}$ - 3x3 transition matrix from $S_{p}$ into $S$; $U^{(i)}, \vartheta^{(j)}$ - independent coordinates of the helicoid $\Sigma_i^{(j)}$; $\varphi_i$ - parameter of meshing; $\overline{n}_{i,s}^{(j)}$ - normal vector at $\Sigma_i^{(j)}$ in arbitrary point; $n_{i,s}^{(j)}$ - column-matrix of $\overline{n}_{i,s}^{(j)}$, written in $S$; $n_{i,s}^{(j)}$ - column-matrix of $\overline{n}_{i,s}^{(j)}$, written in the co-ordinate system $S_{p}$; $\overrightarrow{V}_{12}$ - sliding velocity vector in arbitrary point from the mesh region of the rack drive.

For the case of convolute rack drive, the equation system, describing action surface/mesh region, obtains the following type [5 - 7]:

$$\begin{align*}
\begin{bmatrix}
x^{(j)} \\
y^{(j)} \\
z^{(j)} \\
1
\end{bmatrix} &= M_{m_s} \begin{bmatrix}
r_0^{(j)} \cos \vartheta^{(j)} + U^{(i)} \sin \vartheta^{(j)} \\
r_0^{(j)} \sin \vartheta^{(j)} + U^{(i)} \cos \vartheta^{(j)} \\
p_1 \vartheta^{(j)} + U^{(i)} \cos \vartheta^{(j)} \\
1
\end{bmatrix}, \\
n_{i,s}^{(j)} &= \overline{n}_{i,s}^{(j)}(j_{12} \cos \vartheta_i, j_{12} \sin \vartheta_i),
\end{align*}$$

where $j_{12} = V_{12} / \omega_i$ ($V_{12}$ is a magnitude of translation velocity $\overrightarrow{V}_{12}$, $\omega_i$ - magnitude of angular velocity $\overrightarrow{\omega}_i$, see Fig. 2).

$$\begin{align*}
\begin{bmatrix}
n_{i,s}^{(j)} \\
n_{i,s}^{(j)} \\
n_{i,s}^{(j)}
\end{bmatrix} &= M_{m_s} \begin{bmatrix}
\mp h^{(j)} \cos \vartheta^{(j)} - U^{(i)} \cot \xi^{(j)} \sin \vartheta^{(j)} \\
\mp h^{(j)} \sin \vartheta^{(j)} + U^{(i)} \cot \xi^{(j)} \cos \vartheta^{(j)} \\
U^{(i)}
\end{bmatrix},
\end{align*}$$

$$M_{m_s} = \begin{bmatrix}
\cos \varphi_i & -\sin \varphi_i & 0 & 0 \\
\sin \varphi_i & \cos \varphi_i & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix},$$

$$L_{m_s} = \begin{bmatrix}
\cos \varphi_i & -\sin \varphi_i & 0 \\
\sin \varphi_i & \cos \varphi_i & 0 \\
0 & 0 & 1
\end{bmatrix},$$

$$H^{(j)} = p_1 + r_0^{(j)} \cot \xi^{(j)}, \quad U^{(i)} = u^{(j)} \sin \xi^{(j)} \neq 0,$$

where $h^{(j)}$ is a parameter of distribution of the cylindrical convolute helicoid.

For the action surface of spatial convolute rack drive, after a calculation, it is obtained:
The tooth surfaces \( \Sigma_i^{(2)} \) of the gear rack of these types of rack mechanisms are determined by the system of equations:

\[
\begin{pmatrix}
x_i^{(j)} \\
y_i^{(j)} \\
z_i^{(j)} \\
1
\end{pmatrix} = M_{s_i} \begin{pmatrix}
x_{j_i}^{(j)} \\
y_{j_i}^{(j)} \\
z_{j_i}^{(j)} \\
1
\end{pmatrix} = p_s - j_{2i} \cos \Sigma_i, \quad \frac{1}{j_{2i}} \sin \Sigma_i, \quad (10)
\]

where

\[
M_{s_i} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & j_{2i} \varphi_i \sin \Sigma_i \\
0 & 0 & 1 & j_{2i} \varphi_i \cos \Sigma_i \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

(11)

