# $H_{\infty}$ Controller Design for Robust Control in MMC-HVDC System

Eun-Sung Gil, Hui Song, and Kwan-Ho Chun Chungnam National University, Daejeon, Korea Email: {gileunsung90, thomas\_songhui, khchun}@cnu.ac.kr

Abstract— In this paper, a Modular–Multilevel–Converter based high voltage direct current (MMC-HVDC) system is considered. It is assumed that the grid is strong. The effects of load change and, PLL errors can cause voltage, current, and active/reactive power changes on the grid side. It is assumed that active/reactive power and q-axis voltage are disturbances. Based on the  $H_{\infty}$  theory, a robust controller is designed to deal with external disturbances. Purpose of controller maintains active/reactive power at the inverter station. The designed  $H_{\infty}$  controller is validated by a simulation using MATLAB/Simulink.

Index Terms—MMC-HVDC system, Strong Grid,  $H_{\scriptscriptstyle \infty}$  control, PLL

## I. INTRODUCTION

Recently, renewable energy has seen an increase in development because of increasing power requirements, environmental problems, and the exhaustion of existing resources. Because most windfarms, tidal power generation sources, and wave power generation sources are located remotely from the power grid, there is a significant loss of power in transmission lines. To solve this problem, High-Voltage Direct Current (HVDC) has been increasingly investigated [1]–[3]. In general, HVDC systems can be classified as three types: Line-commutated Current Source Converter (LCC), Voltage Source Converter (VSC), and Modular Multilevel Converter (MMC).

An LCC-HVDC system based on thyristors can transfer large amounts of current and power. This system is low cost, but it absorbs a large amount of reactive power. Thus, it requires a large reactive power compensator such as a Static VAr Compensators (SVC) [2]–[3].

A VSC-HVDC system based on insulated-gate bipolar transistors (IGBTs) can control active power and reactive power independently. It requires little reactive power compensator than LCC-HVDC. However, it is expensive because it requires many IGBTs. The system also requires LC-filters because of its harmonics [4]–[5].

An MMC-HVDC system consists of many submodules (SMs). The SMs can be controlled individually. A SM consists two IGBTs and one capacitor. It can output

three-stage voltages such as +V,-V,0. As a result, an MMC can output scalable voltages according to the number of SMs. If there are infinite SMs, the output voltage takes on the form of a sinewave form. Then, the MMC does not require LC-filters. However, an MMC is very expensive because it has many IGBTs and capacitors. In addition, its control is very complex, and its harmonics affect the output current, voltage, and power [6], [7].

There have been several efforts to apply robust control techniques to power systems. In [8], a robust  $H_{\infty}$  theory was applied to design a reactive power compensator. In [9], an  $H_{\infty}$  controller was designed for a VSC-HVDC system. By use of  $H_{\infty}$  theory, constant DC voltage  $H_{\infty}$  controller and constant AC voltage controller are designed. In addition, design of  $H_{\infty}$  controller is evaluated for different power system operating conditions like the three-phase short-circuits in the converter and inverter, short-circuit on the DC-line, loading changes [10].

This paper presents the design of an  $H_{\infty}$  controller that deals with external disturbances in an MMC-HVDC system. The paper is organized as follows: Section 2 develops a MMC-HVDC system model, presents a statespace equation including disturbances and PLL errors, and presents the design of the  $H_{\infty}$  controller. Section 3 provides simulation results for the designed  $H_{\infty}$  controller using MATLAB/Simulink

## II. MODELING AND CONTROL

# A. Modeling

Fig. 1 shows an MMC-HVDC system with a strong grid. In the Fig. 1  $V_{sa}$ ,  $V_{sb}$ , and  $V_{sc}$  are the instantaneous values of the three-phase voltage of the inverter output;  $I_{sa}$ ,  $I_{sb}$ , and  $I_{sc}$  are the instantaneous values of the three-phase current of the inverter output; and  $V_{ra}$ ,  $V_{rb}$ , and  $V_{rc}$  are the instantaneous values of the three-phase voltage of the load. R, L represent the impedances of the inverter side.  $\omega$  is the angular frequency. P and Q represent the active power and reactive power flowing to the load, respectively.

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Figure 1. MMC-HVDC system with strong grid.

