Effect of Unbalanced Force on the Crack Breathing Mechanism

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***Abstract*—The dependence of crack breathing behaviors on the crack location was investigated under the effect of unbalanced force. A parameter known as the effectual bending angle is introduced to describe the nonlinear relationship between crack direction and bending direction for balanced and unbalanced shafts along the shaft length. The breathing behavior of cracks was visualized by examining the duration of each crack status (open, closed, and partially open/closed) during a full shaft rotation. It is shown that a crack in an unbalanced shaft has more breathing patterns than a crack in a balanced shaft, including a single status (fully open/never closed or fully closed/never open) and dual statuses. Two pairs of interesting locations along the shaft length were identified, where the crack shows specific breathing behaviors. Further, the angular range, during which a crack remains fully closed, partially open/closed, or fully open, changes significantly with the crack location. The analytical model developed in this work can be further utilized to obtain the time-varying stiffness matrix of the cracked shaft element under the influence of unbalanced force.**

***Index Terms*—Crack breathing, crack location, unbalanced force, rotating shaft.**

I. Introduction

Breathing of fatigue cracks is considered to be one of the main rotor faults in rotating machinery. It attracted a great deal of attention in the literature as one of the main causes of dangerous damage in rotor systems [1]. During the last decade alone, a wide variety of analytical and practical methods have been used or developed for the detection of transverse rotor cracks [2–4]. There are some papers in the literature on cracked shafts used gaping crack models, where the crack was considered to be always fully open [5, 6]. However, such a model does not represent the actual breathing of a fatigue crack. Improvements on the nonlinear nature of crack breathing are seen through a switching crack model [7, 8]. In the switching crack model, the crack is considered to be either fully open or fully closed. Further, switching crack models are associated with chaotic and quasiperiodic vibrations that are not seen during experimental testing. Recently, a number of papers have used more realistic trigonometric functions to describe the crack breathing mechanism of a rotating shaft [9, 10].

As mentioned earlier, numerous studies on cracked rotors are performed on the basis of a crack breathing mechanism under static loading, which disallows accurate modeling of crack breathing behaviors under unbalanced force. Moreover, almost all existing models are not applicable near the shaft's critical speed because equations of motion developed under the assumption of rotor static load dominance are no longer suitable for analysis near the critical speed. As such, a few studies have examined some facets of nonlinear crack breathing [9–13]. Bachschmid et al. [12] studied the nonlinear breathing mechanism by significantly reducing the damping of the cracked rotor system so as to amplify the influence of the breathing behavior of the crack. Cheng et al. [9] found that unbalance can restore the stability of a rotor. Rubio et al. [13] used commercial finite element methoding software to model the effects of mass unbalance on the breathing behavior of a cracked Jeffcott rotor with cracks. Further, the key aspect of any crack model is the reduction in stiffness introduced by the crack. The localized reduction in stiffness is directly related to the crack depth, whereas the global reduction in stiffness is influenced by both the crack depth and the crack location along the shaft. Unfortunately, all researchers opt to either ignore crack location or mitigate its effects.

It is clear that an accurate analytical model is still absent, which considers the coupling influence of both crack location and unbalanced force on the crack breathing mechanism. In this paper, we aim to develop an unbalanced shaft model to study the effect of crack location on the breathing behavior of fatigue cracks under an unbalanced force. Firstly, the proposed model describes the relative angle between the crack and shaft bending directions and is then used to find the breathing behavior of the crack at different crack locations by examining the duration of each crack status (fully open, fully closed, and partially open/closed). Results are also compared with those obtained using a balanced model, which do not include the effect of unbalanced force.

II. Determination of the Effectual Bending Angle

The model seen in Fig. 1 was developed to describe the periodic relationship between the bending load and the crack direction of a system subject to cracking and

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mass unbalance for different crack locations. Regardless of the type of loading (static and/or dynamic), the breathing of a fatigue crack is governed by the effectual bending angle, φ, the proximity of the direction of the bending load on the system relative to the crack direction.

