Position Control of a Quadrotor Using State Transformation Technique

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***Abstract*—In this paper, we propose a state transformation technique for the position control of a quadrotor. First, we derive the dynamics of a quadrotor using the Newton-Euler formulation. Second, we present a state transformation technique to derive the position dynamics of a quadrotor. Then, we present backstepping based position control of a quadrotor. The stability analysis based on Lyapunov theorem shows that the proposed control method can realize a quadrotor system that is asymptotically stabilized. Finally, we verify the performance of the proposed position control method through the comparison simulations.**

***Index Terms*—quadrotor, position control, state transformation, backstepping control, small angle assumption**

I. Introduction

Unmanned aerial vehicles (UAVs) received a lots of attention in the past few years due to huge potential application such as exploration, transportation, reconnaissance and surveillance. Among all the UAVs, the quadrotor is the most widely applied. Since it has the advantage of easy implementation compared to other aerial vehicles, and it has the vertical take-off and landing (VTOL) ability, high agility and maneuverability.

Recently, much interest in quadrotor applications has been increased, so there has been attention on the studies that focus on the control of a quadrotor. Meanwhile, quadrotor has have complex nonlinear dynamics and coupled states. Therefore, linear control method such as proportional-integral-differential (PID) control and linear quadratic (LQ) control methods [1, 2], which focus on local behavior of the system, are difficult to expect good performance in a quadrotor system having the nonlinear dynamics. For these reasons, various nonlinear control methods have been proposed such as nested saturations [3], feedback linearization [4], sliding mode control [5], backstepping control [6], integral predictive nonlinear H_∞ control [7], and neural network based adpative control [8].

On the other hand, the position control of a quadrotor is not easy because the quadrotor position system is an underactuated system unlike the attitude system. Therefore, it is generally difficult to directly control the x, y position of a quadrotor by controlling the speed of the motors. Most previous studies use the small angle assumption (SAA) or linearize method [9-10], when deriving reference attitude angles for position control. However, in this case, if the change of the attitude angle is large, the performance of the position control is not good.

In this paper, we derive a reference attitude angles for position control using state transformation technique without special assumption. First, we derive the dynamics of the quadrotor using the Newton-Euler formulation. Then, we present the state transformation technique to derive the altitude and position control of a quadrotor. From the Lyapunov stability theory, we prove that all signals of a quadrotor system are asymptotically stable. Finally, through comparison simulations with other position control method, we verify the performance of the proposed position control method.

II. Dynamic Model of A Quadrotor

To derive the equation for the movement of a quadrotor, we should consider the coordinates of a quadrotor system. The generalized coordinates of a quadrotor system is shown in Figure 1, where B and E denote the body-fixed and earth inertial frames, respectively.

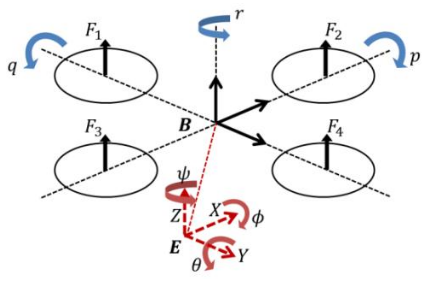

Figure 1. The coordinates and thrusts of a quadrotor.

Let us assume that the generalized velocity vectors with respect to the earth inertial and body-fixed frames

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are in the form of $\dot{\xi} = [\dot{\Gamma}^E \quad \dot{\Theta}^E]^T$ and $\nu = [V^B \quad \omega^B]^T$, respectively. Here, $\Gamma^E = (X \quad Y \quad Z)$ represents the position of the center of mass of a quadrotor and $\Theta^E = (\phi \quad \theta \quad \psi)$ are the Euler angles representing the orientation of a quadrotor, namely roll-pitch-yaw, with respect to the earth inertial frame. Similarly, $V^B = (u \quad v \quad w)$ and $\omega^B = (p \quad q \quad r)$ represents the linear and angular velocity of the quadrotor with respect to the body-fixed frame, respectively.

