Stability Enhancement of MLB System via Servo Approach

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Abstract—A magnetic levitation ball (MLB) is required to operate over large variations in the air gap. Specifically, it may be difficult to design a linear controller that can provide satisfactory performance, stability and disturbance rejection over a wide range of operating points. Conventional controllers with the linearisation of a nonlinear system exhibit a low-efficiency control performance. To simplify a complex control system, in this paper, we analyse the robustness and stability of disturbance for a magnetic levitation system using Jacobian linearisation from the equilibrium point, by adding one integrator in an open-loop model and servo control system design with a state observer. The experimental results show that this method can be effectively controlled by dynamic response criteria of the control system.

Index Terms—MLB, stability, linearisation, servo, observer.

I. INTRODUCTION

Many physical systems are nonlinear in their behaviour and exhibit nonlinear responses. Many designs work satisfactorily with such linearised system models. The problem is viewed as trying to follow moving targets. Most of the problems that may be described as the servo type come from a field other than chemical industry. Tracking of missiles and aircraft and the automatic machining of intricate parts from a master pattern are a well-known example of the servo problem.

The control area, the task of the nonlinear system, is one of the major problems owing to the complex behaviour, coupling, control and stability requirement. The robustness and stability of the nonlinear system have been widely studied [1]. Magnetic levitation ball (MLB) systems are inherently open-loop unstable systems and rely on feedback control for producing the desired levitation action. They are highly nonlinear and open-loop unstable systems. An unpredictable aspect of the MLB and its inherent nonlinearities make the modelling and control problems very challenging. Linear system models only work well over a small region of the operating point. The tracking performance of the linear control strategies rapidly deteriorates with the increase in deviations from the nominal operating point. Several approaches ensure the consistency of using independent operating points, which have been reported in the literature. After that, several methods have been proposed to solve this problem. In particular, some useful methods and linearisation techniques have been developed. Nonlinear control [2, 3], sliding mode control [4], neural networks [5], fuzzy logic control [6], which include linearisation, small gain theorem, Lyapunov theory, backstepping and feedback are all very successful [7, 8].

This paper is organised into five parts as follows. The first part presents an introduction to control systems and problems of the MLB system. The second part treats the mathematical modelling of the MLB system that is shown in the state-space model and the development of a linearisation technique for the MLB. The third part illustrates the design and compensation technique using a servo system with a state observer. The fourth part shows an experimental setup and discusses the experimental results of the MLB system that can control the distance of the permanent magnet ball and examines the stability of the monitoring system. Finally, the last part concludes the discussions on the control system design for the MLB system.

II. THE MLB SYSTEM

MLB is a magnetic ball suspension system that is used to levitate a steel ball in the air by electromagnetic force generated by an electromagnet. The force (current function) and the air gap relationship of magnetic suspensions are nonlinear, which places significant demands on the control techniques used in magnetic suspensions. The current technology in many magnetic bearings limits the travel of the actuator to a small region around a nominal operating point in the magnetic field. These narrow gap devices employ linear control strategies that are based on the Taylor series expansion of
that there is a unique equilibrium point, that is, the sensor induced by the levitating magnet can be written as

\[ mg - k \frac{i}{y^2} + f_d = m \frac{d^2y}{dt^2}, \quad u = Ri + L \frac{di}{dt}, \]

where \( y \) is the vertical position of the levitating magnet measured from the bottom of the electromagnet, \( m \) is the mass of the permanent magnet ball, \( k \) is the acceleration due to gravity and \( f_d \) is the disturbing force caused by the Hall-effect sensor. \( f_d \) is a function of constants that depend on the Hall-effect geometry of the magnetic field strength. \( R \) and \( L \) are the resistance and inductance of the coil, respectively, \( i \) is the current through the electromagnet and \( u \) is the control input. Let

\[ x = [x_1, x_2, x_3]^T = [y, \dot{y}, i]^T \]

be the state of the system, \( z \) be the controller output and \( v \) be the measured output of the electromagnet, which are a function of constants that depend on the Hall-effect sensor \( c_0, c_1 \) and \( c_2 \), respectively, which can be written in the state equation as follows:

\[ \dot{x} = f(x) + bu + df_d, \]
\[ z = v = c_0 + c_1 \frac{1}{x_1^2} + c_2 x_3 \]

and

\[ f(x) = \left[ x_2, g - \frac{k}{m} \frac{x_1}{x_1^2} - \frac{R}{L} x_3 \right]^T, \]
\[ b = \left[ 0, 0, \frac{1}{L} \right]^T, \quad d = \left[ 0, \frac{1}{m}, 0 \right]^T \]

where \( b \) is the input vector with the Jacobian disturbing vector force \( d \) and \( f(x) \) is called the system vector of the MLB system.