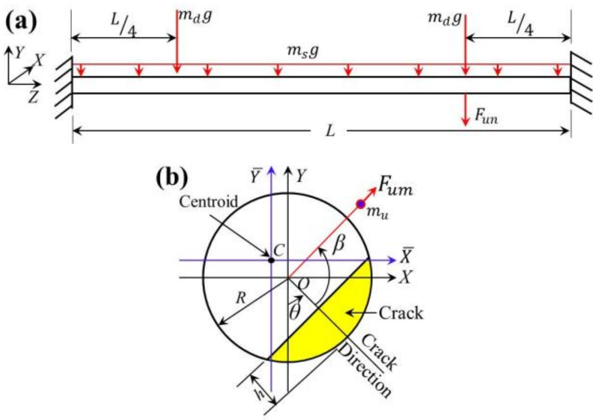

Figure 1. (a) A two-disk rotor supported rigidly and (b) relative orientation between an unbalanced force and the crack on the shaft cross section plane.

In this analysis, a two-disk rotor model supported rigidly by two bearings has been considered. The model consists of a straight front oriented crack on a plane normal to the axis of the shaft. As shown in Fig. 1(b), h is the crack depth in the radial direction and R is the shaft radius. The parameters of the rotor model are given in Table I. The static load due to the shaft weight, $m_s g$, is uniformly distributed, where m_s is the mass of the shaft. The weights of the two disks, $2m_d g$, are static point loads, and m_d is the mass of each disk. The unbalanced force, $F_{un} = m_u \omega^2 d$, is considered as the rotational load due to the unbalance mass, m_u, located on the right disk as an additional mass at a radial distance d from the center of the shaft, where ω is the shaft rotation speed [see Fig. 1(b)]. The direction of the static forces due to the disks and the shaft always acts on the negative Y-axis. Therefore, the gravitational moments M_{mg1} and M_{mg2} due to $m_s g$ and $2m_d g$, respectively, act perpendicularly to the load direction (clockwise) along the negative X-axis. The rotational load F_{un} acts in the radial direction with a fixed angular position β relative to the crack direction. Therefore, the direction of the rotational unbalance load F_{un} is $(\theta + \beta)$, where θ is the shaft rotation angle and β is the angular position of the unbalanced force relative to the crack direction at time t. The direction of the unbalance moment M_{un} due to F_{un} is perpendicular to F_{un} (see Fig. 2) and rotates anticlockwise by ω.

TABLE I. PARAMETERS OF THE UNBALANCED CRACK ROTOR SYSTEM.

Description	Value
Shaft length, L	724 mm
Shaft radius, R	6.35 mm
Density, ρ	7800 kg/m^3
Disk mass, m_d	0.50 kg

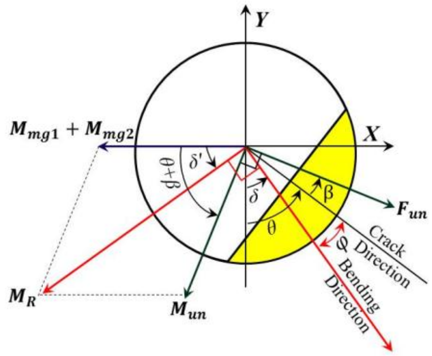

Figure 2. Effectual bending angles for different crack locations.

The gravitational moments M_{mg1} and M_{mg2} and unbalance moment M_{un} along the shaft length are described in Eqs. (1)–(6), where l_0 is the crack location.

$$M_{mg1} = \frac{m_s g}{12L}(6Ll_0 - L^2 - 6l_0^2) \quad \text{when } 0 \le l_0 \le L \tag{1}$$

$$M_{mg2} = m_d g l_0 - \frac{3m_d gL}{16} \quad \text{when } 0 \le l_0 \le \frac{L}{4} \tag{2}$$

$$M_{mg2} = \frac{m_d gL}{16} \quad \text{when } \frac{L}{4} < l_0 < \frac{3L}{4} \tag{3}$$

$$M_{mg2} = m_d g(L - l_0) - \frac{3m_d gL}{16} \quad \text{when } \frac{3L}{4} \le l_0 \le L \tag{4}$$

$$M_{un} = \frac{5F_{un}l_0}{32} - \frac{3F_{un}L}{64} \quad \text{when } 0 \le l_0 \le \frac{3L}{4} \tag{5}$$

$$M_{un} = \frac{27F_{un}}{32}(L - l_0) - \frac{9F_{un}L}{64} \quad \text{when } \frac{3L}{4} \le l_0 \le L \tag{6}$$