It is assumed that the phase change caused by R, L is compensated by a phase compensator. The output voltage of the three-phase inverter can be expressed as

$$V_{sj} = Ri_{sj} + L\frac{d}{dt}i_{sj} + V_{rj}$$
(1)

where j = a, b, c. By using the PARK transformation, Equation (1) can be changed to

$$\frac{d}{dt}(I_{sd} + \delta i_{sd}) = -\frac{R}{L}(I_{sd} + \delta i_{sd}) + \omega(I_{sd} + \delta i_{sd}) - \frac{1}{L}(V_{rd} - V_{sd}),$$
(2)  
$$\frac{d}{dt}(I_{sq} + \delta i_{sq}) = -\frac{R}{L}(I_{sq} + \delta i_{sq}) - \omega(I_{sq} + \delta i_{sq}) - \frac{1}{L}(V_{rq} - V_{sq}),$$

where  $\delta i_{sd}$ ,  $\delta i_{sq}$  are the current perturbations caused by load changes [11]. If the three-phase system is effectively balanced under a steady-state condition, then  $V_{rd} = V_{ra}$ , and  $V_{rq} = 0$  by PLL [5]. Then, the active power and reactive power can be expressed as

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} V_{rd}(I_{sd} + \delta i_{sd}) + V_{rq}(I_{sq} + \delta i_{sq}) \\ -V_{rd}(I_{sq} + \delta i_{sq}) + V_{rq}(I_{sd} + \delta i_{sd}) \end{bmatrix}$$
(3)

# B. Design of $H_{\infty}$ Controller

If PLL errors exist, then  $V_{rq}$  since is not zero ( $V_{rq} = 0 + \delta v_{rq}$ ), Equation (3) can be expressed as

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} P \\ Q \end{bmatrix} = \begin{bmatrix} V_{rd}(I_{sd} + \delta i_{sd}) + \delta v_{rq}(I_{sq} + \delta i_{sq}) \\ -V_{rd}(I_{sq} + \delta i_{sq}) + \delta v_{rq}(I_{sd} + \delta i_{sd}) \end{bmatrix}.$$
 (4)

By substitution of  $i_{sq} = I_{sq} + \delta i_{sq}$  and differentiation, we obtain the following equation:

$$\dot{x} = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \dot{P} \\ \dot{Q} \end{bmatrix} = \begin{bmatrix} \dot{V}_{rd}\dot{i}_{sd} + V_{rd}\dot{i}_{sd} + \delta\dot{v}_{rq}\dot{i}_{sq} + \delta\dot{v}_{rq}\dot{i}_{sq} \\ -\dot{V}_{rd}\dot{i}_{sq} - V_{rd}\dot{i}_{sq} + \delta\dot{v}_{rq}\dot{i}_{sd} + \delta\dot{v}_{rq}\dot{i}_{sq} \end{bmatrix}$$
(5)

Since  $\dot{V}_{rd} = 0$ , the Equation (5) is simplified as

$$\dot{x} = \begin{bmatrix} V_{rd}\dot{s}_{sd} + \delta\dot{v}_{rq}\dot{s}_{sq} + \delta\dot{v}_{rq}\dot{s}_{sq} \\ -V_{rd}\dot{s}_{sq} + \delta\dot{v}_{rq}\dot{s}_{sd} + \delta\dot{v}_{rq}\dot{s}_{sq} \end{bmatrix}$$
(6)

If the load change is slow, then  $\delta \dot{v}_{rq}$ ,  $\delta \dot{i}_{sd}$ , and  $\delta \dot{i}_{sq}$  are very small. Thus  $\delta \dot{v}_{rq}$ ,  $\delta \dot{i}_{sd}$ , and  $\delta \dot{i}_{sq}$  can be ignored. Equation (5) can be expressed as

$$\begin{bmatrix} \dot{x}_{1} \\ \dot{x}_{2} \end{bmatrix} = \begin{bmatrix} V_{rd}\dot{s}_{sd} + \delta v_{rq}\dot{s}_{sq} \\ -V_{rd}\dot{s}_{sq} + \delta v_{rq}\dot{s}_{sq} \end{bmatrix}$$
(7)

