# Fault Diagnosis for Composite Asynchronous Sequential Machines with Cascade Composition

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***Abstract*—In this paper, we investigate fault diagnosis of composite asynchronous sequential machines with cascade composition. An adversarial input can infiltrate one of two submachines comprising the composite asynchronous machine, causing an unauthorized state transition. The main objective is to specify the condition under which the controller can diagnose any fault occurrence. Two control configurations, state feedback and output feedback, are considered in this paper. In the case of output feedback, the exact estimation of the state of the front machine is impossible since the current state is inaccessible. Due to feature of cascade composition, we must consider the case where the rear submachine undergoes a faulty transition caused by the adversarial input occurring to the front submachine. Fault detectability is also addressed in the case of output feedback.**

***Index Terms*—asynchronous sequential machines, cascade composition, corrective control, fault diagnosis**

## I. Introduction

Corrective control is a novel automatic control theory that is used to compensate for the stable-state behavior of asynchronous sequential machines. It has been mainly applied to correcting faulty operations of the machine, e.g., critical races [1], infinite cycles [2], nondeterministic transitions [3], etc. Recently, it has been successfully applied to tolerating various faults occurring to the machine; refer to [4], [5] for theoretical development of this topic, and to [6], [7] for experimental verification on asynchronous digital systems.

In this paper, we study fault diagnosis for a cascaded asynchronous sequential machine, which is a composite system made of two single asynchronous sequential machines, termed front and rear machines, in a series connection. The motivation of our study is that many built-in asynchronous sequential machines are combined into a cascaded one for various purposes [8]. When cascaded systems are operated in hazardous environments, adversarial inputs causing unwanted transitions may happen to the systems. In comparison with the case of single asynchronous sequential machines [4]-[7], it is more difficult to diagnose fault situations occurring to cascaded asynchronous sequential machines. The major reason is that owing to the structure of the cascaded system, the adverse effect of a fault occurring to the front machine can be propagated to the rear machine. Moreover, often only output feedback of the cascaded asynchronous sequential machine is available to the controller. The foregoing constraint makes it impossible for a controller to conduct exact fault diagnosis on the cascaded asynchronous sequential machine.

In this study, two control configurations, state feedback and output feedback, are considered separately in fault diagnosis. When state feedback is available, the controller knows the state at which the fault occurs as well as the state reached by the machine as the result of the fault. On the other hand, the controller cannot derive the exact state of the machine only with the use of output feedback. Instead, we derive the change of state uncertainty throughout the unauthorized state transition. Due to feature of cascade composition, we must consider the case where the rear submachine undergoes a faulty transition caused by the adversarial input occurring to the front submachine. Note that the construction of a fault tolerant controller is not discussed in this paper. A simple example is provided to demonstrate the proposed methodology of fault diagnosis. Recent results of modeling and control of composite asynchronous sequential machines can be found in, e.g., [9], [10].

## II. Preliminaries

A composite asynchronous sequential machine $\Sigma$ is made of two single input/state asynchronous sequential machines $\Sigma_f$ and $\Sigma_r$ whose dynamics are described as

$$\Sigma_f = (A \cup W, X, s_f)$$
$$\Sigma_r = (X \cup W, Y, s_r)$$

where $A$ is the set of external inputs, $W$ is the set of adversarial inputs, $X$ and $Y$ are the state sets, and

$$s_f: X \times (A \cup W) \rightarrow X$$
$$s_r: Y \times (X \cup W) \rightarrow Y$$

are the stable recursion function of $\Sigma_f$ and $\Sigma_r$, respectively. In our paper, each machine is represented by a stable-state machine in which a transition involves no transient states. Let us discuss the feature of single asynchronous sequential machines using $\Sigma_f$ (the following is equally applied to $\Sigma_r$). If $s_f(x,u)=x$, $x \in X$ is a stable state of $\Sigma_f$ with respect to the input $u \in A$; else if $s_f(x,u) \neq x$, it is a transient

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state with $u$. Owing to the absence of a synchronizing clock, $\Sigma_f$ stays at a stable combination $(x,u')$ indefinitely. If the input $u'$ changes to another value $u$ for which $(x,u)$ is a transient combination, $\Sigma_f$ engages in the transition from $x$ to

$$s_f(x,u) = x'$$

asynchronously where $x'$ is called the next stable state of $(x,u)$.

For later usage, we extend the mapping of $s_f$ and $s_r$ as follows. Let $\chi \subset X$ be a subset of $X$. Then $s_f: P(X) \times A \rightarrow P(X)$ is defined as follows, where $P(X)$ is the power set of $X$.

$$s_f[\chi,u] = \{s_f(x,u) | \, x \in \chi\}$$

where '[ · ]' is used to highlight that $s_f[\chi,u]$ is a set value.