Such a linear system is equivalent to the nonlinear system considered within a limited operating range. We developed what is known as Jacobian linearisation of a nonlinear system. We can linearise the equilibrium point of the system as follows:

\[ x_{e,0} = \left( \frac{ku_a}{gmR} \right)^{1/3}, \quad x_{2,0} = 0 \quad \text{and} \quad x_{3,0} = \frac{u_0}{R} \]

We then defined \( u_0 \) as the equilibrium electromagnet voltage to suspend the levitating magnet at \( x_{1,0} = z_0 \). Note that there is a unique equilibrium point, that is, \( \delta x = A \delta x + B \delta u + d, \)
\[ \delta z = C \delta x \]

where \( A, B \) and \( C \) are called the system matrix, the input vector and the output vector of the Jacobian, respectively.

\[ \delta x = \begin{bmatrix} x_1 - x_{1,0} & x_2 - x_{2,0} & x_3 - x_{3,0} \end{bmatrix}^T \]

is a state vector at equilibrium, \( \delta u = u - u_0 \) is the voltage control and \( \delta z = z - z_0 \) is the output of the MLB system.

In the next part, we will present state-space representations of system (4) in a controllable canonical form:

\[ A = \frac{\partial f(x)}{\partial x}|_{x=x_0} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ A_{31} & A_{32} & A_{33} \end{bmatrix}, \]
\[ B = \left[ \frac{\partial(bu)}{\partial u} \right]|_{u=u_0} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^T, \]
\[ C = \left[ \frac{\partial v}{\partial x} \right]|_{x=x_0} = \begin{bmatrix} C_{11} & 0 & C_{13} \end{bmatrix}, \]

where

\[ A_{31} = \frac{R}{L}, \quad A_{32} = \frac{3g\left(gmR\right)^{\frac{1}{3}}}{\left(ku_a\right)^{\frac{1}{3}}}, \quad A_{33} = -\frac{3Rg\left(gmR\right)^{\frac{1}{3}}}{L\left(ku_a\right)^{\frac{1}{3}}}, \]
\[ C_{11} = \frac{2c_1 g^2 m R^3}{Lku_a^3} - \frac{3c_2 g\left(gmR\right)^{\frac{1}{3}}}{L\left(ku_a\right)^{\frac{1}{3}}} \quad \text{and} \quad C_{13} = \frac{c_2}{L} \]

The eigenvalues of the matrix, \( \det(sI - A) = 0 \), are the values of \( s \); one of the poles is in the right-half-plane (RHP). It has a positive real part, which means the system is unstable in the open loop.

## II. SERVO CONTROL BASED ON STATE OBSERVER

![Diagram](https://via.placeholder.com/150.png)

Figure 2. A schematic of a state feedback servo system on a state observer.

A controller design using state-variable feedback is used to create or test the function of the control design. It must contain information on all signals of the state vector of the system. However, conventional systems can measure the state variables using control state estimation. Different methods are used to design an observer that is used to estimate the state variables of the system fast and correctly.
In this section, we discuss the pole placement approach for the design of type servo control (integral control) [10]. Here, we will limit our systems to have a control signal \( \delta u \) and output \( \delta z \). If the MLB system has no integrator (type 0 plant), the principle of the design of a servo control will be to insert an integrator in the feed-forward path between the error comparator and the MLB plant, as shown in Fig. 2. Using a state observer, the control and estimation of the system’s state variables need to ensure that the system exhibits controllability and observability. We can determine an nth-order completely controllable system if the controllability matrix is available [see Eq. (4)]. The observation of the system without disturbances can be represented as follows:

\[
\begin{align*}
\dot{\delta x} &= A\delta x + B\delta u + K_o(\delta z - \dot{\delta z}) \\
\dot{\delta z} &= C\delta x
\end{align*}
\]

(5)

which can be rewritten as

\[
\dot{\epsilon}_o = [A - K_o C]\epsilon_o
\]

(6)

where \( \epsilon_o = \delta x - \dot{\delta z} \) is the difference (\( \epsilon_o \to 0 \)). If the dual system is completely controllable, then the state feedback gain \( K_o \) can be determined. Consequently, the matrix \( A - K_o C \) yields a set of the desired eigenvalues. A set of \( \lambda_1, \lambda_2, \lambda_3, \) and \( \lambda_4 \) are the selected eigenvalues for the state observer matrix, resulting in an effective gain

\[
\lambda_1, \lambda_2, \lambda_3, \lambda_4 > 0
\]

for the design of type servo control (integral control) [10].

\[
\text{To design a control law with feedback servo control, the observation is given by}
\]

\[
\begin{align*}
\delta u &= -K\dot{\delta x} + k_1e \\
\dot{e} &= \delta_{ref} - \delta z = \delta_{ref} - C\delta x
\end{align*}
\]

(8)

(9)

where \( e \) is a signal at the output of integrals, \( \delta_{ref} \) is the input reference, \( k_1 \) is an integral gain of servo control and

\[
K = [K_1 \ K_2 \ K_3]
\]

is the gain state feedback matrix from the observer.

