A Quadratic Regression Model with Interaction to Optimize the Turning Conditions of Mild Carbon Steel

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***Abstract*—Surface roughness of turned products is an important quality measure in machining operations. In order to investigate the dependence of surface roughness on the turning process variables in the case of mild carbon steel (MCS), experiments were carried out according to the Design of Experiments methodology. Three process variables were studied: depth of cut, feed rate, and spindle speed. A 3 ×3 ×4 full factorial design with three replicates was generated and conducted. The average surface roughness of machined specimens was measured in these experimental runs. The results of surface roughness were then used to develop a quadratic regression model with interaction. This model was then examined using factorial plots and hypothesis tests. Accordingly, the model was revised and used to identify the optimal conditions of depth of cut, feed rate, and spindle speed that minimize the surface roughness of turned parts.**

***Index Terms*—optimization, design of experiments, regression analysis, ANOVA, turning process**

I. Introduction

Traditional machining operations such as the turning process are characterized by leaving their distinct and imperfect surface covered with grooves and irregularities, which is called surface roughness. In most cases, this roughness is considered unacceptable because it may lead to excessive wear, act as local stress raisers, or compromise safety. Although it is unavoidable, it can be significantly reduced by manipulating the relevant process conditions that have a large effect on it.

Many researchers have investigated this problem because of its significance. Some researchers have utilized the Design of Experiments methodology in their investigations because it is very efficient in generating designs that are more economical and maintain the effect of nuisance and time-related factors on the data at minimum levels [1]–[3]. In trying to optimize process conditions, different methods can be found in use by researchers in the literature. While the genetic algorithm is a commonly used method of optimization [4]–[6], some have used approaches such as surface response methodology [7], [8]. Other researchers have found use in approaches like the generalized pattern search algorithm [1], Taguchi technique [9], [10], or even fuzzy logic [7].

Research studies in the literature show a rich variety of materials being investigated, which reflect their importance in industrial applications. Although the list is long, materials such as AISI D2 steel, 6061-T6 aluminum, glass-fiber-reinforced plastic, Inconel 718 superalloy, and brass are widely encountered in the literature.

In this work, the aim is to optimize process conditions for the turning of mild carbon steel (MCS) in order to minimize the surface roughness of turned parts. This will be done through conducting designed experiments to collect the required surface roughness data. Then, a quadratic regression model with interaction will be constructed and employed to estimate the best subset of process conditions to use for turning of MCS parts.

II. Experimental Work

Turning experiments were conducted on a CNC lathe machine. A single-edge cutting tool that is made of high carbon steel was used in all runs. At each setting of depth of cut, feed rate, and spindle speed, a specimen was machined (i.e., turned). The specimen's surface was then scanned with a stylus-type profilometer to measure the average surface roughness (R_a). The profilometer then provides a digital readout of R_a in units of μm, which is calculated according to the following equation [11]:

$$R_a = \frac{1}{L}\int_0^L | z(x) dx, \tag{1}$$

where L is the evaluation length, z is the asperity height, and x is the position along the scanned length. The depth of cut was studied with three fixed levels (0.05, 0.1, and 0.2 mm) as well as the feed rate (0.04, 0.08, and 0.16 mm/rev). Spindle speed had four fixed levels (170, 305, 555, and 745 rpm). For each combination of the three process variables, three replicates were also generated. Thus, a total of 108 (3 × 3 × 4 × 3) experimental runs were conducted.

Minitab® statistical software was used to generate a randomized factorial design with replicates. The produced design was used to guide the run order of all experiments. It should be noted that, at each experimental

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run, a depth of cut of 1 mm was initially removed from the specimen for cleaning and surface conditioning before any observation was made.

III. Data Results and Analysis

Due to the large amount of data for the average surface roughness, they are listed in Table III of the Appendix. In this table, the "Std. order" refers to the standard order of the runs. Each R_a value represents the average of three readings of the average surface roughness. This was done because of the high variability nature of roughness measurements. One advantage of using a factorial design is that it allows analyzing the data using two types of factorial effect plots: the main effect plot and the interaction effect plot. The first one is shown in Fig. 1 for each factor individually (generated with the aid of Minitab®). In a main effect plot, a plotted point represents the average of all data points that correspond to a particular factor level. Thus, a data point on the main effect plot for the depth of cut represents the average of 36 observations (because we have 108 observations divided equally over three factor levels). It can be seen in Fig. 1 that changes in the level of the depth of cut have a little effect on the average surface roughness. However, an increase in the feed rate or a decrease in the spindle speed is seen to increase the average surface roughness overall.