Now, we describe the kinematics of a generic six-DOF rigid-body as follows:

$$\dot{\xi} = \begin{bmatrix} R & 0_{3\times3} \\ 0_{3\times3} & T \end{bmatrix} \nu, \tag{1}$$

where, $0_{3\times3}$ is a 3 by 3 submatrix filled with all zeros, matrices R and T are defined respectively, as follows:

$$R = \begin{bmatrix} c_\psi c_\theta & -s_\psi c_\phi + c_\psi s_\theta s_\phi & s_\psi s_\phi + c_\psi s_\theta c_\phi \\ s_\psi c_\theta & c_\psi c_\phi + s_\psi s_\theta s_\phi & -c_\psi s_\phi + s_\psi s_\theta c_\phi \\ -s_\theta & s_\phi c_\theta & c_\phi c_\theta \end{bmatrix},$$

$$T = \begin{bmatrix} 1 & s_\phi t_\theta & c_\phi t_\theta \\ 0 & c_\phi & -s_\phi \\ 0 & s_\phi / c_\theta & c_\phi / c_\theta \end{bmatrix},$$

where $c_k = \cos(\mathrm{k})$, $s_k = \sin(\mathrm{k})$, and $t_k = \tan(\mathrm{k})$.

The dynamic model of a quadrotor is described by

$$\begin{bmatrix} m & 0_{3\times3} \\ 0_{3\times3} & I_{3\times3} \end{bmatrix} \begin{bmatrix} \dot{V}^B \\ \dot{\omega}^B \end{bmatrix} + \begin{bmatrix} \omega^B \times (mV^B) \\ \omega^B \times (I\omega^B) \end{bmatrix} = \begin{bmatrix} F^B \\ \tau^B \end{bmatrix}, \tag{2}$$

where m is the mass of a quadrotor, $I_{3\times3}$ means a 3 by 3 identity matrix. F^B is the force vector and τ^B is the torques vector of the body-fixed frame.

The nonlinear dynamics of a quadrotor can be described as follows [11]:

$$\begin{aligned} &\ddot{X} = (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\frac{u_1}{m}, \\ &\ddot{Y} = (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)\frac{u_1}{m}, \\ &\ddot{Z} = (\cos(\phi)\cos(\theta))\frac{u_1}{m} - g, \\ &\ddot{\phi} = \dot{\theta}\dot{\psi}\frac{(I_{yy} - I_{zz})}{I_{xx}} - \frac{J}{I_{xx}}\dot{\theta}\Omega_r + \frac{u_2}{I_{xx}}, \\ &\ddot{\theta} = \dot{\phi}\dot{\psi}\frac{(I_{zz} - I_{xx})}{I_{yy}} - \frac{J}{I_{yy}}\dot{\phi}\Omega_r + \frac{u_3}{I_{yy}}, \\ &\ddot{\psi} = \dot{\theta}\dot{\phi}\frac{(I_{xx} - I_{yy})}{I_{zz}} + \frac{u_4}{I_{zz}}, \end{aligned} \tag{3}$$

where J is the rotor inertia, I_{kk} $(kk = x, y, z)$ is the total inertia moment of each axis of a quadrotor, g is the gravitational acceleration, and the $u_i (i = 1,2,3,4)$ are the altitude, the roll, the pitch and the yaw control inputs represented by

$$\begin{aligned} u_1 &= b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2) \\ u_2 &= l \cdot b(\Omega_1^2 - \Omega_3^2) \\ u_3 &= l \cdot b(\Omega_2^2 - \Omega_4^2) \\ u_4 &= d(\Omega_1^2 - \Omega_2^2 + \Omega_3^2 - \Omega_4^2) \\ \Omega_r &= b(\Omega_1 + \Omega_1 + \Omega_1 + \Omega_1). \end{aligned}$$

Here, $\Omega_i (i = 1,2,3,4)$ is the propeller speed of the i-th rotor, l is the distance between the center of a quadrotor and the center of a propeller, b and d are the thrust and drag factors of a quadrotor, respectively.

III. Design of the Position Controller

In this section, we design an altitude, position and attitude controller of the quadrotor based on the backstepping control method. Specially, we use the state transformation technique to derive the reference attitude angles for position control without special assumption.