When (11) is substituted in (10), the following system is obtained:

\[
x_i^{(j)} = x^{(j)} = r_0^{(j)} \cos (\vartheta_i^{(j)} + \varphi_i) \pm U^{(j)} \sin (\vartheta_i^{(j)} + \varphi_i),
\]

\[
y_i^{(j)} = y^{(j)} = j_{2i} \varphi_i \sin \Sigma_i = r_0^{(j)} \sin (\vartheta_i^{(j)} + \varphi_i),
\]

\[
z_i^{(j)} = z^{(j)} = j_{2i} \varphi_i \cos \Sigma_i = \begin{pmatrix}
p_s - j_{2i} \cos \Sigma_i \\
p_s - j_{2i} \sin \Sigma_i
\end{pmatrix}, \quad (12)
\]

Figure 3. Spatial cylindrical convolute rack mechanism with velocity ratio \( j_{2i} = 2.29 \text{ mm/rad} \) and with number of teeth \( z_i = 1 \) : a) cylindrical convolute right-hand helicoid \( \Sigma_i^{(2)} \Rightarrow \xi_i^{(2)} = 98^\circ \), \( r_0^{(2)} = 4.15 \text{ mm} \), \( \vartheta_i^{(2)} \in [24,03-35,34] \), \( \varphi_i^{(2)} \in [0-10\pi] \); b) \( \Sigma_i^{(2)} \Rightarrow \xi_i^{(2)} = 120^\circ \), \( r_0^{(3)} = 17.68 \text{ mm} \), \( \vartheta_i^{(2)} \in [24,03-35,34] \), \( \varphi_i^{(2)} \in [0-10\pi] \); c) mesh region \( MR_i^{(2)} \); d) \( MR_i^{(2)} \); surfaces \( \Sigma_2^{(1)} \) conjugated with \( \Sigma_1^{(1)} \); e) surfaces \( \Sigma_2^{(2)} \) conjugated with \( \Sigma_1^{(2)} \).

B. **Cylindrical Archimedean Rack Drive**

The system of equations, describing analytically the cylindrical Archimedean helicoids, is obtained from (3), after substituting \( r_0^{(j)} = 0 \) \([1, 2]\):

\[
n_i^{(j)} = \mp p_i^{(j)} \cos \vartheta_i^{(j)} - U^{(j)} \cos \xi_i^{(j)} \sin \vartheta_i^{(j)},
\]

\[
n_i^{(j)} = \mp p_i^{(j)} \sin \vartheta_i^{(j)} + U^{(j)} \cos \xi_i^{(j)} \cos \vartheta_i^{(j)},
\]

(13)

Analogically, by substituting \( h^{(j)} = p_s \) (for \( r_0^{(j)} = 0 \) ) in the system (5) for the coordinates of the normal vector at any point of the cylindrical Archimedean helicoid, it is obtained:

\[
n_i^{(j)} = \mp p_i^{(j)} \cos \vartheta_i^{(j)} - U^{(j)} \cos \xi_i^{(j)} \sin \vartheta_i^{(j)},
\]

\[
n_i^{(j)} = \mp p_i^{(j)} \sin \vartheta_i^{(j)} + U^{(j)} \cos \xi_i^{(j)} \cos \vartheta_i^{(j)},
\]

(14)
The system of equations (14) shows unequivocally, that the cylindrical Archimedean helicoid is composed only of ordinary points, because \( \vec{n}_j^{(1)} \neq \vec{0} \).

The analytical description of the action surface of the cylindrical Archimedean rack drive is received from the equations system (9), when there \( r_0^{(1)} = 0 \) and \( h^{(1)} = p_s \) are substituted correspondingly, i.e.:

\[
\begin{align*}
\Theta^{(1)} &= \pm U^{(1)} \sin(\theta^{(1)} + \phi_j), \\
\phi^{(1)} &= \mp U^{(1)} \cos(\theta^{(1)} + \phi_j), \\
\epsilon^{(1)} &= p_s \theta^{(1)} \pm U^{(1)} \cot \epsilon^{(1)} \\
\mp p_s \sin(\theta^{(1)} + \phi_j) + U^{(1)} \cot \epsilon^{(1)} \cos(\theta^{(1)} + \phi_j) = \\
= \frac{p_s - j_{21} \cos \Sigma_r}{j_{21} \sin \Sigma_r}. \\
\end{align*}
\]

On Fig. 4 [1] the generated cylindrical Archimedean helicoids \( \Sigma_i^{(j)} \) \( (j = 1, 2) \), representing the active tooth surfaces of the rotating link \( i = 1 \); the mesh regions \( \mathcal{MR}^{(j)} \) \( (j = 1, 2) \) of the cylindrical Archimedean rack drive and the tooth surfaces \( \Sigma_j^{(j)} \) \( (j = 1, 2) \) of the link \( i = 2 \), realizing rectilinear translation [1] are graphically illustrated.

