Computational Simulation of Conjugated Problem of External Aerodynamics and Internal Heat and Mass Transfer in High-Speed Aircraft Composite Constructions

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Abstract—A conjugated problem statement for aerogasdynamics, internal heat and mass transfer and thermal strength of heat shield structures of hypersonic vehicles is formulated. A method for numerical solving of the problem is suggested. It is based on the iterative solution of the three types of detached problems: a gasdynamics problem for viscous heat-conducting flows, internal heat and mass transfer and thermoelasticity of shell constructions. An example of the numerical solution of the conjugated problem is given. It is shown that due to the high temperatures of the aerodynamic heating of the structure made of a polymer composite material there can appear a polymer phase thermodecomposition and intensive internal gas generation into the structure of the material.

Index Terms—conjugated problem, computational fluid dynamics, aerothermodynamics, hypersonic flows, heat and mass transfer, thermomechanics, polymer composites, thermodecomposition structures, heat shield

I. Introduction

The learning of hypersonic speeds is one of the most promising complex problems of the high-tech development. This problem can be identified such components as: the study of hypersonic aerodynamics of flights, study of the heat transfer on surfaces of aircraft constructions, thermophysics research of constructional materials, thermal strength study, analysis and development of constructional materials for hypersonic aircrafts, and the problems of hypersonic aeroelasticity, control and others. Significant amount of work (e.g. [1]) described researches of hypersonic aerodynamics conditions, less studied problems of heat transfer [2] under hypersonic speeds. The more difficult problem is the high-temperature behavior of composite materials based on heat-resistant filaments and masters [3]. The complex conjugated problem of aerothermodynamics, heat transfer, thermal physics and thermal strength of hypersonic constructions is still practically uninvestigated, and there are relatively recent work [4]-[5] that studied aeroelasticity of constructions under hypersonic speeds.

However under the actual operating conditions of hypersonic vehicles the problems of aerothermodynamics, heat transfer and thermophysics of the constructions are conjugated through the boundary conditions on the surface of the constructions, so the parameters of the heat flux acting on the materials depend on the properties of these materials. By turn, thermal properties of the materials at high temperatures may depend on the mode of deformation of the constructions. So the significant level of thermal stresses in composite materials leads to the filament microcracking long before the full macrodestruction of constructions, thereby the gas permeability and thermal conductivity materials and temperature fields in the constructions are changed. Thus the development of the methods for solving the conjugated problem for the study of the real processes taking place in the constructions of hypersonic vehicles is necessary.

The general formulation of the conjugated problem of aerothermodynamics and thermomechanics consists of the three systems of equations

- The Navier-Stokes equations of an external gas flow,
- The internal heat and mass transfer equations,
- The equations of thermoelasticity of a shell.

II. Mathematical Formulation

A. System of Gasdynamics Equations

Consider the system of equations of a viscous heat-conducting gas (the Navier-Stokes equations)

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \rho \mathbf{v} = 0,$$

$$\frac{\partial \rho \mathbf{v}}{\partial t} + \nabla \cdot \left(\rho \mathbf{v} \otimes \mathbf{v} + p\mathbf{E} - \mathbf{T}_v \right) = 0, \qquad (1)$$

$$\frac{\partial \rho \varepsilon}{\partial t} + \nabla \cdot \left((\rho \varepsilon + p) \mathbf{v} - \mathbf{T}_v \cdot \mathbf{v} + \mathbf{q} \right) = 0,$$

where ρ is the gas density, t is the time, $\mathbf{v}$ is the velocity vector, p is the pressure, $\mathbf{E}$ is the identity tensor, ε is the total energy per unit volume.

This system adds relations for the perfect gas, viscous stress tensor and heat flux vector

$$p = \rho \frac{R}{M} \theta, \quad e = c_v \theta, \quad E = e + |\mathbf{v}|^2 / 2,$$
$$\mathbf{T}_v = \mu_1 (\nabla \cdot \mathbf{v}) \mathbf{E} + \mu_2 (\nabla \otimes \mathbf{v} + \nabla \otimes \mathbf{v}^T), \quad (2)$$
$$\mathbf{q} = -\lambda \nabla \theta,$$

where R is the universal gas constant, M is the molecular weight of gas, θ is the gas temperature, c_v is the specific heat at constant volume, μ_1 and μ_2 are the coefficients of viscosity, λ is the thermal conductivity of gas. The coefficients of viscosity and thermal conductivity are functions of temperature.