In association with $s_f$ and $s_r$, we define the inverse functions $s^{-1}_f: X \times X \rightarrow P(A)$ and $s^{-1}_r: Y \times Y \rightarrow P(X)$ as

$$s^{-1}_f(x,x') = \{v \in A | \, s_f(x,v)=x'\}$$
$$s^{-1}_r(y,y') = \{x \in X | \, s_r(y,x)=y'\}.$$

Note that the set of adversarial inputs $W$ are excluded from the range of $s^{-1}_f$ and $s^{-1}_r$.

Since $\Sigma_f$ and $\Sigma_r$ are combined in a series connection, $X$ serves as the external input set of $\Sigma_r$ and $Y$ as the output set of the cascaded asynchronous sequential machine $\Sigma$. As $\Sigma_f$ undergoes a transition, the next stable state of $\Sigma_f$ is transmitted to $\Sigma_r$ as the new input. In particular, assume that $\Sigma_f$ and $\Sigma_r$ have been staying at a stable state $x$ and $y$, respectively, with the control input $u$. This means that $s_f(x,u)=x$ and $s_r(y,x)=x$. Assume further that the control input changes to $u'$. Then $\Sigma_f$ first undergoes a stable transition from $x$ to $s_f(x,u')$. If $s_f(x,u') \neq x$, the transition of $\Sigma_f$ also induces that of $\Sigma_r$, namely $\Sigma_r$ transfers from $y$ to $s_r(y,s_f(x,u'))$. Hence it can be said that in response to $u'$, $\Sigma$ transfers from $(x,y)$ to $(s_f(x,u'), s_r(y,s_f(x,u')))$.

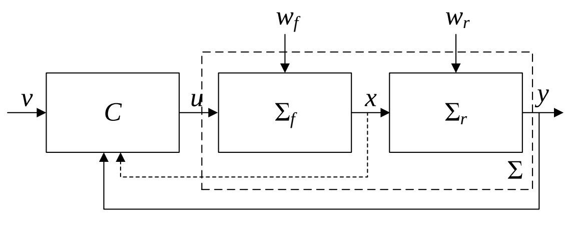


Figure 1. Control configuration of the composite asynchronous sequential machine Σ with cascade composition.

Fig. 1 illustrates the corrective control system for a composite asynchronous sequential machine $\Sigma$, where $C$ is the corrective controller that has the form of an input/output asynchronous sequential machine, $v \in A$ is the external input, $u \in A$ is the control input generated by $C$, $x \in X$ and $y \in Y$ are the state of $\Sigma_f$ and $\Sigma_r$, respectively, and $w_f \in W$ and $w_r \in W$ are the adversarial input to $\Sigma_f$ and $\Sigma_r$. We denote by $\Sigma_c$ the closed-loop system composed of $C$ and $\Sigma$.

If $w_f$ occurs to $\Sigma_f$ that has been staying at a stable state $x$, $\Sigma_f$ is forced to transfer to $s_f(x,w_f)$ regardless of the current external input. $\Sigma_r$ can be also influenced by an occurrence of $w_f$ since its input, or the state of $\Sigma_f$, is switched by an occurrence of $w_f$. On the other hand, $w_r$ causes $\Sigma_r$ to undergo an unauthorized state transition without affecting $\Sigma_f$.

In this paper, we consider two control configuration separately — (i) state feedback and (ii) output feedback. To stress this setting, the route of state feedback is marked in dashed lines in Fig. 1. In the case of state feedback, both $x$ and $y$ are transmitted to $C$. Hence the formulation of $C$ is written as

$$C = (X \times Y \times A, A, \Xi, \xi_0, \phi, \eta) \text{ with } (x,y).$$

where $X \times Y \times A$ is the input set ($x$, $y$, and $v$), $A$ is the output set serving as the control input $u$, $\Xi$ is the state set, $\xi_0 \in \Xi$ is the initial state, $\phi: \Xi \times X \times Y \times A \rightarrow \Xi$ is the recursion function, and $\eta: \Xi \rightarrow Z$ is the output function. In the case of output feedback, on the other hand, only $y$ from $\Sigma_r$ is relayed to $C$ as the output feedback. Hence the formulation of $C$ is

$$C = (Y \times A, A, \Xi, \xi_0, \phi, \eta) \text{ with } y.$$

The objective of fault diagnosis by C is also determined depending on the control configuration as follows.