The spatial non-orthogonal Archimedean rack mechanism, realized by a partial application of 3D software technology [8, 9] is shown on Fig. 5.

![Figure 5. Spatial non-orthogonal cylindrical Archimedean rack mechanism](image)

**C. Cylindrical Involute Rack Drive**

The systems of equations (2) and (3) describe cylindrical involute helicoid, for which the condition of development of linear surface is fulfilled [2]:

\[
h^{(1)} = p_s^{(1)} + r_0^{(1)} \cot \epsilon^{(1)} = 0. \tag{17}
\]

The normal vector to the cylindrical involute helicoid is described by the system of equations

\[
\begin{align*}
n_{i_s}^{(j)} &= -U^{(j)} \cot \epsilon^{(j)} \sin \theta^{(j)}, \\
n_{i_r}^{(j)} &= +U^{(j)} \cot \epsilon^{(j)} \cos \theta^{(j)}, \\
n_i^{(j)} &= U^{(j)}. \tag{18}
\end{align*}
\]

The system of equations (18) shows that if the condition \( U^{(j)} = u^{(j)} \sin \theta^{(j)} = 0 \) is fulfilled, then the \( \vec{n}_i^{(j)} = \vec{0} \) \( (j = 1, 2) \). Hence, these points from cylindrical involute helicoids are undercutting points.
The points, which fulfill the above condition, are contact points between the generatrix $L^i$ and the basic cylinder, which radius is $r_0^{(i)}$. These points shouldn’t belong to the active tooth surfaces of the cylindrical involute worm.

From the equations system (9), for the action surface of cylindrical involute rack drive, after a substitution $h^i = 0$ it is obtained [1]:

$$
\begin{align*}
    x^i &= r_0^{(i)} \cos(\theta^i + \varphi_i) \pm U^{(i)} \sin(\theta^i + \varphi_i), \\
    y^i &= r_0^{(i)} \sin(\theta^i + \varphi_i) \mp U^{(i)} \cos(\theta^i + \varphi_i), \\
    z^i &= p_s^{(i)} U \pm U^{(i)} \cot \xi^i, \\
    U^{(i)} \cot \xi^i \cos(\theta^i + \varphi_i) &= \frac{p_s - j_{2i} \cos \Sigma_i}{j_{2i} \sin \Sigma_i}.
\end{align*}
$$

On Fig. 6 the generated cylindrical involute helicoids $\Sigma_{i}^{(j)} (j = 1, 2)$, representing the active tooth surfaces of the rotating link $i = 1$, mesh regions $MR^{(j)} (j = 1, 2)$ of the cylindrical involute rack drive and the tooth surfaces $\Sigma_{2}^{(j)} (j = 1, 2)$ of link $i = 2$, which performs rectilinear translation are graphically illustrated.

### III. ANALYSIS OF THE GEOMETRY OF ACTION SURFACE OF CYLINDRICAL RACK MECHANISM

An analytical approach that defines the geometric character of the action surface of cylindrical rack drive will be presented in this part of the article. Here and further the study will treat only spatial rack mechanisms, which rotating links have linear helicoids [1].

Let, the system (9), describing parametrically the action surface of cylindrical involute rack drive, is presented in the form:

$$
\begin{align*}
    x &= r_0 \cos \theta \pm U \sin \theta, \\
    y &= r_0 \sin \theta \mp U \cos \theta, \\
    z &= p_s \theta \pm U \cot \xi, \\
    U &= \pm \frac{h \sin \theta}{\cot \xi \cos \theta - A},
\end{align*}
$$

where $\theta = \theta + \varphi_i$, $A = \frac{p_s - j_{2i} \cos \Sigma_i}{j_{2i} \sin \Sigma_i} = \text{constant}$.