A. *Altitude Control of a Quadrotor*

The altitude dynamics in (3) can be rewritten in state-space form using the following states:

$$x_5 = Z, \qquad x_6 = \dot{Z}.$$

We have the altitude state-space equation of a quadrotor as follows:

$$\begin{cases} \dot{x}_5 = x_6 \\ \dot{x}_6 = \frac{\cos\phi\cos\theta}{m}\left(u_1 - g\frac{m}{\cos\phi\cos\theta}\right) = T_1(\phi,\theta)\cdot v_1 \end{cases} \tag{4}$$

where v_1 is the pseudo altitude control input defined by

$$v_1 := \left(u_1 - g\frac{m}{\cos\phi\cos\theta}\right), \tag{5}$$

and $T_1(\phi,\theta)$ is the first transformation variable given by

$$T_1(\phi,\theta) = \frac{\cos\phi\cos\theta}{m}. \tag{6}$$

First of all, the first error e_1 and its derivative are defined as follows:

$$e_1 = x_5 - z_d, \ \dot{e}_1 = x_6 - \dot{z}_d, \tag{7}$$

where z_d is the reference altitude of a quadrotor.

To converge the first error e_1 to zero, The virtual altitude control input for x_6 is defined by

$$\bar{x}_6 = -k_1 e_1 + \dot{z}_d, \tag{8}$$

where k_1 is a positive constant.

Secondly, the second error e_2 and its derivative are defined as follows:

$$e_2 = x_6 - \bar{x}_6, \ \dot{e}_2 = T_1(\phi,\theta)\cdot v_1 - \dot{\bar{x}}_6. \tag{9}$$

To converge the second error e_2 to zero, The virtual altitude control input for v_1 is defined by

$$v_1 = T_1(\phi,\theta)^{-1}(-k_2 e_2 + \dot{\bar{x}}_6 - e_1), \tag{10}$$

where k_2 is a positive constant.

For the invertibility of $T_1(\phi,\theta)$, throughout the paper, we use the following assumption.

Assumption 1: The attitude angles $\phi, \theta,$ and ψ satisfy the following conditions:

$$\|\phi,\theta,\psi\| < \frac{\pi}{2} \qquad (\forall \phi,\theta,\psi \in \mathbb{R})$$

Therefore, based on the definition of v_1 in (4) and (5), we can derive the actual altitude control input as follows:

$$u_1 = v_1 + g\frac{m}{\cos\phi\cos\theta}. \tag{11}$$

Theorem 1: Consider the altitude system (4) of a quadrotor controlled by the virtual and pseudo altitude control inputs (8), (10), respectively. Then, there exist the design parameters k_1 and k_2 such that the actual altitude control input (11) of a quadrotor asymptotically stabilizes the altitude system (4) of a quadrotor.

Proof: Let us consider the following Lyapunov function candidate:

$$V_z(e_1,e_2)=\frac{1}{2}(e_1^2+e_2^2). \quad (12)$$

Differentiating $V_z(e_1,e_2)$ with respect to the time and substituting (7) ~ (10), we obtain

$$\dot{V}_z(e_1,e_2)=e_1(-k_1e_1+e_2)+e_2(-k_2e_2-e_1)$$
$$=-k_1e_1^2-k_2e_2^2<0 \quad (13)$$

Therefore, by the Lyapunov stability theorem, the altitude system (4) of a quadrotor is asymptotically stable with the actual altitude control input (11).

B. Position Control of a Quadrotor

In this subsection, to realize the position control of a quadrotor, we define the variable $\tilde{u}_1$, using the actual altitude control input u_1, as follows:

$$\tilde{u}_1=T_1(\phi,\theta)u_1=\frac{cos\phi cos\theta}{m}v_1+g. \quad (14)$$

To design the position control of a quadrotor, the dynamics of a quadrotor (2) can be rewritten in the state-space form using the following states:

$$x_1=X, x_2=\dot{X}, x_3=Y, x_4=\dot{Y}.$$

We obtain the (x,y) position state-space equations of a quadrotor as follows:

$$\begin{cases}\dot{x}_1=x_2\\ \dot{x}_2=u_1(\cos\phi\sin\theta\cos\psi+\sin\phi\sin\psi)/m\\ \dot{x}_3=x_4\\ \dot{x}_4=u_1(\cos\phi\sin\theta\sin\psi-\sin\phi\cos\psi)/m\end{cases} \quad (15)$$