The system can be described by an equation that is a combination of Eqs. (5) and (9):

\[
\begin{bmatrix}
\dot{\delta x} \\
\dot{e}
\end{bmatrix} =
\begin{bmatrix}
A & 0 \\
-C & 0
\end{bmatrix}\begin{bmatrix}
\delta x \\
e
\end{bmatrix} +
\begin{bmatrix}
B \\
0
\end{bmatrix}\delta u +
\begin{bmatrix}
0 \\
1
\end{bmatrix}\delta_{ref}
\]

(10)

We designed an asymptotically stable system [11]. The linear state feedback controls laws (8) and (10) can be rewritten as follows:

\[
\begin{align*}
\delta u &= -K\dot{\delta x} = -\hat{B}^TPX \\
\dot{X} &= \hat{A}X + \hat{B}\delta u + R
\end{align*}
\]

(11)

(12)

when

\[
X = \begin{bmatrix}
\delta x \\
e
\end{bmatrix}, \hat{A} = \begin{bmatrix}
A & 0 \\
-C & 0
\end{bmatrix}, \hat{B} = \begin{bmatrix}
B \\
0
\end{bmatrix}
\]

\[
R = \begin{bmatrix}
0 \\
1
\end{bmatrix}\delta_{ref}, \hat{K} = [K - k_1]
\]

For all eigenvalues of \( \hat{A} \), if \( \Re\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} < 0 \), then the equilibrium point is asymptotically stable, where \( P \) is a positive definite symmetric solution of the Riccati algebra matrix equation, which is given by

\[
P\hat{A} + \hat{A}^TP - \hat{P}\hat{B}^TP - \hat{P} = -Q
\]

(13)

Substituting the linear state feedback control law (11) into the new linearisation system (12), we can see that

\[
\dot{X} = (\hat{A} - \hat{B}^TP)X + R
\]

(14)

We can form a Lyapunov function as follows:

\[
V(X) = X^TPX
\]

(15)

Then, along the trajectory of system (15), its derivative is

\[
\dot{V}(X) = -X^TQX + 2X^TPR - X^T\hat{P}\hat{B}^TX
\]

(16)

Let \( V_1 = -X^TQX + 2X^TPR \) and \( V_2 = -X^T\hat{P}\hat{B}^TX \).

We should know the function \( R \) that satisfies

\[
\lim_{|X| \to \infty} \frac{|V_1|}{|X|^2} = \varepsilon \Rightarrow |R| < \varepsilon |X|
\]

where \( \varepsilon > 0 \), \( \delta_{ref} \leq \varepsilon < \delta_{ref}^* \) and \( \delta_{ref}^* \) and \( \delta_{ref}^* \) are the minimum and maximum reference for control, respectively. According to the matrix theory, we should know

\[
V_2 < 0
\]

(17)

If \( V_1 < 0 \), \( \dot{V} \leq 0 \), thus

\[
X^TQX \geq \lambda_{\text{min}}(Q)\|X\|^2, \quad 2X^TPR \leq 2\sqrt{\lambda_{\text{min}}(P^T)}\|X\|\|R\|
\]

When \( |X| = \sqrt{\lambda_{\text{min}}(X^TX)} \), \( |R| = \sqrt{\lambda_{\text{min}}(R^TR)} \) and

\[
\varepsilon < \frac{\lambda_{\text{min}}(Q)}{2\sqrt{\lambda_{\text{min}}(P^T)}}
\]

(18)

where \( P \) and \( Q \) are symmetric positive definite matrices and \( \lambda_{\text{min}}(Q) \) and \( \lambda_{\text{min}}(P^T) \) are positive. Thus, we can have

\[
V_1 < 0
\]

Combining Eqs. (17) and (18), we know that \( \dot{V} < 0 \) is asymptotically stable.