(i) In the case of state feedback, $C$ must identify not only the original state of $\Sigma$ at which the unauthorized state transition initiates, but also the deviated state reached by $\Sigma$ as the result of the fault.

(ii) In the case of output feedback, the exact observation of the state of $\Sigma_f$ is impossible. Instead of deriving the current state of $\Sigma_f$, we must specify a subset of $X$, one element of which $\Sigma_f$ stays at the moment of the fault occurrence, and another subset of $X$ that represents all the possible states that can be reached by $\Sigma_f$ as the result of the fault.

The final purpose of fault diagnosis is to conduct fault tolerant control that drives $\Sigma$ to return to the normal input/state or input/output behavior. In this paper, however, we only discuss fault diagnosis and leave fault tolerant control as a future topic.

To prevent unpredictable results caused by the absence of a synchronizing clock, the closed-loop system $\Sigma_c$ is supposed to preserve the principle of fundamental mode operations [11] whereby a variable must change its value when both $C$ and $\Sigma$ are in stable states, and no two or more variables can be altered simultaneously. Under this principle, an adversarial input can happen only when both $\Sigma_f$ and $\Sigma_r$ stay at stable states, and $\Sigma_r$ can engage in a transition only after the transition of $\Sigma_f$ terminates and vice versa.

## II. Main Result

### A. State Feedback

We first study the problem of fault diagnosis in the control configuration with access to state feedback, i.e., both $x$ and $y$ are delivered to $C$ as feedback. As mentioned before, an unauthorized state transition of $\Sigma$ may stem from either an occurrence of $w_f$ or that of $w_r$. First, assume that $\Sigma$ has been staying at stable states $(x,y)$ when $w_f$ occurs, enforcing $\Sigma_f$ to reach $s_f(x,w_f)=x'$. One can perceive the occurrence of $w_f$ by observing that the state feedback of $\Sigma_f$ changes to $x'$ while the external input $v$ remains fixed. Since the state of $\Sigma_f$ enters $\Sigma_r$ as its input, $\Sigma_r$ may also experience an unauthorized state transition from $y$ to $s_r(y,x')=y'$ if $(y,x')$ is a transient combination of $\Sigma_r$. In this case, one will observe almost simultaneous change of state feedback from $(x,y)$ to $(x',y')$, as asynchronous sequential machines have zero transient time.

Next, suppose that $w_r$ occurs to $\Sigma_r$ when $\Sigma$ stays at stable states $(x,y)$. Referring to Fig. 1, the output channel of $\Sigma_r$ is detached from $\Sigma_f$. Hence, only $\Sigma_r$ experiences the unauthorized state transition from $y$ to $s_r(y,w_r)$ while $\Sigma_f$ stands still.

In either occurrence of $w_f$ or $w_r$, we can identify with certainty both the original state at which an adversarial input occurs and the next stable state reached by $\Sigma_f$ and $\Sigma_r$ as the result of the fault. Let us summarize our analysis on fault diagnosis with full state feedback as follows.

(i) $(x,y)\rightarrow(x',y')$: $w_f$ occurs to $\Sigma_f$ such that $s_f(x,w_f)=x'$ and $s_r(y,x')=y'$.
(ii) $(x,y)\rightarrow(x',y)$: $w_f$ occurs to $\Sigma_f$ such that $s_f(x,w_f)=x'$ and $s_r(y,x')=y$.
(iii) $(x,y)\rightarrow(x,y')$: $w_r$ occurs to $\Sigma_r$ such that $s_r(y,w_r)=y'$.

### B. Output Feedback

Since the exact identification of the current state of $\Sigma_f$ is impossible in the control configuration with output feedback, in this paper we introduce the notion of state uncertainty of $\Sigma_f$. Provided that $\Sigma$ stays at a stable combination with the control input $u\in A$ and the output feedback value $y\in Y$, denote by

$$X_n(u,y) \subset X$$

state uncertainty about $\Sigma_f$ with $u$ and $y$ in the normal behavior. Explicitly, $X_n(u,y)$ is defined as

$$X_n(u,y) = \{x\in X|\ s_f(x,u)=x,\ \ s_r(y,x)=y\}.$$

$X_n(u,y)$ implies that the exact state of $\Sigma_f$ is unknown; but it stays at a stable combination with a state of $X_n(u,y)$.