Rearranging these equations, we finally obtain the following position dynamics of a quadrotor as follows:

$$\begin{cases}\dot{P}=V\\ \dot{V}=\tilde{u}_1\begin{bmatrix}cos\psi & sin\psi\\ sin\psi & -cos\psi\end{bmatrix}\begin{bmatrix}tan\theta\\ sec\theta\, tan\phi\end{bmatrix},\\ \quad =T_2(e_1,e_2,\phi,\theta,\psi)\,u_p\end{cases} \quad (16)$$

where $P:=[X \quad Y]^T, V:=\dot{P}$. u_p is the actual position control input vectors given by

$$u_p=\begin{bmatrix}u_{11}\\ u_{22}\end{bmatrix}=\begin{bmatrix}tan\theta\\ sec\theta\, tan\phi\end{bmatrix}, \quad (17)$$

and $T_2(e_1,e_2,\phi,\theta,\psi)$ is the state transformation matrix given by

$$T_2(e_1,e_2,\phi,\theta,\psi)=\tilde{u}_1\begin{bmatrix}cos\psi & sin\psi\\ sin\psi & -cos\psi\end{bmatrix}. \quad (18)$$

To design the position control input using backstepping technique, the third error vector e_3 and its derivative are defined as follows:

$$e_3=P-P_d,\ \dot{e}_3=V-\dot{P}_d, \quad (19)$$

where P_d is the target position vector of a quadrotor.

To converge the third error e_3 to zero, The virtual position control input vector for V is defined by

$$\bar{V}=-k_3e_3+\dot{P}_d, \quad (20)$$

where k_3 is a positive constant.

The forth error vector e_4 and its derivative are defined as follows:

$$e_4=V-\bar{V},\ \dot{e}_4=T_2(e_1,e_2,\phi,\theta,\psi)u_p-\dot{\bar{V}}. \quad (21)$$

To converge the forth error vector e_4 to zero, the actual position control input vector u_p is defined by

$$u_p=T_2(e_1,e_2,\phi,\theta,\psi)^{-1}\left(-k_4e_4+\dot{\bar{V}}-e_3\right), \quad (22)$$

where k_4 is a positive constant.

Theorem 2: Consider the position system (15) of a quadrotor controlled by the actual position control input (22). Then, there exist the design parameters k_3 and k_4 such that the actual altitude control input (22) of a quadrotor asymptotically stabilizes the position system (15) of a quadrotor.

Proof: Let us consider the following Lyapunov function candidate:

$$V_p(e_3,e_4)=\frac{1}{2}(\|e_3\|^2+\|e_4\|^2). \quad (23)$$

Differentiating $V_p(e_3,e_4)$ with respect to the time and substituting (19) ~ (22), we obtain

$$\dot{V}_p(e_3,e_4)=e_3(-k_3e_3+e_4)+e_4(-k_4e_4-e_3)$$
$$=-k_3\|e_3\|^2-k_4\|e_4\|^2<0. \quad (24)$$

Therefore, by the Lyapunov stability theorem, the position system (15) of a quadrotor is asymptotically stable with the actual position control input (22).

The motion along the x and y axes is related to the pitch and roll angles, respectively. The reference roll(θ_d) and pitch (ϕ_d) angles of a quadrotor that enable a quadrotor to converge in the desired position are obtained from the position control input u_p designed in the subsection 3.2 [12, 13]. Therefore, the reference roll(θ_d) and pitch(ϕ_d) angles are obtained by using the position control input (17) as follows:

$$\begin{cases}\theta_d=arctan(u_{11})\\ \phi_d=arctan\left(\frac{u_{22}}{sec\theta_d}\right)\end{cases}. \quad (25)$$

C. Attitude Control of a Quadrotor

In (3), the attitude dynamics about roll movement can be rewritten in the state-space form using the following states:

$$x_7=\phi, x_8=\dot{\phi}.$$

We have the attitude about roll movement state-space equation of a quadrotor as follows:

$$\begin{cases}\dot{x}_7=x_8\\ \dot{x}_8=f_1+\frac{u_2}{I_{xx}}\end{cases} \quad (26)$$

where $f_1=\dot{\theta}\dot{\psi}\frac{(I_{yy}-I_{zz})}{I_{xx}}-\frac{J}{I_{xx}}\dot{\theta}\Omega_r$.