III. SERVO CONTROL SYSTEM DESIGN

In this section, we consider an MLB system using the pole placement based on Riccati matrix for warranting
asymptotical stability using the observer approach. We use
the following design procedure:
1. A state-space model of the MLB system (2).
2. Obtain the mathematical model of the MLB system
using Jacobian linearisation, which is stated in a
controllable canonical form (4).
3. Prove the observability condition by determining
the desired observer poles (closed-loop poles) for
pole placement and calculate the observer gains (7).
4. Prove the controllability condition by choosing a
matrix $Q$ with a new linearisation system (12).
5. Solve Eq. (13), the matrix Riccati equation, to
obtain the matrix $P$, and substitute this matrix $P$ into
Eq. (11) to solve matrix $\hat{K}$, which is an optimal
matrix.

The considered system rank $[C^T : A^T C : (A^T)^2 C^T]$ is
3. Thus, this system is observable. The design consists of
a gain matrix of the observer, which has to be determined.
We make such a choice because a set of closed-loop
poles result in a reasonable or acceptable transient
response. The specifications and system parameters of the
proposed MLB system are listed in the Table I.

### Table I. Parameters of the Magnetic Levitation System.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameters</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic coil</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mass of ball (barium-strontium)</td>
<td>$m$</td>
<td>$41.30 \times 10^{-3}$</td>
<td>kg</td>
</tr>
<tr>
<td>Resistance</td>
<td>$R$</td>
<td>$1.71$</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>Inductance</td>
<td>$L$</td>
<td>$15.10 \times 10^{-3}$</td>
<td>$H$</td>
</tr>
<tr>
<td>Electromagnetic constant</td>
<td>$k$</td>
<td>$3.10 \times 10^{-6}$</td>
<td>kg·m/s²/A</td>
</tr>
<tr>
<td>Hall-effect sensor</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Output voltage (equilibrium)</td>
<td>$\mu_o$</td>
<td>$1.79$</td>
<td>V</td>
</tr>
<tr>
<td>Sensor gain</td>
<td>$c_0$</td>
<td>$2.48$</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>$c_1$</td>
<td>$4.25 \times 10^4$</td>
<td>V·m²/V</td>
</tr>
<tr>
<td></td>
<td>$c_2$</td>
<td>$0.31$</td>
<td>V/A</td>
</tr>
</tbody>
</table>

In the conventional approach to design a SISO system,
we designed a compensator where the dominant closed-loop
poles have the desired damping ratio $\zeta$ and natural
frequency $\omega_n$. In this approach, we assume that the effects
on the responses of nondominant closed-loop poles are
negligible. We have the pole of the observer at
$\lambda_{o1,2} = -210 \pm 42i$ and $\lambda_{o3} = -200$, which can be written
in a characteristic equation as follows:

$$s^3 + 620s^2 + (1.2986 \times 10^5)s + 9.1729 \times 10^6 = 0$$

The gain matrix of the observation yielded $[sI - A + K_o C]$, which is equal to zero and can solve the
matrix gain observer as follows:

$$K_o = [0.0067 \ 1.2367 \ 13.0654]^T.$$  

The next step is to verify the controllability of the
system. The new system’s rank $[\hat{B} : A\hat{B} : (A^T)^2 \hat{B} : (A^T)^3 \hat{B}]$
is 4. Thus, this system has the ability $Q$ of the arbitrarily
positive defined matrix to control the design matrix gain of
the controller.

$$Q = 10^3 I$$

We used MATLAB to compute the matrix Riccati equation, in order to obtain the matrix $P$.

$$P = \begin{bmatrix}
2.1123 \times 10^6 & 5.6018 \times 10^7 & 1.8507 \times 10^6 & \text{Symmetric} \\
3.9428 \times 10^6 & 1.2157 \times 10^6 & 85.3441 & \\
-3.3794 \times 10^6 & -2.1352 \times 10^6 & -31.6228 & 201.8905
\end{bmatrix}$$

The resulting matrix

$$\hat{K} = 10^3 \cdot [3.9428 \ 0.1216 \ 0.0009 \ -0.0003]$$
is the optimal matrix.

### IV. EXPERIMENTAL RESULTS AND DISCUSSION

The experiment setup of the MLB system is shown in
Fig. 3 on an electrodynamic shaker system. The MLB
system is manufactured by Zeltom LLC [12]. This system
consists of an electromagnet, a permanent magnet ball, a
Hall-effect sensor, a data acquisition analogue-to-
digital/digital-to-analogue (AD/DA) board and a control
computer. The Hall-effect sensor is connected to one of
the analogue inputs of a RABCON control board, and the
electromagnet is driven by one of the H-bridges of the
same board. Two forces act on the steel ball: gravity and
the electromagnetic force from the coil. Additionally,
the system is connected to a computer with a PCI serial card
that can implement the controller in a Simulink model.
The control circuitry consists of a set of power supplies
and amplifiers connected to a control computer. The
control computer has a dedicated digital signal processor,
programmed through it. The power supplies and
amplifiers receive the signal from the position sensor and
send power to the actuator. Input and output boxes wired
to the AD and DA converters on the computer allow the
power electronics and the computer to talk to each other.