Suppose now that $\Sigma$ has been staying at stable states with $u$, $y$, and uncertainty $X_n(u,y)$ when an adversarial input occurs so that $y$ changes to $y'$. As addressed before, the unauthorized state transition from $y$ to $y'$ may stem from one of two faults: First, $w_f$ may have happened to $\Sigma_f$ so that $\Sigma_f$ transfers from a (unknown) state $x\in X_n(u,y)$ to $s_f(x,w_f)=x'$ and $\Sigma_r$ in turn transfers from $y$ to $s_r(y,x')=y'$. Next, $w_r$ may have happened to $\Sigma_r$ so that $\Sigma_r$ undergoes the transition from $y$ to $s_r(y,w_r)=y'$. Since the current state of $\Sigma_f$ is still uncertain in either case, we have to estimate it using the current information, i.e., using $u$, $y$, $X_n(u,y)$, and $y'$. For this purpose, let us define two subsets $W_f(x,x')\subset W$ for $x,x'\in X$ and $W_r(y,y')\subset W$ for $y,y'\in Y$ as

$$W_f(x,x') = \{w_f\in W|\ s_f(x,w_f)=x'\}$$
$$W_r(y,y') = \{w_r\in W|\ s_r(y,w_r)=y'\}.$$

$W_f(x,x')$ and $W_r(y,y')$ symbolize the set of adversarial inputs causing the unauthorized state transition from $x$ to $x'$ in $\Sigma_f$ and from $y$ to $y'$ in $\Sigma_r$, respectively. At the end of the unauthorized state transition, the procedure of state estimation of $\Sigma_f$ must be conducted by checking $W_r(y,y')$ and $s^{-1}_r(y,y')$.

1) $W_r(y,y')\neq\varnothing$ and $s^{-1}_r(y,y')=\varnothing$: First of all, if $W_r(y,y')\neq\varnothing$ and $s^{-1}_r(y,y')=\varnothing$, the unauthorized state transition from $y$ to $y'$ must be caused solely by an occurrence of $w_r\in W_r(y,y')$, since no state of $X$ exists that takes $\Sigma_r$ from $y$ to $y'$. Hence $\Sigma_f$ stays at the same state of $X_n(u,y)$ during the unauthorized state transition and the uncertainty $X_n(u,y)$ remains unchanged.

2) $W_r(y,y')=\varnothing$ and $s^{-1}_r(y,y')\neq\varnothing$: Second, if $W_r(y,y')=\varnothing$ and $s^{-1}_r(y,y')\neq\varnothing$, the unauthorized state transition must be relayed from $\Sigma_f$, which has undergone an unauthorized state transition caused by an occurrence of $w_f\in W$. To address this case further, we remind that $\Sigma_f$ stays at a state of $X_n(u,y)$ before the fault occurrence. Since $s^{-1}_r(y,y')\neq\varnothing$, we have to examine which states among $s^{-1}_r(y,y')\neq\varnothing$ can be reached by $\Sigma_f$ as the result of the unauthorized state transition. A subset of $s^{-1}_r(y,y')$ defined with respect to $u$, $y$, and $y'$, termed $X_d(u,y,y')$, represents such states.

$$X_d(u,y,y') = \{x\in s^{-1}_r(y,y')|\ \exists w_f\in W,\ x'\in X_n(u,y) \text{ such that } s_f(x',w_f)=x\}.$$

$X_d(u,y,y')$ equals the updated uncertainty about the state of $\Sigma_f$ after the fault occurrence.

3) $W_r(y,y')\neq\varnothing$ and $s^{-1}_r(y,y')\neq\varnothing$: Finally, if $W_r(y,y')\neq\varnothing$ and $s^{-1}_r(y,y')\neq\varnothing$, the unauthorized state transition may be caused either by indirect influence by $w_f$ or direct influence by $w_r$. In this case, uncertainty about the state of $\Sigma_f$ must be the sum of $X_n(u,y)$ and $X_d(u,y,y')$. Let

$$\Omega(u,y,y') \subset X$$

be uncertainty about the state of $\Sigma_f$ induced after the unauthorized state transition from $y$ to $y'$ with the input $u$. Then, assembling the foregoing discussions, we derive $\Omega(u,y,y')$ as follows.

$$\Omega(u,y,y') = \begin{cases} X_n(u,y) & W_r(y,y')\neq\varnothing, s_r^{-1}(y,y')=\varnothing \\ X_d(u,y,y') & W_r(y,y')=\varnothing, s_r^{-1}(y,y')\neq\varnothing \\ X_n(u,y)\cup X_d(u,y,y') & W_r(y,y')\neq\varnothing, s_r^{-1}(y,y')\neq\varnothing \end{cases}$$

## III. EXAMPLE

To address applicability of the presented analysis of fault diagnosis, consider an example machine $\Sigma$ shown in Fig. 2. Here, $A=\{a,b,c,d\}$, $X=\{x_1,x_2,x_3,x_4\}$, $Y=\{y_1,y_2,y_3,y_4\}$, and $W=\{w_1,w_2,w_3\}$ where $w_1$ and $w_3$ may occur to $\Sigma_f$ at $x_3$ and $x_2$, respectively, and $w_2$ to $\Sigma_r$ at $y_2$. Since fault diagnosis is self-evident in the case of state feedback, we investigate only fault occurrence and its diagnosis for the closed-loop system $\Sigma_c$ with access to output feedback $y$.