As in the case of altitude control, to design the attitude about roll movement control input using backstepping technique, the roll error e_7 and its derivative are defined as follows:

$$e_7 = x_7 - \phi_d,\ \dot{e}_7 = x_8 - \dot{\phi}_d, \tag{27}$$

where ϕ_d is the reference roll angle of a quadrotor.

The virtual control input about roll movement $\bar{x}_8$ is defined by

$$\bar{x}_8 = -k_7 e_7 + \dot{\phi}_d, \tag{28}$$

where k_7 is a positive constant.

The eighth error e_8 and its derivative are defined as follows:

$$e_8 = x_8 - \bar{x}_8,\ \dot{e}_8 = f_1 + \frac{u_2}{I_{xx}} - \dot{\bar{x}}_8, \tag{29}$$

To converge the second error e_8 to zero, The actual control input about roll movement u_2 is defined by

$$u_2 = I_{xx}(-k_8 e_8 - f_1 + \dot{\bar{x}}_8 - e_7), \tag{30}$$

where k_8 is a positive constant.

Similarly, the actual control inputs about pitch and yaw movement u_3 and u_4 are defined by

$$u_3 = I_{yy}(-k_{10} e_{10} - f_2 + \dot{\bar{x}}_{10} - e_9), \tag{31}$$

$$u_4 = I_{zz}(-k_{12} e_{12} - f_3 + \dot{\bar{x}}_{12} - e_{11}), \tag{32}$$

where k_{10} and k_{12} are positive constants, $f_2 = \dot{\phi}\dot{\psi}\frac{(I_{zz}-I_{xx})}{I_{yy}} - \frac{J}{I_{yy}}\dot{\phi}\Omega_r$, $f_3 = \dot{\theta}\dot{\phi}\frac{(I_{xx}-I_{yy})}{I_{zz}}$, and virtual control inputs about pitch and yaw movement $\bar{x}_{10}$ and $\bar{x}_{12}$ are defined by

$$\bar{x}_{10} = -k_9 e_9 + \dot{\theta}_d, \tag{33}$$

$$\bar{x}_{12} = -k_{11} e_{11} + \dot{\psi}_d, \tag{34}$$

where θ_d, ψ_d are the reference pitch and yaw angles of a quadrotor, respectively, and k_9, k_{11} are positive constants.

The stability proof for the quadrotor attitude system is derived the same as in Theorem 1, by the Lyapunov stability theorem, the attitude system in (3) is asymptotically stable with the attitude control inputs (30) ~ (32).

IV. Simulation Results and Analysis

In order to verify the effectiveness of the proposed position controller of a quadrotor using state transformation technique, we perform some computer simulations. The dynamics (3) of a quadrotor is employed in the simulations. The simulation is performed with the following parameters presented in Table 1. In addition, the sampling time was fixed at 0.01 [s] for simulation.

TABLE I. The Simulation Parameters.

Parameter	**Value**
m	1.0 [kg]
g	9.806 $[\mathrm{m}/s^2]$
I_{xx}, I_{yy}	2.3×10^{-3} $[\mathrm{kg}\cdot m^2]$
I_{zz}	5.09×10^{-3} $[\mathrm{kg}\cdot m^2]$
J	6.5×10^{-5} $[\mathrm{kg}\cdot m^2]$

In this simulation, we compare the performance of the position control using SAA with that of the proposed method using state transformation technique when deriving the reference angles for position control. Here, SAA means to assume that the range of roll and pitch angles of a quadrotor are small. In this simulation, the ascending circular trajectory is set as the reference trajectory, and the initial position of a quadrotor is $(\mathrm{x},\mathrm{y},\mathrm{z}) = (0, 1, 0)[\mathrm{m}]$.

The control performance depends on several gains. Therefore, selection of the gain is important for the proper performance comparison. To select the proper gains, the range of the reference attitude angle and the magnitude of the altitude input should be similar between two cases. Since, the range of the reference attitude angles for the position control changes according to the gains for position control.