Here, we achieved real-time operation with a sampling
rate of 2.048 kHz. Comparisons also highlighted the
importance of servo controller procedures and
investigated the boundary stability of control system on
the observer from position control to be provided to
demonstrate effectiveness.

#### A. Examining the Stability of the Control System

The operation of experimentation must define the
boundary conditions and the function of each control type.
The ramp signal is the condition of reference input to
trace the performance of the servo controller for
controlling the MLB system as follows:

Figure 3. Experiment setup of the MLB system.

\[ \delta_{ref} = 17.9 \pm \frac{t}{150} \text{ mm.} \]

Ten seconds from a set point until the magnet is out of control and can be defined as the lower and upper limit boundary of the control system as shown in Figs. 4 and 5. The signal can adapt to the reference input signal, which is a linear relationship.

B. Tracking Responses

The experimental result demonstrates the effects that respond to the dynamics of the system when there is a change in the input step without external force interference. The reference system is the input signal. The system functions by allowing signals in the voltage to enter into the induction coils to generate a magnetic field, and the levitating ball responds to the input signal.
The test responses to the step input and the sinusoidal input references are as follows:

- **Case 1.** Set point at 17.9 mm and increase by 0.5 mm, every 10 s, and then return to the set point.
- **Case 2.** Change the sinusoidal range for high frequencies. Amplification is functionalities at $t > 10$ s from the set point:

$$\delta_{ref} = 17.9 + t/150\sin(0.4\pi t) \text{ mm}.$$ 

- **Case 3.** Similarly, the sinusoidal variation range in Case 2 is changed to be $\sin(0.2\pi t)$ mm.

Figs. 6, 7 and 8 show the response caused by the operation of the servo controller with a state observer. The observation state estimates the state of a variable of the system to use in feedback for effective control. The following behaviour of observation is consistent with the actual system and responds in the same direction. The lifting of the magnetic ball is swung with a harmonic because of the nature of the system. The signal must be controlled in the manner of a harmonic magnetic force to keep the displacement of the lifting of magnetic levitation.

### C. Robust Disturbance

In the final test, the experimental result with a disturbance response was studied with the response effect to the dynamics of the system when there is a change of the external force interference. The reference system is the input signal. The system functions by allowing signals in the voltage to enter into the induction coils, and the levitating ball responds to the input signal at the equilibrium point. This regulation is controlled within the range of the linear boundaries as mentioned in the previous section. The disturbance of response shown in Fig. 9 is a sinusoidal function with a peak amplitude of ±1.5 mm and ramped at $t > 10$ s from the set point. This is considered the robust external interference that occurred. Figure 10 shows the response of the servo controller with the disturbance. As a result, servo controllers are also robust to external disturbances.

The fundamental feature of the system is nonlinearity, as well as instability. Therefore, controls have a dramatic effect on the effectiveness of the system response. On the contrary, the servo controller has advantages regarding its ability to track the input reference. According to types, this control is an integral component that increased the system type and reduces error values at equilibrium. The responsiveness of the system needs to meet the requirements of the users, and it needs to be robust to external disturbances under control conditions such as limited settings, and the error at the equilibrium point must be zero. The state observation of the system can also substitute the sensing instruments effectively.

![Figure 9. Responding to disturbance.](image)

![Figure 10. Response of the servo controller with a disturbance.](image)

### V. Conclusions

The servo techniques is also becoming increasingly applicable in the industry, which places significant demands on accurate positioning. Moreover, it proved superior to other approaches, such as precision motion control systems, since it provides large travel, high bandwidth and high accuracy (limited only by the sensing technology). An additional advantage of this techniques can apply to operate a precision work and a noncontact actuator such as the magnetic bearing system, transportation, magnetic suspension, vibration isolation, etc.

The MLB system is a nonlinear, dynamic, open-loop, unstable system. The control system design for the MLB system should be uncomplicated and easy to use. In this paper, we demonstrated examining the stability and servo control system design with a state observer. The mathematical model of the MLB system was linearised on the basis of Jacobian linearisation with the equilibrium point. The selected linearised model of the MLB system is used to design a controller for suspending the magnet ball away from the electromagnet. In addition, these closed-loop poles correspond to the desired poles in the Lyapunov theory via the estimation of immeasurable state variables using a state observer. The experimental results verified the robustness and stability of disturbance.
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