The boundary conditions on the solid surface, which is the interface of the gas and solid domains, are as follows

$$\mathbf{v} = \mathbf{0}, \quad \theta = \theta_w,$$
$$-\lambda \nabla \theta \cdot \mathbf{n} = -\lambda_s \nabla \theta_s \cdot \mathbf{n} + \varepsilon_g \sigma \theta_{max}^4 - \varepsilon_s \sigma \theta_w^4, \quad (3)$$

where θ_w is the temperature of the solid surface, θ_{max} is the maximum temperature into the boundary layer, $\nabla \theta_s$ is the temperature gradient on the solid wall from the construction, ε_g and ε_s are the emissivity of the heated gas and solid surface respectively and σ is the Stefan-Boltzmann constant.

B. System of Equations of Internal Heat and Mass Transfer

We consider a typical element which is made of a heat-resistant composite material consisting of a polymer master with heat-resistant filaments. There are physical and chemical processes of thermodecomposition in such composite master under high temperatures of aerodynamic heating. In these processes the gaseous products of thermal decomposition are generated, then they are accumulated in the pores of the material and filtered into the outer gas flow, as well as a new solid phase is formed. It is the phase of pyrolytic master which has significantly lower elastic-strength properties than the original polymer phase. The four-phase model to describe the internal heat and mass transfer and deformations of such composite is proposed in [3]. This model consists of

- The equation of change of mass of polymer master phase

$$\rho_b \frac{\partial \varphi_b}{\partial t} = -J \quad (4)$$

- The equation of filtration of gaseous products of thermodestruction in pores of the composite material

$$\frac{\partial \rho_g \varphi_g}{\partial t} + \nabla \cdot \rho_g \varphi_g \mathbf{v}_g = J\Gamma \quad (5)$$

- The heat transfer equation in the thermodestruction composite

$$\rho c \frac{\partial \theta}{\partial t} = -\nabla \cdot \mathbf{q} - c_g \nabla \theta \cdot \rho_g \varphi_g \mathbf{v}_g - J \Delta e^0 \quad (6)$$

where φ_b, φ_g are the volume concentrations of the phase of the initial polymer master and gas phase; ρ_b is the density of the phase of the initial polymer master which is assumed to be constant; ρ_g is the average pore density of the gas phase (variable); c_g is the specific heat of the gas phase at constant volume, ρ and c are the density and specific heat of the composite as a whole, $\mathbf{q}$ is the heat flux vector, θ is the composite temperature for all phases in common; $\mathbf{v}_g$ is the velocity vector of the gas phase; Δe^0 is the specific heat of the master thermodecomposition; J is the mass velocity of the master thermodecomposition and Γ is the gas-producing factor of the master.

The equations (4)-(6) are added the relations between the heat flux vector $\mathbf{q}$, the velocity vector of the gas in pore $\mathbf{v}_g$ with the temperature gradient $\nabla \theta$ and the pressure gradient ∇p using the Fourier and Darcy laws, as well as the Arrhenius relation for the mass velocity of the master thermodecomposition J and Mendeleev-Clapeyron equation for the pore pressure of the gas phase p

$$\mathbf{q} = -\mathbf{\Lambda} \cdot \nabla \theta, \quad \rho_g \varphi_g \mathbf{v}_g = -\mathbf{K} \cdot \nabla p,$$
$$J = J_0 \exp\left(-\frac{E_A}{R\theta}\right), \quad p = \frac{\rho_g R \theta}{M_g}, \quad (7)$$

where J_0 is the pre-exponential factor, E_A is the activation energy of the thermodecomposition process, M_g is the molecular weight of the gas phase, $\mathbf{\Lambda}$ is the thermal conductivity tensor, $\mathbf{K}$ is the permeability tensor of the composite. They depend on the phase concentration.

The relations for the density and specific heat of the composite are

$$\rho = \rho_f \varphi_f + \rho_b \varphi_b + \rho_p \varphi_p + \rho_g \varphi_g,$$
$$c\rho = c_f \rho_f \varphi_f + c_b \rho_b \varphi_b + c_p \rho_p \varphi_p + c_g \rho_g \varphi_g, \quad (8)$$

where φ_f, φ_p are the volume concentrations of reinforcement filaments and pyrolytic phase of master, ρ_f, ρ_p are the densities of reinforcement filaments and pyrolytic phase of master (assumed to be constant); c_f, c_b, c_p are the specific heats of the solid phases (reinforcement filaments, phase of initial polymer master, pyrolytic phase of master) at constant deformation. The specific heats of the solid phases are assumed to be constant and temperature independent.