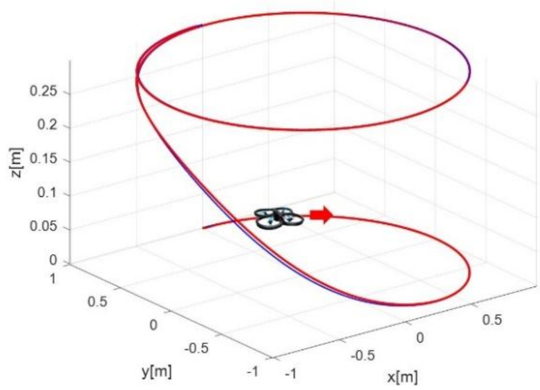

Figure 2. Backstepping based position control with state transformation technique (720-degree circular trajectory)

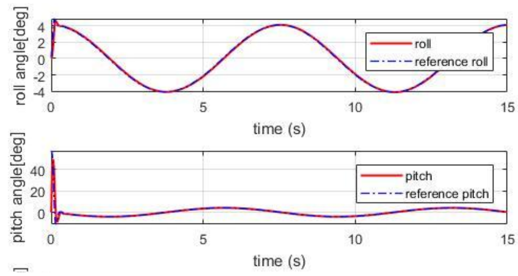

Figure 3. The reference angles for the backstpping based position control (720-degree circular trajectory)

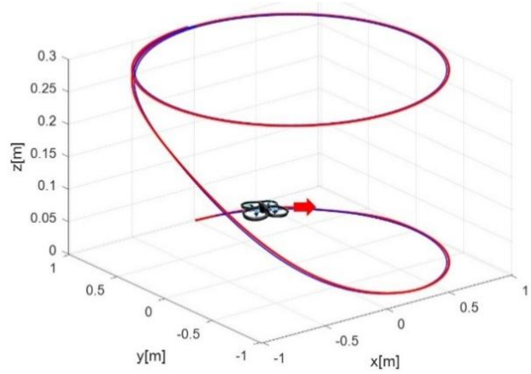

Figure 4. PD position control with small angle assumption (720-degree circular trajectory)

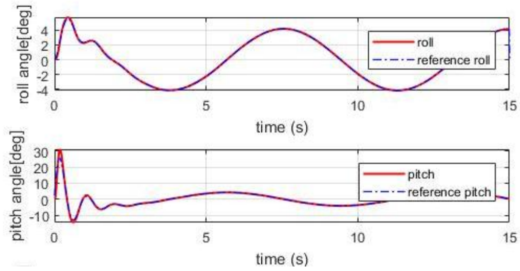

Figure 5. The reference angles for the PD position control (720-degree circular trajectory)

Figs. 2 and 3 show the simulation results obtained by applying the backstepping based position control of a quadrotor with state transformation technique. And Figs. 4 and 5 show the simulation results for the position control by applying PD controller with SAA. As shown in Figs. 2~5, there is no significant difference between two cases when the reference angles for the position control are small.

Figs. 6 and 7 show the simulation results for 2160-degree circular trajectory obtained by applying the backstepping based position control of a quadrotor with state transformation technique. And Figs. 8 and 9 show the simulation results for 2160-degree circular trajectory for the position control by applying PD controller with SAA. As shown in Figs. 7 and 9, the larger reference angles are required to track the circle trajectory faster. When the change of the attitude angle is large, in the case of deriving the reference angle using SAA, the position tracking control performance is degraded as shown in Fig. 8. On the other hand, when the reference angles for position control are derived using the proposed state transformation technique, the position tracking control performance is better as shown in Fig. 6. From the simulation rusults, we verify that the proposed position control method is effective and efficient.