For instance, suppose that $\Sigma$ has been staying at a stable combination with $u=c$ and $y=y_2$. Then, uncertainty about the current state of $\Sigma_f$ is

$$X_n(c,y_2) = \{x\in X|\ s_f(x,c)=x,\ \ s_r(y_2,x)=y_2\} = \{x_2,x_3\}.$$

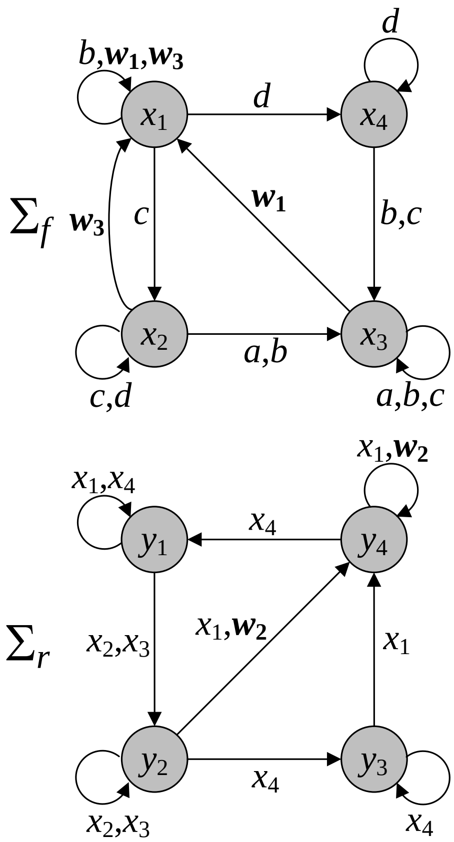


Figure 2. Composite asyncronous sequential machine Σ with $\Sigma_f$ and $\Sigma_r$.

Suppose further that the output feedback changes from $y_2$ to $y_4$ while the control input remains unchanged. Referring to Fig. 2, this unauthorized state transition can be caused either by an occurrence of $w_2$ in $\Sigma_r$, i.e., $s_r(y_2,w_2)=y_4$, or by an occurrence of $w_1$ or $w_3$ in $\Sigma_r$, i.e., $s_f(x_3,w_1)=s_f(x_2,w_3)=x_1$ and $s_r(y_2,x_1)=y_4$. $s^{-1}_r(y_2,y_4)$ and $X_d(c,y_2,y_4)$ are derived as

$$\begin{aligned} s^{-1}_r(y_2,y_4) &= \{x\in X \mid s_r(y_2,x)=y_4\} \\ &= \{x_1\} \\ X_d(c,y_2,y_4) &= \{x\in s^{-1}_r(y_2,y_4) \mid \exists w_f\in W, x'\in X_n(c,y_2) \\ &\quad \text{such that } s_r(y_2,x')=y_4\} \\ &= \{x_1\}. \end{aligned}$$

Using the derived formula of state uncertainty, we easily derive $\Omega(c,y_2,y_4)$, uncertainty about the current state of $\Sigma_f$ estimated at the end of the unauthorized transition, as follows.

$$\begin{aligned} \Omega(c,y_2,y_4) &= X_n(c,y_2) \cup X_d(c,y_2,y_4) \\ &= \{x_2,x_3\} \cup \{x_1\} \\ &= \{x_1,x_2,x_3\}. \end{aligned}$$

The above analysis can be said that upon observing the unauthorized transition from $y_2$ to $y_4$, the state uncertainty of $\Sigma_f$ changes from $X_n(c,y_2)=\{x_2,x_3\}$ to $\Omega(c,y_2,y_4)=\{x_1,x_2,x_3\}$.

## IV. Conclusion

We have investigated fault diagnosis of a class of composite asynchronous sequential machines made of cascade composition of two single asynchronous machines. We have examined whether an unauthorized state transition can be identified in the closed-loop system of the composite asynchronous sequential machine with access to state feedback or output feedback. While state identification of front and rear submachines is feasible in the case of sate feedback, it is impossible in the case of output feedback. Instead, we update uncertainty about the current state of the front machine in the course of an unauthorized state transition according to the available information of the machine. The proposed methodology has been validated using a simple example instance.

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