According to the principle of the superposition theory, the total moments at crack location on the X-axis and on the Y-axis are described in Eqs. (7), and (8), respectively.

$$\sum Mx = M_{mg1} + M_{mg2} + M_{un}\cos(\theta + \beta) \tag{7}$$

$$\sum My = M_{un}\sin(\theta + \beta) \tag{8}$$

As shown in Fig. 2, the bending direction of the shaft is perpendicular to the resultant moment direction. The angle δ' of the resultant moment with the X-axis is the same as δ of the bending direction with the negative Y-axis. It should be pointed out that the unbalance load F_{un} is not always located at the crack plane. Figure 2 only represents a projection of F_{un} on the crack plane. The effectual bending angle, φ, described in Eq. (10), solely determines the breathing of the crack at any crack location, where δ as described in Eq. (9), is the bending direction with the negative Y-axis. Modifications were made to ensure that δ and φ are within the codomain of full rotation of shaft values between 0 and 2π.

$$\delta = \tan^{-1}\left(\frac{\sum My}{\sum Mx}\right) \tag{9}$$

$$\varphi = \theta - \tan^{-1}(\frac{\sum My}{\sum Mx}) \tag{10}$$

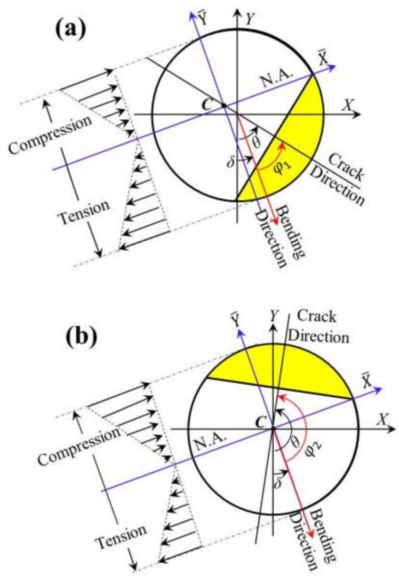

Figure 3. Crucial crack rotating angles for an unbalanced shaft: (a) the crack begins to close; (b) the crack becomes fully closed.

The crack starts to close at a certain shaft rotation angle, θ, when the effectual bending angle $\varphi = \varphi_1$, where the upper end of the crack edge reaches the compression stress field as shown in Fig. 3(a), and the crack becomes fully closed at a certain shaft rotation angle when the effectual bending angle $\varphi = \varphi_2$, where the crack fully reaches the compression stress field as shown in Fig. 3(b). Table 2 shows the full status of the crack breathing for a complete rotation. φ_1 and φ_2 are given in Eqs. (11), and (12), respectively, where μ is the nondimensional crack depth ratio h/R and e is the location of the centroid at time zero as given in Eq. (13). Equations (11), and (12), are initially developed for the balanced shaft [14].

$$\varphi_1 = \tan^{-1}(\frac{e + R(1-\mu)}{R\sqrt{\mu(2-\mu)}}) \tag{11}$$

$$\varphi_2 = \frac{\pi}{2} + \cos^{-1}(1-\mu) \tag{12}$$

$$e = \frac{2R^3}{3A_1}[\mu(2-\mu)]^{\frac{3}{2}} \tag{13}$$

TABLE II. STATUS OF CRACK BREATHING FOR A COMPLETE ROTATION.

Effectual bending angle (°)	Status of the crack
$0 \le \varphi < \varphi_1$	Fully open
$\varphi_1 \le \varphi \le \varphi_2$	Partially open/closed
$\varphi_2 \le \varphi < 2\pi\text{-}\varphi_2$	Fully closed
$2\pi\text{-}\varphi_2 \le \varphi \le 2\pi\text{-}\varphi_1$	Partially open/closed
$2\pi\text{-}\varphi_1 < \varphi \le 360$	Fully open

III. RESULTS AND DISCUSSION

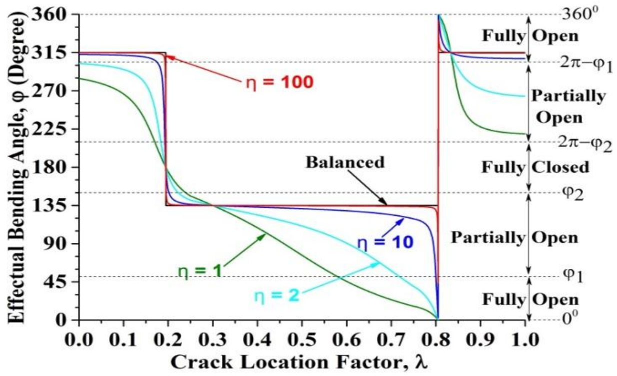

Figure 4. Effectual bending angle along the shaft length for different force ratios, η, where $\theta = 135°$.