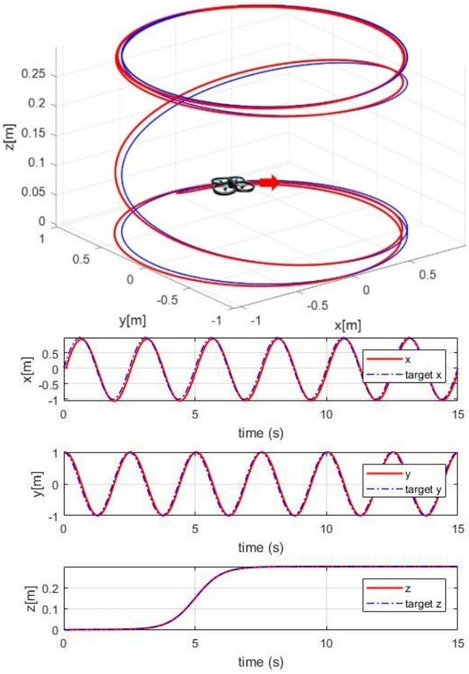

Figure 6. Backstepping based position control with state transformation technique (2160-degree circular trajectory)

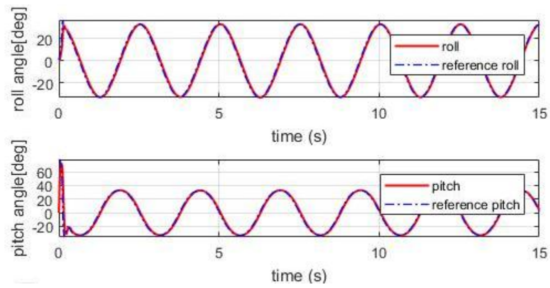

Figure 7. The reference angles for the backstpping based position control (2160-degree circular trajectory)

V. Conclusions

In this paper, we focused on the state transformation technique for the position control of a quadrotor. First, we derived the dynamics of a quadrotor using the Newton-Euler formulation. Then, we presented backstepping based position control of a quadrotor by using state transformation technique. The stability analysis based on Lyapunov theorem showed that the proposed control method resulted in the asymptotically stability. Finally, from the simulation results, we verified that the proposed position control method is effective and efficient.

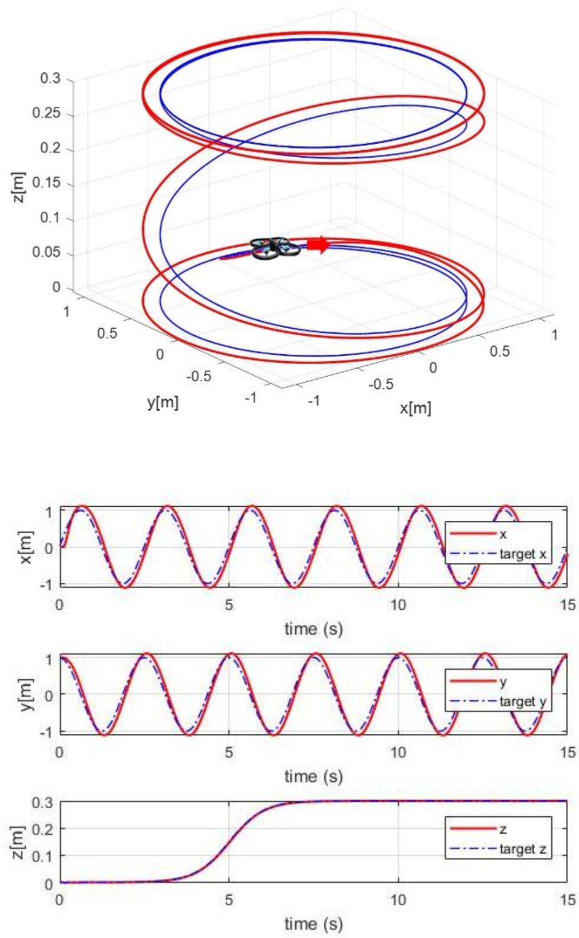

Figure 8. PD position control with small angle assumption (2160-degree circular trajectory)

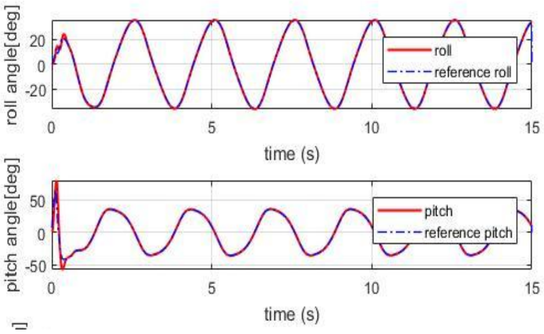

Figure 9. The reference angles for the PD position control (2160-degree circular trajectory)

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