The second type is the interaction effect plot, which is shown in Fig. 2 for all possible pairs of factors. Since there are three factors in this study, we have only three possible pairs and three interaction effect plots. However, Fig. 2 shows six plots because each interaction effect plot can be constructed in two ways depending on which of the two factors occupies the *x*-axis. In an interaction effect plot, a plotted point represents the average of all data points that correspond to a particular combination of the two factor levels. Thus, a data point on the interaction effect plot between the depth of cut and spindle speed (plot of 3rd row and 1st column) represents the average of nine observations (because we have 108 observations divided equally over 12 possible combinations of the two factor levels). In general, an interaction effect plot that shows curves that are mostly parallel to each other reflects a weak interaction effect between the two considered factors. This rule applies to interaction effect plots between the depth of cut and feed rate. However, the other two interaction effect plots show some crossing between the curves, which reflects significant interaction effects between the depth of cut and spindle speed, or between the feed rate and spindle speed.

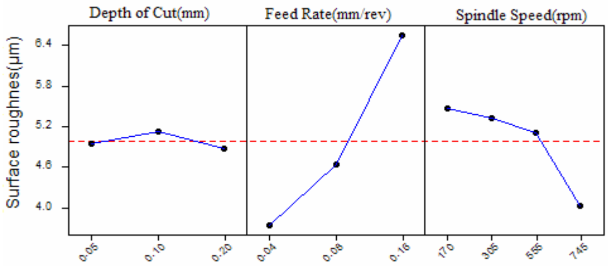

Figure 1. Main effect plots for all individual factors.

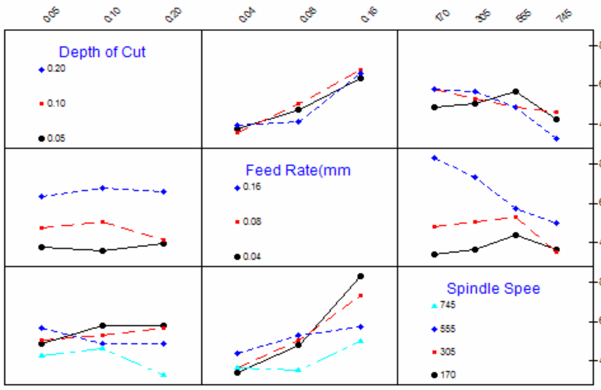

Figure 2. Interaction effect plots for all possible pairs of factors.

IV. Regression and Hypothesis Testing

It can be inferred from the previous discussion about the main effect plots that both the feed rate and the depth of cut have a significant effect on the average surface roughness. Therefore, these two factors need to be included in the regression model to be constructed. Although the effect for the depth of cut seems negligible, it has an indirectly significant effect through its interaction with the spindle speed. Therefore, the depth of cut should also be added to the regression model. In this study, a general quadratic regression model with interaction is proposed according to the following statistical model:

$$R_a = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_1^2 + \beta_5 x_2^2 + \beta_6 x_3^2 + \beta_7 x_1 x_2 + \beta_8 x_1 x_3 + \beta_9 x_2 x_3 + \in \quad (2)$$

where $\beta_o, \beta_1, ..., \beta_9$ are the regression coefficients, x_1 denotes the depth of cut, x_2 denotes the feed rate, x_3 denotes the spindle speed, and $\in$ denotes the random error.

The model in Eq. (2) was fitted using Minitab® on the basis of the data in Table III of the Appendix. The estimated values for the regression coefficients are shown in Table I. These values are calculated on the basis of the least squares method (LSM) [12]. The terms in this general model can be tested for significance using the following hypotheses:

$$\begin{aligned} H_o &: \quad \beta_j = 0, \quad j = 0,1,2,\cdots,9 \\ H_1 &: \quad \beta_j \neq 0 \end{aligned} \quad (3)$$

Calculating the test statistic values for each test and converting these into equivalent *P*-values, the results are obtained as shown in Table I. Let us use a value of 10% for the probability of type I error (α), that is, the probability of rejecting H_o given H_o is true. Now, any model term that corresponds to a *P*-value that is greater than α is considered insignificant and should be eliminated from the model given in Eq. (2). Excluding the constant, the terms corresponding to x_2*x_2, x_3*x_3, and x_1*x_2 are all insignificant and should be dropped. This revision of the model is in line with the conclusions drawn from analyzing the main and interaction effect plots. The revised model now becomes

$$R_a = \beta_o + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4^2 + \beta_5 x_1 x_3 + \beta_6 x_2 x_3 + \in \quad (4)$$

TABLE I. FITTED REGRESSION COEFFICIENTS AND *P*-VALUES FOR THE GENERAL MODEL.