We also introduce the notation for the density and specific heats of the cage of composite (the set of all solid phases)

$$\rho_s = \rho_f \varphi_f + \rho_b \varphi_b + \rho_p \varphi_p,$$
$$c_s = \frac{1}{\rho_s} (c_f \rho_f \varphi_f + c_b \rho_b \varphi_b + c_p \rho_p \varphi_p). \quad (9)$$

The volume concentration of pyrolytic phase of master φ_p can be expressed analytically

$$\varphi_p = \left(\varphi_b^0 - \varphi_b\right)(1-\Gamma)\frac{\rho_b}{\rho_p} \quad (10)$$

The boundary conditions for the equations (4)-(6) on the heated surface of the construction are as follows

$$p = p_e, \quad \theta = \theta_e \tag{11}$$

where p_e, θ_e are the pressure and temperature of the flow on the surface.

The boundary conditions of tightness and thermal insulation are specified on the rest of the composite surface

$$\mathbf{n} \cdot \nabla p = 0, \quad -\mathbf{n} \cdot \mathbf{\Lambda} \cdot \nabla \theta = 0 \tag{12}$$

C. System of Equations of Thermoelasticity

In curvilinear coordinates $Oq_1q_2q_3$ associated with the middle surface of a thermoelastic shell composite structure the system of equations consists of [3]

- The equilibrium equations of the shell

$$\frac{\partial}{\partial q_\alpha}\left(A_\beta T_{\alpha\alpha}\right) + \frac{\partial}{\partial q_\beta}\left(A_\alpha T_{\alpha\beta}\right) - \frac{\partial A_\beta}{\partial q_\alpha}T_{\beta\beta} +$$

$$+ \frac{\partial A_\alpha}{\partial q_\beta}T_{\alpha\beta} + A_\alpha A_\beta k_\alpha Q_\alpha - A_\beta \frac{\partial P_g}{\partial q_\alpha} = 0,$$

$$\frac{\partial}{\partial q_\alpha}\left(A_\beta M_{\alpha\alpha}\right) + \frac{\partial}{\partial q_\beta}\left(A_\alpha M_{\alpha\beta}\right) - \frac{\partial A_\beta}{\partial q_\alpha}M_{\beta\beta} + \tag{13}$$

$$+ \frac{\partial A_\alpha}{\partial q_\beta}M_{\alpha\beta} - A_\alpha A_\beta Q_\alpha - A_\beta \frac{\partial P_g}{\partial q_\alpha} = 0,$$

$$-A_1 A_2\left(k_1 T_{11} + k_2 T_{22}\right) + \frac{\partial A_2 Q_1}{\partial q_1} + \frac{\partial A_1 Q_2}{\partial q_2} -$$

$$p_e A_1 A_2 - -\left(k_1 + k_2\right)A_1 A_2 \varphi_g P_g = 0;$$

- The kinematic relations

$$e_{\alpha\alpha} = \frac{1}{A_\alpha}\frac{\partial U_\alpha}{\partial q_\alpha} + \frac{1}{A_1 A_2}\frac{\partial A_\alpha}{\partial q_\beta}U_\beta + k_\alpha W,$$

$$2e_{\alpha 3} = \frac{1}{A_\alpha}\frac{\partial W}{\partial q_\alpha} + \gamma_\alpha - k_\alpha U_\alpha,$$

$$2e_{12} = \frac{1}{A_2}\frac{\partial U_1}{\partial q_2} + \frac{1}{A_1}\frac{\partial U_2}{\partial q_1} - \frac{1}{A_1 A_2}\left(\frac{\partial A_1}{\partial q_2}U_1 + \frac{\partial A_2}{\partial q_1}U_2\right), \tag{14}$$

$$\kappa_{\alpha\alpha} = \frac{1}{A_\alpha}\frac{\partial \gamma_\alpha}{\partial q_\alpha} + \frac{1}{A_1 A_2}\frac{\partial A_\alpha}{\partial q_\beta}\gamma_\beta, \quad 2\kappa_{\alpha 3} = -k_\alpha \gamma_\alpha,$$

$$2\kappa_{12} = \frac{1}{A_2}\frac{\partial \gamma_1}{\partial q_2} + \frac{1}{A_1}\frac{\partial \gamma_2}{\partial q_1} - \frac{1}{A_1 A_2}\left(\frac{\partial A_1}{\partial q_2}\gamma_1 + \frac{\partial A_2}{\partial q_1}\gamma_2\right);$$