By transformation  $u_1 = V_{cd} - V_{rd}$ ,  $u_2 = V_{cq} - V_{rq}$ , the system can be expressed with disturbances( $\delta v_{rq}$ ,  $\delta i_{sd}$ , and  $\delta i_{sq}$ ):

$$\begin{bmatrix} \dot{x}_{1} \\ \dot{x}_{2} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega \\ \omega & \frac{R}{L} \end{bmatrix} \begin{bmatrix} \dot{x}_{1} \\ \dot{x}_{2} \end{bmatrix} + V_{rd} \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & -\frac{1}{L} \end{bmatrix} \begin{bmatrix} u_{1} \\ u_{2} \end{bmatrix}$$

$$+ \begin{bmatrix} -\frac{R}{L} & \omega & \dot{I}_{sq} \\ \omega & L & (\frac{V_{rd}}{L} + \dot{I}_{sd}) \end{bmatrix} \begin{bmatrix} \delta P \\ \delta Q \\ \delta v_{rq} \end{bmatrix}$$
(8)

where  $\delta i_{sd}$ ,  $\delta i_{sq} = 0$ ,  $\hat{P} = V_{rd}I_{sd}$ ,  $\hat{Q} = V_{rd}I_{sq}$ , and  $\hat{P}$ ,  $\hat{Q}$  are the operating points.

To solve the problem using an  $H_{\infty}$  theory, it is required to have controllability and observability matrices. The controllability matrices are calculated as

$$B_{1} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, AB_{1} = \begin{bmatrix} -\frac{R}{L} \\ \omega \end{bmatrix}, \rho(\begin{bmatrix} B_{1} & AB_{1} \end{bmatrix}) = \rho \begin{bmatrix} 1 & -\frac{R}{L} \\ 0 & \omega \end{bmatrix} = 2$$

$$B_{2} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, AB_{2} = \begin{bmatrix} \omega \\ \frac{R}{L} \end{bmatrix}, \rho(\begin{bmatrix} B_{2} & AB_{2} \end{bmatrix}) = \rho \begin{bmatrix} 0 & \omega \\ 1 & \frac{R}{L} \end{bmatrix} = 2$$
(10)

Since the controllability matrices have full rank, it satisfies the conditions for controllability [12]. The observability matrices are also obtained as

$$C_{1} = \begin{bmatrix} 1 & 0 \end{bmatrix}, C_{1}A = \begin{bmatrix} -\frac{R}{L} & \omega \end{bmatrix}, \rho \begin{pmatrix} C_{1} \\ C_{1}A \end{pmatrix} = \rho \begin{pmatrix} 1 & 0 \\ -\frac{R}{L} & \omega \end{pmatrix} = 2$$

$$C_{2} = \begin{bmatrix} 0 & 1 \end{bmatrix}, C_{2}A = \begin{bmatrix} \omega & \frac{R}{L} \end{bmatrix}, \rho \begin{pmatrix} C_{2} \\ C_{2}A \end{pmatrix} = \rho \begin{bmatrix} 0 & 1 \\ \omega & \frac{R}{L} \end{bmatrix} = 2$$

$$(11)$$

Thus the system is controllable and observable, it can be represented as the following Equation (12) by  $H_{\infty}$  control theory [13]–[15].

$$\dot{x}(t) = Ax(t) + B_1 d(t) + B_2 u(t)$$

$$z(t) = C_1 x(t) + D_{11} d(t) + D_{12} u(t)$$

$$y(t) = C_2 x(t) + D_{21} d(t) + D_{22} u(t)$$
(12)

where x(t) is the state variable, u(t) is the control input, z(t) is the controlled output, y(t) is the output state, and d(t) represents disturbances from the outside.

Using Equation (12), Equation (9) can be expressed as

$$\begin{bmatrix} \dot{x}_{1} \\ \dot{x}_{2} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega \\ \omega & \frac{R}{L} \end{bmatrix} \begin{bmatrix} \hat{x}_{1} \\ \hat{x}_{2} \end{bmatrix} + \begin{bmatrix} -\frac{R}{L} & \omega & \dot{i}_{sq} \\ -\frac{R}{L} & \frac{R}{L} & (\frac{V_{rd}}{L} + \dot{i}_{sd}) \end{bmatrix} \begin{bmatrix} \delta P \\ \delta Q \\ \delta v_{rq} \end{bmatrix} + V_{rd} \begin{bmatrix} \frac{1}{L} & 0 \\ 0 & -\frac{1}{L} \end{bmatrix} \begin{bmatrix} u_{1} \\ u_{2} \end{bmatrix},$$

$$z = \alpha \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} x + \beta \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix} x.$$
(13)