Model term	Regression coefficient	*P*-value
Constant	−1.02	0.463
X_1	20.14	0.165
X_2	44.93	0.014
X_3	0.01	0.002
X_1*X_1	−41.85	0.419
X_2*X_2	9.61	0.905
X_3*X_3	−0.00	0.072
X_1*X_2	5.68	0.883
X_1*X_3	−0.02	0.007
X_2*X_3	−0.05	0.000

Fitting the model in Eq. (4) using Minitab® on the basis of the same data in Table III, the regression coefficients and *P*-values are estimated as shown in Table II. It can be seen from this table that all *P*-values are less than α, which indicates that all the terms are significant.

TABLE II. FITTED REGRESSION COEFFICIENTS AND *P*-VALUES FOR THE REVISED MODEL.

Model term	Regression coefficient	*P*-value
Constant	−0.6315	0.501
X_1	9.9032	0.021
X_2	47.5694	0.000
X_3	0.0115	0.002
X_3*X_3	−6.54E-6	0.069
X_1*X_3	−0.0239	0.006
X_2*X_3	−0.0541	0.000

V. OPTIMAL PROCESS CONDITIONS

The model in Eq. (4) can be used to find the best combination of depth of cut, feed rate, and spindle speed, at which the average surface roughness is minimum within the range of the levels used for each factor. Due to the fact that the model includes three independent variables, it cannot be presented graphically in three dimensions. However, if one of the variables is held constant, then this becomes feasible. For example, if the feed rate is held constant at 0.04 mm/rev, then the function in Eq. (4) appears as a surface, as shown in Fig. 3. Another useful way of presenting the model is through plotting two-dimensional contour curves, as shown in Fig. 4. These curves represent constant values for the average surface roughness, which makes it easier to determine how the average surface roughness increases or decreases.

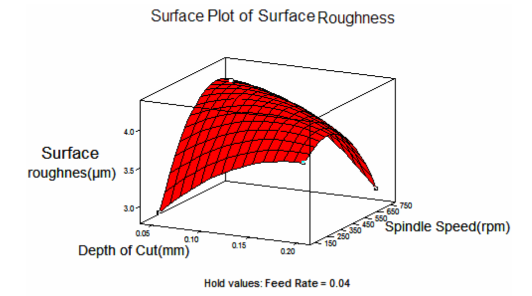

Figure 3. Response surface as the feed rate is held constant at 0.04 mm/rev.

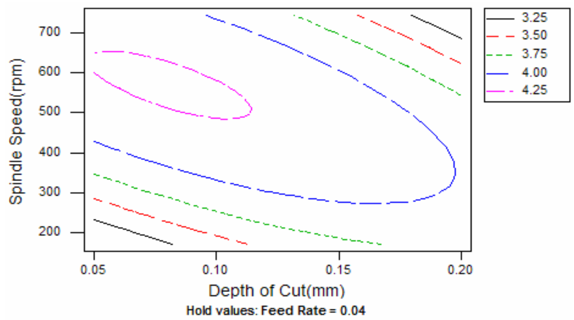

Figure 4. Contour plot for the average surface roughness as the feed rate is held constant at 0.04 mm/rev.

The problem of finding the optimal turning conditions at which the average surface roughness is minimum is considered an unconstrained multivariate minimization problem. This can be solved by defining the model shown in Eq. (4) using MATLAB®, followed by using the "fminunc" command. By trying different initial solutions covering the range of the factor levels, an optimal minimum of 3.016 µm was achieved. This was obtained at values of 0.2 mm, 0.04 mm/rev, and 745 rpm for the depth of cut, feed rate, and spindle speed, respectively.

VI. CONCLUSIONS

In light of this work, the following conclusions can be drawn regarding the turning process of MCS:

(1) The relationship between the average surface roughness and turning conditions, such as depth of cut, feed rate, and spindle speed, is nonlinear because it includes quadratic and interaction terms.
(2) The average surface roughness is significantly affected by both the feed rate and the spindle speed.
(3) The depth of cut has an indirect effect on the average surface roughness through its interaction with the spindle speed.
(4) Significant interaction effects exist between the spindle speed, on the one hand, and the feed rate or depth of cut, on the other hand.
(5) The average surface roughness is minimum when the depth of cut, feed rate, and spindle speed are set at 0.2 mm, 0.04 mm/rev, and 745 rpm, respectively. At these values, an expected value of 3.016 µm can be obtained for the average surface roughness.

APPENDIX

TABLE III. DATA RESULTS FOR THE AVERAGE SURFACE ROUGHNESS.