- The defining relationships of the thermoelastic shell

$$T_{\alpha\alpha} = \sum_{\beta=1}^{2}(C_{\alpha\beta}e_{\beta\beta} + N_{\alpha\beta}\kappa_{\beta\beta}) - P_{g\alpha} - \hat{T}_\alpha,$$

$$T_{12} = 2(C_{66}e_{12} + N_{66}\kappa_{12}),$$

$$M_{\alpha\alpha} = \sum_{\beta=1}^{2}(N_{\alpha\beta}e_{\beta\beta} + D_{\alpha\beta}\kappa_{\beta\beta}) - M_{g\alpha} - \hat{M}_\alpha, \tag{15}$$

$$M_{12} = 2(N_{66}e_{12} + D_{66}\kappa_{12}),$$

$$Q_\alpha = \bar{C}_{\alpha+3,\alpha+3}e_{\alpha 3},$$

where $T_{\alpha\alpha}$, $T_{\alpha\beta}$, $M_{\alpha\alpha}$, $M_{\alpha\beta}$ are the forces and moments in the shell; Q_α are the shear forces; $e_{\alpha\alpha}$, $e_{\alpha 3}$, e_{12} are the deformations of the middle surface; $\kappa_{\alpha\alpha}$, $\kappa_{\alpha 3}$, κ_{12} are the curvatures of the middle surface; U_α, γ_α, W are the displacement, angles of curvature and deflection of the middle surface; A_α, k_α are the parameters of the first quadratic form and principal curvatures of the middle surface, $\alpha,\beta = 1,2$; and P_g, M_g are the forces and moments of the pore pressure in the shell

$$P_g = \int\limits_{-h/2}^{h/2} \varphi_g p \, dq_3, \quad M_g = \int\limits_{-h/2}^{h/2} \varphi_g p q_3 \, dq_3 \tag{16}$$

The forces and moments of the thermal stresses $\hat{T}_\alpha$, $\hat{M}_\alpha$ depending on the thermal deformations $\hat{\varepsilon}_\alpha$ for the shell are introduced as follows

$$\hat{T}_\alpha = \sum_{\beta=1}^{3} C_{\alpha\beta}\hat{\varepsilon}_\beta^{(0)}, \quad \hat{M}_\alpha = \sum_{\beta=1}^{3} C_{\alpha\beta}\hat{\varepsilon}_\beta^{(1)},$$

$$\hat{\varepsilon}_\beta^{(j)} = \int\limits_{-h/2}^{h/2} a_{01}\hat{\varepsilon}_\beta q_3^j \, dq_3, \quad \hat{\varepsilon}_3^{(j)} = \int\limits_{-h/2}^{h/2} a_{02}\hat{\varepsilon}_3 q_3^j \, dq_3, \tag{17}$$

$$\hat{\varepsilon}_\gamma = (\alpha_f \varphi_f B_\gamma + \alpha_b \varphi_b \Omega_\gamma)(\theta - \theta_0) +$$

$$+ \alpha_p \Omega_\gamma \int\limits_0^t (\theta(t) - \theta(\tau))\hat{\varphi}_p \, d\tau - \beta_p \varphi_p \Omega_\gamma,$$

where α_f, α_b, α_p are the coefficients of the thermal expansion of the filament, polymer and pyrolysis phases, β_p is the shrinkage ratio, B_γ, Ω_γ are the coefficients depending on the location of the filaments in the composite [3], $\gamma = 1,2,3$.

The forces and moments of the interphase interaction $P_{g\alpha}$, $M_{g\alpha}$ are

$$P_{g\alpha} = \int\limits_{-h/2}^{h/2} p\tilde{f}_\alpha \, dq_3, \quad M_{g\alpha} = \int\limits_{-h/2}^{h/2} p\tilde{f}_\alpha q_3 \, dq_3 \tag{18}$$

where $\tilde{f}_\alpha$ are the coefficients of interphase interaction.