The  $H_{\infty}$  state feedback controller equation is expressed as

$$u = -B_2^T P(t)x = \begin{bmatrix} -B_2^T P(t) & 0 \end{bmatrix} \begin{bmatrix} x^* \\ \omega^* \end{bmatrix}$$
(14)  
$$K^* = \begin{bmatrix} -B_2^T P(t) & 0 \end{bmatrix}$$

In Equation (14), P(t) is the positive definite solution of the Riccati differential equation [13][15] - [17]:

$$-\dot{P} = PA + A^{T}P - P(B_{2}B_{2}^{T} - \gamma^{-2}B_{1}B_{1}^{T})P + C_{1}^{T}C_{1}, \ \gamma > 0$$

or

$$A^{T}P + PA + P(\gamma^{-2}B_{1}B_{1}^{T} - B_{2}B_{2}^{T})P + C_{1}^{T}C_{1} = 0$$
(15)

## III. SIMULATION

The designed  $H_{\infty}$  controller is validated using MATLAB/Simulink. The parameters of the simulation are as follows: ( $R = 0.15[\Omega]$ , L = 2[mH], V = 1[p.u.] = 230[kV], f = 60[Hz],  $\dot{I}_{sd} = 0.4[\text{p.u.}]$ ,  $\dot{I}_{sq} = 0.1[\text{p.u.}]$ ). In Fig. 2, the variations in active power and reactive power, the effect of disturbances  $\delta P$ ,  $\delta Q$ , and  $\delta v_{rq}$ , are depicted. The q-axis voltage perturbation  $\delta v_{rq}$  is generally induced by PLL errors. The values of three disturbances are given as 0.1, 0.3, and 0.1, respectively. After 0.025s, active power is increased but reactive power is decreased.

The  $H_{\infty}$  controller is designed using  $\alpha = 0.6$ ,  $\beta = 0.2$ . The designed controller and the cost are as follows:

$$\gamma = 0.25, \quad u = \begin{bmatrix} -6 & -9 \\ 9 & 31 \end{bmatrix}$$
 (16)

Fig. 3 depicts the frequency response of the closed-loop system. Since the maximum singular values of the closed-loop system is less than  $\gamma$ , the designed  $H_{\infty}$  controller satisfies the performance requirement. The active and reactive powers are shown in Fig. 4 and the designed  $H_{\infty}$  controller is validated by simulation results.





## **IV. CONCLUSION**

This paper presents the robust control of an MMC-HVDC using an  $H_{\infty}$  controller. An MMC-HVDC exhibits good efficiency and storage energy its capacitors. However, an MMC is very expensive because of its many IGBTs and capacitors. The MMC also has many harmonics because of its switching function. The effects of load changes and PLL errors can cause voltage, current, and active/reactive power changes on the grid side. It is suggested that active power and reactive power are states in state-space. Based on the  $H_{\infty}$  theory, a robust controller is designed to deal with external disturbances. Purpose of controller maintains active/reactive power at the inverter station. The designed  $H_{\infty}$  controller is validated by a simulation using MATLAB/Simulink. This paper assumes that the load change is very slow. In future,  $H_{\infty}$  controller may be designed when load change is fast.

The  $H_{\infty}$  controller also may be designed that load is weak grid.

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Eun-Sung Gil was born in 1990, Korea. He received the B.S. degree in Electrical Engineering from Chungnam National University, in 2015. He is currently the M.S. course at Chungnam National University. His research interests include power system, HVDC system, and automatic control.



Hui Song was born in 1990, China. He received the B.S. degree in Electrical Engineering from Chungnam National University, Daejeon, Korea, in 2015. He is currently the M.S. course at Chungnam National University. His research interests include switched system and power system.



Kwan-Ho Chun was born in 1970, Korea. He received the B.S., M.S. and the Ph.D. degrees in Electrical Engineering from Seoul National University, in 1993, 1995 and 2002, respectively. From 2002 to 2004, he worked for SAMSUNG Electronics Co., KOREA as a senior Engineer. From 2004 to 2013, he was a senior engineer of TOSHIBA SAMSUNG Storage Technology Co., KOREA. Since 2013, he has been with the Department of Electrical

Engineering at Chungnam National University, Daejeon, Korea, where he is currently an Associate Professor. His research interests include nonlinear systems theory, switched system control theory and applications in robotics, renewable energy and electrical power systems.