R_a (μm)	Std. order	Run order	Depth of cut (mm)	Feed rate (mm/rev)	Spindle speed (rpm)
3.37	3	1	0.05	0.04	555
5.99	82	2	0.05	0.16	305
3.79	72	3	0.2	0.16	745
4.27	1	4	0.05	0.04	170
2.56	36	5	0.2	0.16	745
5.91	78	6	0.05	0.08	305
2.81	13	7	0.1	0.04	170
3.03	61	8	0.2	0.04	170
8.27	23	9	0.1	0.16	555
6.23	27	10	0.2	0.04	555
3.15	38	11	0.05	0.04	305
3.67	6	12	0.05	0.08	305
5.23	12	13	0.05	0.16	745
6.61	108	14	0.2	0.16	745
8.21	9	15	0.05	0.16	170
2.51	28	16	0.2	0.04	745
9.51	94	17	0.1	0.16	305
4.84	90	18	0.1	0.08	305
7.9	34	19	0.2	0.16	305
4.22	49	20	0.1	0.04	170
6.12	45	21	0.05	0.16	170
5.55	22	22	0.1	0.16	305
4.95	107	23	0.2	0.16	555
2.66	25	24	0.2	0.04	170
4.91	24	25	0.1	0.16	745
5.28	96	26	0.1	0.16	745
3.83	75	27	0.05	0.04	555
4.56	102	28	0.2	0.08	305
5.54	81	29	0.05	0.16	170
2.65	52	30	0.1	0.04	745
6.76	21	31	0.1	0.16	170
5.17	66	32	0.2	0.08	305
6.28	91	33	0.1	0.08	555
8.06	46	34	0.05	0.16	305
7.85	105	35	0.2	0.16	170
6.5	67	36	0.2	0.08	555
3.86	76	37	0.05	0.04	745
5.19	42	38	0.05	0.08	305
6.29	92	39	0.1	0.08	745
6.49	60	40	0.1	0.16	745
4.96	101	41	0.2	0.08	170
3.42	26	42	0.2	0.04	305
5.5	10	43	0.05	0.16	305
5.58	55	44	0.1	0.08	555
6.25	70	45	0.2	0.16	305
4.31	40	46	0.05	0.04	745
3.53	100	47	0.2	0.04	745
5.7	54	48	0.1	0.08	305
4.9	99	49	0.2	0.04	555
4.11	8	50	0.05	0.08	745
4.18	51	51	0.1	0.04	555
3.34	73	52	0.05	0.04	170
3.44	50	53	0.1	0.04	305
3.97	4	54	0.05	0.04	745
5.65	7	55	0.05	0.08	555
6.16	30	56	0.2	0.08	305
4.1	48	57	0.05	0.16	745
2.41	37	58	0.05	0.04	170
11.92	33	59	0.2	0.16	170
3.8	80	60	0.05	0.08	745
3.29	14	61	0.1	0.04	305
4.22	19	62	0.1	0.08	555
3.8	62	63	0.2	0.04	305
4.29	15	64	0.1	0.04	555
1.89	32	65	0.2	0.08	745
3.03	103	66	0.2	0.08	555
4.86	88	67	0.1	0.04	745
7.02	11	68	0.05	0.16	555
4.06	35	69	0.2	0.16	555
3.76	65	70	0.2	0.08	170
4.46	74	71	0.05	0.04	305
4.22	18	72	0.1	0.08	305
5.46	83	73	0.05	0.16	555
5.15	17	74	0.1	0.08	170
3.8	97	75	0.2	0.04	170
2.21	59	76	0.1	0.16	555
2.29	104	77	0.2	0.08	745
2.09	68	78	0.2	0.08	745
5.29	53	79	0.1	0.08	170
3.55	87	80	0.1	0.04	555
4.89	29	81	0.2	0.08	170
7.91	58	82	0.1	0.16	305
9.53	57	83	0.1	0.16	170
9.57	93	84	0.1	0.16	170
4.94	41	85	0.05	0.08	170
5.15	95	86	0.1	0.16	555
4.29	5	87	0.05	0.08	170
4.91	89	88	0.1	0.08	170
4.33	39	89	0.05	0.04	555
4.4	31	90	0.2	0.08	555
4.47	63	91	0.2	0.04	555
3.35	2	92	0.05	0.04	305
5.34	79	93	0.05	0.08	555
5.87	84	94	0.05	0.16	745
6.66	43	95	0.05	0.08	555
5.07	20	96	0.1	0.08	745
4.68	98	97	0.2	0.04	305
2.87	16	98	0.1	0.04	745
4.75	77	99	0.05	0.08	170
3.09	86	100	0.1	0.04	305
9.05	47	101	0.05	0.16	555
4.96	71	102	0.2	0.16	555

8.94	106	103	0.2	0.16	305
9.06	69	104	0.2	0.16	170
3.65	85	105	0.1	0.04	170
2.75	44	106	0.05	0.08	745
4.08	64	107	0.2	0.04	745
3.07	56	108	0.1	0.08	745

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