The membrane, bending and mixed stiffnesses of the shell $\bar{C}_{\alpha\beta}$, $N_{\alpha\beta}$, $D_{\alpha\beta}$ from the relations (15) are

$$\bar{C}_{\alpha\beta} = C_{\alpha\beta}^0 a_{01}^{(0)}, \quad N_{\alpha\beta} = C_{\alpha\beta}^0 a_{01}^{(1)}, \quad D_{\alpha\beta} = C_{\alpha\beta}^0 a_{01}^{(2)},$$

$$\bar{C}_{66} = C_{66}^0 a_{01}^{(0)}, \quad N_{66} = C_{66}^0 a_{01}^{(1)}, \quad D_{66} = C_{66}^0 a_{01}^{(2)},$$

$$\bar{C}_{\alpha+3,\alpha+3} = C_{\alpha+3,\alpha+3}^0 a_{02}^{(0)}, \quad \alpha = 1,2; \tag{19}$$

$$a_{0k}^{(j)} = \int\limits_{-h/2}^{h/2} a_{0k}q_3^j \, dq_3, \quad k = 1,2; j = 0,1,2.$$

Due to the softening of the polymer master and its thermodecomposition, stiffnesses of the shell are changing when heated. This change for orthotropic composite shells is taken into account by 2 functions a_{01}, a_{02} [3].

Substituting the defining relationships (15) and kinematic relations (14), we have the 5 equilibrium

equations (13) which form a closed system for the 5 unknown functions U_α, γ_α, W.

The five given values from each pair $(T_{11}-\varphi_g P_g, u_\alpha)$, (T_{22}, u_β), (Q_α, W), $(M_{11}-\varphi_g M_g, \gamma_\alpha)$, (M_{22}, γ_β) are used for the boundary conditions at a line q_α = const.

Deformations $\varepsilon_{\alpha\beta}$ and stresses $\sigma_{\alpha\beta}$ in the shell

$$\varepsilon_{\alpha\beta} = e_{\alpha\beta} + q_3\kappa_{\alpha\beta}, \quad \varepsilon_{33} = 0, \quad \varepsilon_{\alpha3} = e_{\alpha3},$$

$$\sigma_{\alpha\alpha} = -\tilde{f}_\alpha p + a_{\theta1}\sum_{\beta=1}^{3} C_{\alpha\beta}\left(\varepsilon_{\beta\beta} + q_3\kappa_{\beta\beta} - \hat{\varepsilon}_\beta\right), \quad (20)$$

$$\sigma_{12} = a_{\theta1}C_{66}(\varepsilon_{12} + q_3\kappa_{12}), \quad \alpha, \beta = 1, 2.$$

The tangential stresses σ_{13}, σ_{23} and transverse normal stress σ_{33} have the following formulas

$$\sigma_{33} = 6\eta\left(\frac{(p_1 + p_2)}{2} + \frac{1}{h}C_{31}\left(a_{\theta1}^{(0)}e_{11} + a_{\theta1}^{(1)}\kappa_{11} - \hat{\varepsilon}_1^{(0)}\right) - \right.$$

$$\left. -\frac{P_{g1}}{h} + \frac{1}{h}C_{32}^0\left(a_{\theta1}^{(0)}e_{22} + a_{\theta1}^{(1)}\kappa_{22} - \overset{\circ}{\varepsilon}_2^{(0)}\right) - \frac{1}{h}C_{33}^0\hat{\varepsilon}_3^{(0)}\right) +$$

$$+(p_2 - p_1)\xi(q_3) + \frac{p_1 - p_2}{2} + \varphi_g p, \quad (21)$$

$$\sigma_{13} = \frac{12\eta(q_3)}{h}C_{44}^0 e_{13}a_{\theta2}^{(0)}, \quad \sigma_{23} = \frac{12\eta(q_3)}{h}C_{55}^0 e_{23}a_{\theta2}^{(0)},$$

$$\xi(q_3) = \frac{1}{2} - \frac{q_3}{h}, \quad \eta(q_3) = \frac{1}{4} - \left(\frac{q_3}{h}\right)^2,$$

where p_1, p_2 are the pressures on the external surfaces of the shell. The maximum values of the tangential stresses are reached on the middle surface, where $\eta(0) = 0.25$ and

$$\sigma_{13\,\mathrm{max}} = \frac{3}{h}C_{44}e_{13}a_{\theta2}^{(0)}, \quad \sigma_{23\,\mathrm{max}} = \frac{3}{h}C_{55}e_{23}a_{\theta2}^{(0)} \quad (22)$$

III. COMPUTATIONAL METHOD

A step-by-step method with two time scales Δt_1 and Δt_2 is used for the numerical solution of the conjugated problem. The time step Δt_1 is used for changing the boundary conditions for the Navier-Stokes equations and the equations of internal heat and mass transfer. The solution is carried out in 4 stages for each step Δt_1.