In order to know the influences of the location on the crack, the analysis has considered different crack location factors, λ (the ratio of the crack position to the total shaft length) with different force ratios, η (the ratio of the static force to the unbalanced force). A series of analyses have been done using MATLAB. In this study, the shaft rotates anticlockwise and the initial crack direction aligns with the negative Y-axis. A crack with a depth ratio of $\mu = 0.5$ and an angular position of unbalanced force relative to the crack direction, $\beta = 0°$, is chosen to perform the analysis. Throughout the paper, focus is placed on the influences of the crack location on the breathing behavior of a cracked shaft. Figure 4 describes the variation of effectual bending angles along the shaft length. The effectual bending angles of balanced shafts are constant but exhibit a change of 180° at crack locations $\lambda = 0.1946$ and 0.8053, where the bending moment due to the total gravitational force (shaft and disks), $M_{mg} = M_{mg1} + M_{mg2}$, is zero and the moment changes direction across these two inflection points (see Fig. 5). Between the two inflection points, the moment is in the positive X-axis and the bending direction aligns along the negative Y-axis, perpendicular to the moment direction (clockwise). Hence, the relationship between the effectual bending angle and shaft rotation angle is $\varphi = \theta$, which is in agreement with previous results in [14], where the crack is assumed to be in the middle of the shaft. This relationship is clearly explained in the given example in Fig. 5. For the two remaining crack regions, the relationship between the effectual bending angle and shaft rotation angle is $\varphi = 180° + \theta$.

For an unbalanced shaft, the effectual bending angles along the shaft length are remarkably different from the balanced one. There are two pairs of crack locations along the shaft where the bending angle is independent of the force ratios, η. As mentioned earlier, at inflection points $\lambda = 0.1946$ and 0.8053, the gravitational moment is zero (see Fig. 5), and therefore the bending direction is solely determined by the unbalance force moment, M_{un}. It should be pointed out that these two crack locations are in different unbalance force moment regions, that is, negative at the first location and positive at the second location, as shown in Fig. 5.

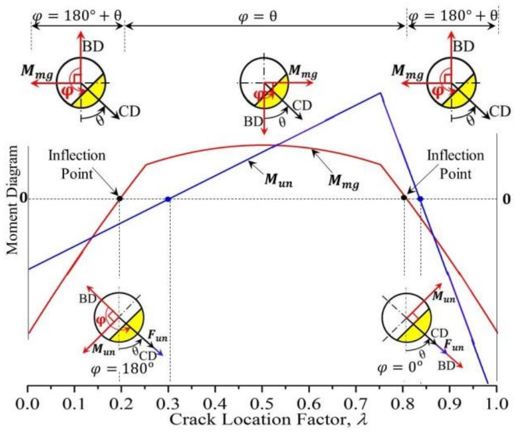

Figure 5. Effectual bending angle due to the gravitational moment (red line) and unbalance moment (blue line). CD represents the crack direction and BD stands for the bending direction of the shaft.

As a result, the effectual bending angle is 180° at the former crack location and 0° at the latter crack location. Further, as shown in Fig. 4, the bending angles at these two locations are independent of the force ratios. A small amount of unbalanced force would have the same effect on the bending angle as a large unbalanced force. Therefore, if the crack is located around these two positions, then the effect of unbalanced force on the crack breathing behavior must be considered. It should also be mentioned that the jump of the bending angle from 0° to 360° at λ = 0.8054 is a result of the crack direction changing from leading to following the bending direction.

The other interesting pair of crack locations are at λ = 0.3 and 0.8335, where the bending angles for all force ratios are same value as the balanced shaft. At these two crack locations, the unbalance force moment, M_{un}, is zero (see Fig. 5) and the gravitational force moment, M_{mg}, is solely responsible for the bending of the shaft. As a result, the cracks will breathe as they would in a balanced shaft. Further, as the unbalance force decreases (force ratio increases), the bending angles will progressively approach those for the balanced shaft, which shows that the unbalanced model will be finally in agreement with a balanced model when the force ratio is large enough.