In equations (21), the above indexes of all the symbols are omitted. $U$ is substituted in the first equation of (21) and it is received:

$$
\begin{align*}
    x &= r_0 \cos \theta \mp \frac{h \sin \theta}{\cot \xi \cos \theta - A}.
\end{align*}
$$

After a substitution of $\cos \theta = t$ the obtained result is a quadratic equation regarding to $t$:

$$
(r_0 \cot \xi - h)t^2 - (Ar_0 + \cot \xi)x t + (Ax + h) = 0
$$

The solution of (23) is of the type:

$$
I_{1,2} = P_i(x) \pm \sqrt{P_i(x)},
$$

where $P_i(x) (i = 1, 2)$ are polynomials of $i$ degree, i.e.
Let, the first equation (21) is multiplied with \( \cos \theta \), the second one – with \( \sin \theta \) and they are summed. Then:
\[
x \cos \theta + y \sin \theta = r_0
\]  
(26)

After substituting (24) into (26), it is obtained:
\[
x \left[ P_1(x) \pm \sqrt{P_2(x)} \right] + 
y \sqrt{1 - P_1(x) \pm \sqrt{P_2(x)}} = r_0
\]  
(27)

Let, the curvilinear coordinate \( \gamma \) is determined by the third equation of (21):
\[
\gamma = \frac{z + U \cot \xi}{p_s}.
\]

Then \( \theta = \frac{z + U \cos \xi}{p_s} + \phi_1 = f(z,u,\phi_1). \)

When \( \cos \theta = \cos[f(z,u,\phi_1)] = t \) is substituted and when the quadratic equation (23) is solved regarding \( t \), the following two solutions - \( t_1 \) and \( t_2 \) (as a function of \( x \)) are obtained. Substituting \( \cos[f(z,u,\phi_1)] = t \) and \( \sin[f(z,u,\phi_1)] = \sqrt{1 - t^2} \) into (26), equation (27) is received. The equation (27), that describes analytically the mesh region of cylindrical convolute rack drive, is an equation of cylindrical surface with generatrix parallel to the \( z \)-axis \( OZ \) of the fixed coordinate system \( S(O,x,y,z) \).

For the cylindrical Archimedean rack mechanism, the polynomials \( P_i(x) \) (\( i = 1, 2 \)) are received, when \( r_0 = 0 \) and \( h = p_s \) are substituted in (25). Then
\[
P_1(x) = -\frac{x \cot \xi}{2p_s},
\]
\[
P_2(x) = \left[ \frac{x \cot \xi}{2p_s} \right]^2 + \frac{A\cdot x + p_s}{p_s},
\]
and equation (27) is of the type:
\[
x \left[ P_1(x) \pm \sqrt{P_2(x)} \right] + 
y \sqrt{1 - P_1(x) \pm \sqrt{P_2(x)}} = 0
\]  
(29)

The action surface of cylindrical involute rack mechanisms is obtained from (21), when it is substituted \( h = 0 \), i.e.:
\[
x = r_0 \cos \theta \pm U \sin \theta,
\]
\[
y = r_0 \sin \theta \pm U \cos \theta,
\]
\[
z = p_s \theta \pm U \cot \xi,
\]
\[
U \left( \cot \xi \cos \theta - A \right) = 0
\]  
(30)

From the fourth equation (30), it is received:
\[
\cos \theta = \frac{A}{\cot \xi}, \quad \sin \theta = \sqrt{1 - \left( \frac{A}{\cot \xi} \right)^2}
\]  
(31)

Then from (30) it is obtained
\[
A \cdot x + \sqrt{\cot^2 \xi - A^2} \cdot y - r_0 \cot \xi = 0.
\]  
(32)

The equation (29) is an equation of cylindrical surface with generatrix parallel to \( z \)-axis \( OZ \) of the fixed coordinate system \( S \). The equation (32) is an equation of a plane parallel to the axis \( OZ \).

IV. CONCLUSION

The study, shown in this paper, presents to the specialists and designers of gear transmissions a new computing method, applicable to the synthesis of spatial cylindrical convolute, Archimedean and involute rack mechanisms. The offered algorithms are thematically related not only to the synthesis and design of the treated gear systems, but also offer an analytical approach to researchers, which can be used for analysis of the basic geometric characteristics of these motions transformers. It is necessary to be mentioned, that the studied mechanical transmissions are characterized with high accuracy, smoothness, noiseless and high loading capacity, when the transformation of rotations into rectilinear translations are realized. The reason for this is that these classes of transmissions are kinematically conjugated and have large number of linear contacting active tooth surfaces, generated in accordance with the second principle of the French geometer T. Olivier.

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