Stage 1. The solution of the equations of internal heat and mass transfer (4)-(6) is carried out by the numerical finite-difference method using the method of linearization and the implicit difference scheme which are described in [6]. The temperature of the solid surface θ_w interacting with the incoming flow is taken from the previous time step Δt_1.

Stage 2. The numerical integration of the Navier-Stokes equations (1) is carried out over the time steps Δt_2 to establish flow. We use the finite volume method based on elements centered on the grid point (vertex-centered volume) [7].

In the boundary condition (3) the heat flux through the wall $q_s = -\lambda_s \nabla\theta_s \cdot \mathbf{n}$ is calculated according to the formula: $q_s = \alpha(\theta_0 - \theta_w)$, where $\alpha = \dfrac{\lambda_s g(\mathrm{Fo})}{h}$ is the heat

transfer coefficient, h is the shell thickness, λ_s is the thermal conductivity of the shell in the transverse direction, g(Fo) is the dimensionless heat transfer coefficient depending on the Fourier number of material construction $\mathrm{Fo} = \dfrac{\lambda_s t_1}{\rho_s c_s h}$ and t_1 is the characteristic time of the construction heating. The function g(Fo) is calculated in advance by the preliminary numerical solution of the one-dimensional heat conduction problem for a plate with different values of the parameter Fo.

Stage 3. The solution of the equations of thermoelasticity for a shell (13)-(15) is carried out by means of the finite-element method which is described in [6]. The input data for this task are pressures on the outer p_1 and inner p_2 surface of the shell, which are calculated after the solution of the Navier-Stokes equations (1), as well as the distributions of the temperature θ, volume concentrations of the phases φ_f, φ_b, φ_p, φ_g and the pore pressure p which are calculated by the solving of the equations of internal heat and mass transfer (4)-(6) for the current time step.

Stage 4. The calculation of thermal stresses in the shell is carried out by means of formulas (20)-(21).

IV. RESULTS AND DISCUSSION

A numerical solution of the conjugated problem was calculated for a hypersonic flow ($M = 6$) around the nose of a vehicle model flying at the altitude of 15 km.

Fig. 1 shows the distribution of the temperature of the flow near the body. The temperature reaches 1.600 ˚K at the stagnation point on the nose of the vehicle and decreases monotonically as the distance from the stagnation point, but it remains rather high value. At the maximum distance the temperature is about 800 ˚K for the edge and lower generatrix and 1.000 ˚K for the upper generatrix of the vehicle which has a large value of the cone angle.

The results of numerical calculations of the fields of internal heat and mass transfer in the shell element of the hypersonic aircraft are shown in Fig. 2-Fig.7. The curves show the distributions along the shell thickness (the dimensionless coordinate $x = 0.5 + q_3/h$ is introduced) at a control point on the bottom surface at the distance of $30r$ from the stagnation point, where r is the blunted body radius at the stagnation point. Different colors correspond to the 7 different time points $t_1,...,t_7$ of aircraft flight.

Fig. 5-Fig. 6 show the distribution of temperature on the outer surface and the thickness maximum pore pressure of the construction at the time of maximum construction heating t_7.

Thermodecomposition of the polymer phase of the composite shell leads to the formation of a large amount of gaseous products in the pores of the material. The produced gases do not have time to percolate into the outer gas flow and create the internal pore pressure due to the low permeability of the composite.

Fig. 8-Fig. 10 show the distribution of the coefficient of changes of elastic properties of the composite $a_{\theta1}$, the thermal longitudinal deformation $\hat{\varepsilon}_1$ and the thermal

transversal deformation $\hat{\varepsilon}_3$ along the shell thickness at the control point for the different time points. A characteristic feature of these functions is their nonmonotonic. The thermal deformations increase in the domain of high temperature and decrease after passing through the conventional point of the beginning of the intensive thermal decomposition due to the formation of pyrolytic residue. This decrease affects the values σ_{11}, σ_{22} of the thermal stresses in the shell. The thermal transversal deformation $\hat{\varepsilon}_3$ becomes negative at a certain moment in contrast to the thermal longitudinal deformation $\hat{\varepsilon}_1$, because there is the shrinkage of the matrix (Fig. 10). The emergence of shrinkage causes the formation of shrinkage stresses σ_{33} in the heated subsurface shell layer.