In Fig. 6, the statuses of the crack over a full shaft rotation for balanced shaft and unbalanced shaft (η = 1) are shown. It is clear that crack opening and closing strongly depend on the crack location. The crack status for the balanced shaft is symmetrical about the first half and second half of the shaft rotation. When the crack is located between two inflection points, the crack follows a sequential change from fully open to partially open/closed, fully closed, partially open/closed, and then fully open again. This is a characteristic of the balanced shaft and was previously observed by many researchers [12, 14]. When the crack is located outside this region, the crack in the balanced shaft follows a sequential change, beginning with a fully closed status at θ = 0°. It should be pointed out that, for both cases, the duration of each crack status remains unchanged (see Fig. 6).

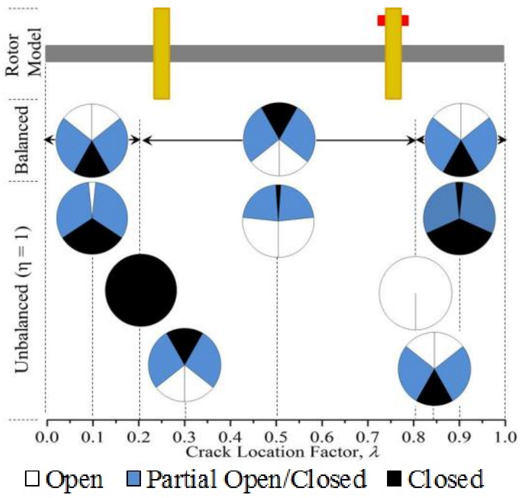

Figure 6. Statuses of the crack over a full shaft rotation at different crack locations.

Notably, different crack breathing behaviors along the shaft length have been identified for an unbalanced shaft. At crack location λ = 0.1946, an unbalanced shaft is just like an uncracked shaft and the crack will never open during shaft rotation. The shaft will have maximum stiffness and it becomes virtually identical to an intact shaft. At crack location λ = 0.8054, the crack behaves just like a notch and will never close during shaft rotation. The shaft will have minimum stiffness. At crack locations λ = 0.3 and 0.8335, the change in the crack status for the unbalanced shaft during a full shaft rotation is consistent with that of the balanced shaft. Therefore, if the crack is located at these two locations, it will breathe completely like one in the balanced shaft and so the shaft will have the same stiffness as the balanced shaft. One key difference is that the initial crack status at λ = 0.3 is fully open; however, it is fully closed at λ = 0.8335. The reason behind this is that the two crack locations are in two different crack breathing regions of the balanced shaft (see Fig. 5). The status of the unbalanced shaft at λ = 0.9 over a full shaft rotation showed a dual crack breathing behavior (i.e., fully closed and partially open/closed). For this case, the unbalanced shaft stiffness will be between the maximum and the minimum values (see Fig. 6).

IV. Conclusion

A new analytical model was developed to study the coupling effects of crack location and unbalance force on the crack breathing mechanism. The effectual bending angle was introduced in the model to describe the breathing behavior of a crack at different crack locations. This effectual bending angle along the shaft length was presented and two pairs of interesting crack locations were identified. At crack locations λ = 0.1946 and 0.8054, the total gravitational moments due to the shaft weight and two disks are equal to zero. At the former location, the crack never opens during shaft rotation and an unbalanced shaft is just like an uncracked shaft, whereas at the latter location, the crack never closes during shaft rotation and the crack behaves just like a notch.

The unbalance force moment is zero at crack locations λ = 0.3 and 0.8335, and the crack in the unbalanced shaft will breathe exactly the same as in the balanced shaft. It was further identified that crack location can significantly change the angular range during which the crack remains fully closed, partially open/closed, or fully open. Finally, as the unbalance force decreases, the breathing behavior of the unbalanced model will gradually approach the breathing behavior of a balanced shaft. The model developed in this work can be further used to obtain the time-varying stiffness matrix of the cracked shaft element consisting of area moments of inertia and then to investigate the vibration behavior of the cracked rotor by solving the equations of motion.

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