The circumferential compressive stress σ_{22} ($h/2$) gradually increases during the heating together with the values of the maximum circumferential tensile stresses at the periphery of the shell closer to the edges of the shell. The positive peak of the tensile stresses σ_{22} occurs due to thermodestruction of the composite at the time of the maximum construction heating t_7 (Fig. 11). As a result the pore pressure of the gaseous products of thermal decomposition of the matrix increases.

Fig. 12 show the distribution of the transversal stresses σ_{33} along the shell thickness at the control point for the different time points of aircraft flight. These graphs show that the peak of the positive (tensile) stress σ_{33} occurs in the subsurface layer due to the increase of the pore pressure and the formation of shrinkage deformation of the shell after the initiation of thermal decomposition of the polymer. The values of transversal stresses are reached values of 0.13 GPa at the bottom of the shell at the time t_6. It is significantly higher than the breaking point of the composite's shell in the transversal direction. The destruction of the bundle type may occur in this part of the shell in which upper layers of the composite's fabric are delaminated from the rest of the material. A similar effect may occur at the top of the shell to the time t_7, as the transversal stresses rise to 0.06 GPa (Fig. 13). It is less than the values at the bottom of the shell, but also comparable to the breaking point of the composite's shell in the transversal direction.
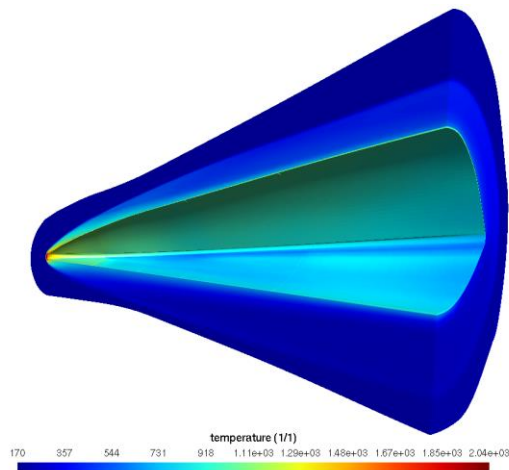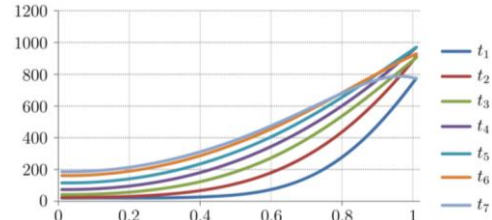

Figure 2. Temperature gradient, $\theta - \theta_0$ (℃).
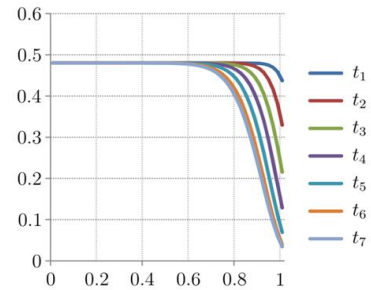

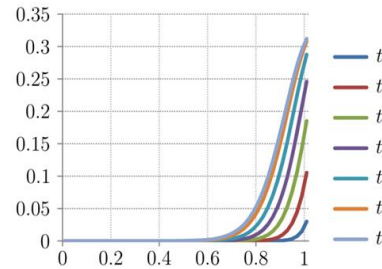Figure 3. Volume concentration of the polymer phase, φ_b.

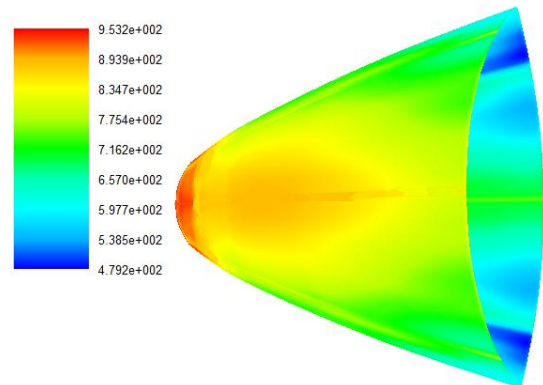Figure 4. Volume concentration of the pyrolytic phase, φ_p.

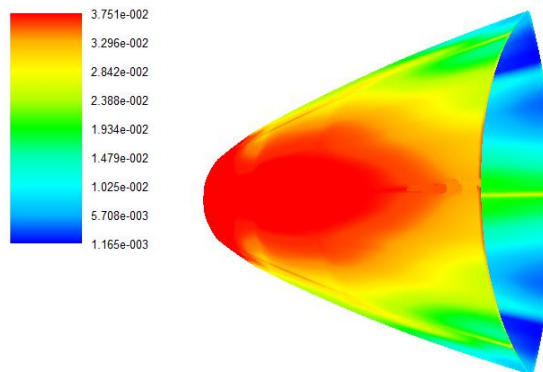Figure 5. Temperature on the outer surface, θ_w (℉K) at the time $t = t_7$.

Figure 6. The thickness maximum pore pressure, p_{max} (GPa) at the time $t = t_7$.

Figure 1. Temperature of the flow, θ (°K) at the time $t = 0$.
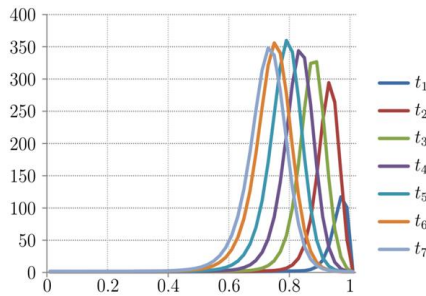
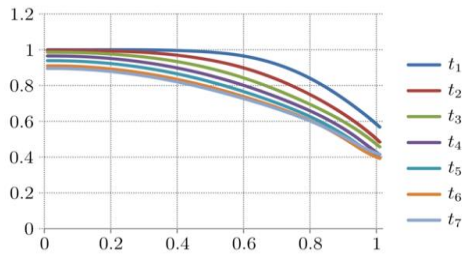Figure 7.   Pore pressure p, atm.

Figure 8.   The coefficient of changes of elastic properties of the composite, $a_{\theta 1}$.
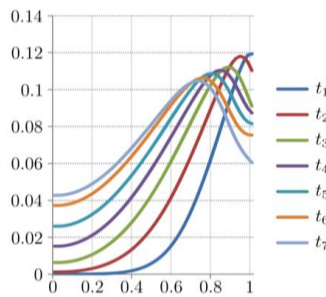

Figure 9.   Thermal longitudinal deformation, $\hat{\varepsilon}_1$.
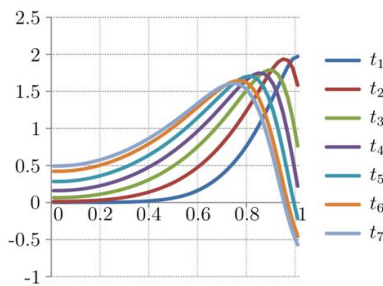

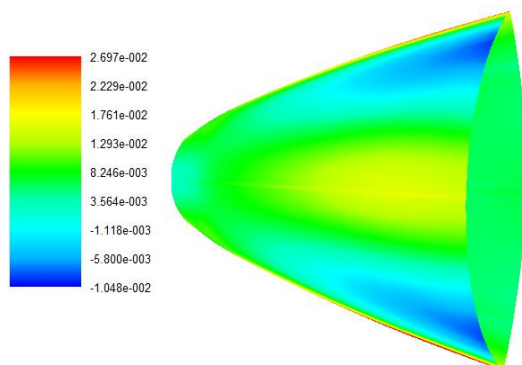Figure 10. Thermal transversal deformation, $\hat{\varepsilon}_3$.

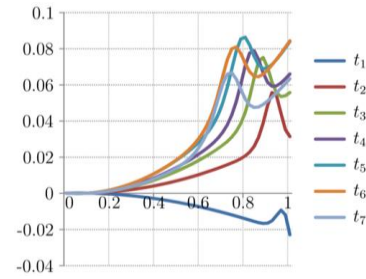Figure 11. Maximum circumferential stress, σ_{22} ($h/2$) (GPa) at the time $t = t_7$.
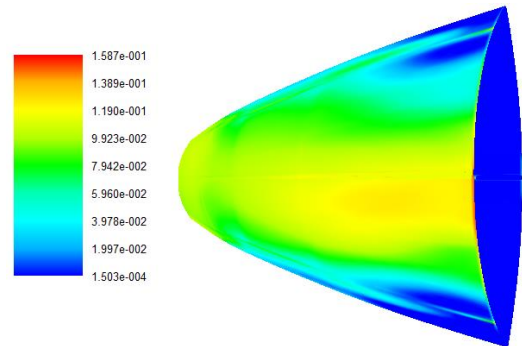

Figure 12. Transversal stress, σ_{33} (GPa).

Figure 13. Maximum transversal stress, σ_{33} (GPa) at the time $t = t_7$.

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