Influence of Module and Pressure Angle on Contact Stresses in Spur Gears

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***Abstract*—Contact stress between meshing gear teeth depends upon the material pair, normal load and the teeth geometry. Normal load at different points of contact changes during contact period, which depends upon the contact ratio. For identical material and operating conditions, contact ratio and the teeth geometry depends upon module and pressure angle of the gear pair. In the present work contact stress and contact fatigue analysis of spur gears has been carried out for identical operating conditions, for different values of module and pressure angle. Results of the investigation indicate decrease in contact fatigue life with increase in module as well as pressure angle.**

***Index Terms*—spur gear, contact stress, involute geometry, module, pressure angle**

I. Introduction

A gear is a component within a transmission device that transmits rotational force to another gear. Depending on their construction and arrangement, geared devices can transmit forces at different speeds and torques in different directions from the power source. When gear teeth profiles are designed so as to produce a constant angular-velocity ratio during meshing, they are said to have conjugate action. The most commonly used conjugate tooth profile is the involute profile because it is easy to manufacture and the centre distance between a pair of involute gears can be varied without changing the velocity ratio.

When two bodies having curved surfaces are pressed together, point or line contact changes to area contact, and the stress developed is three-dimensional, known as contact stress which is also referred to as hertzian stress. Contact stress is based on the compressive load normal to the geometry of contacting surfaces at the contact zone. Contact or hertzian fatigue failure mode is one of the common modes of gear failure. This is formed due to the repeated stress, which causes surface cracks and detachment of metal fragments from the tooth contact surface. The failure of the gear tooth due to pitting occurs when the contact stress between two meshing teeth exceeds the surface endurance strength of the material.

Contact stress in meshing gears depends upon the geometry of mating teeth, which is not constant along the path of contact. The teeth geometry also depends on gear module and pressure angle. The present work is related to study of influence of module and pressure angle on geometric features of mating teeth which in turn affects the contact stresses induced and the contact fatigue life.

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II. Gear Teeth Contact Analysis

A. Contact Stresses in Spur Gears

Gearing is an essential component of many machines. Since gears transmit motion and power through surface contact, good gearing performance depends on the durability of their teeth surfaces [1]. The transfer of power between gears takes place at the contact between the acting teeth. The stresses at the contact point are computed using Hertzian contact stress theory. The theory provides mathematical expressions for stresses and deformations of curved bodies in contact [2].

Contact between two involute spur gear teeth is shown in Fig. 1.

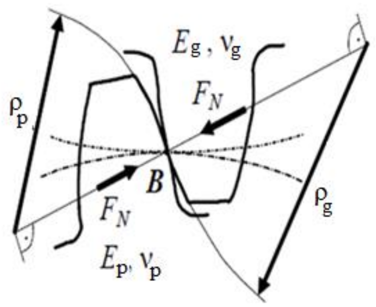

Figure 1. Contact between involute teeth

Maximum contact stress between the two teeth is given by

$$\sigma_H = 0.564\sqrt{\frac{\frac{F_N}{L}*\left[\frac{1}{\rho_p}+\frac{1}{\rho_g}\right]}{\frac{1-\nu_p^2}{E_p}+\frac{1-\nu_g^2}{E_g}}} \qquad (1)$$

where

F_N = Normal force in N

ρ_p and ρ_g = Radii of curvature of pinion and gear teeth at contact in mm

ν_p and ν_g =Poisson's ratio of pinion and gear materials

E_p and E_g= Modulus of elasticity of pinion and gear in GPa

From (1), it can be observed that the contact stress between meshing teeth depends upon normal load

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between the teeth and the radii of curvature of the mating teeth. The normal load acting between two mating teeth changes during contact period, depending upon the contact ratio of the gears. Also the radii of curvature of the mating teeth change continuously from beginning of contact till end of contact. Hence magnitude of contact stress changes continuously during contact phase between two mating teeth.

Stress analysis for gear teeth is regarded as a limiting factor for designers. Stress analysis focuses on the determination of the regions of stress concentration where failure or fracture may be initiated [3].

B. Pressure Angle(φ)

The angle between the pressure line and common tangent to the pitch circle is called Pressure angle or angle of obliquity and this angle is also important as it is related to the forces acting on the gear shaft and the bearings.

Load carrying capacity increases with higher pressure angle and a small number of teeth can be adopted without undercutting. But with a higher pressure angle, the separating force which is undesirable becomes greater. 14.5˚, 20° and 25˚ are commonly used pressure angle in power transmitting gears. The geometry of the tooth profile which influences the contact stress depends upon pressure angle of the gear.

C. Gear Teeth Failure Modess

There are basically two types of gear teeth failure modes: bending fatigue at the teeth root and contact (or surface) fatigue at the teeth flank. The contact fatigue is caused by the stresses developed at the region of contact between the teeth flanks, which, after several cycles, will lead to crack initiation. The contact conditions are responsible for the nucleation of these cracks on the surface or subsurface of the teeth flanks. The crack propagation may result in failure by pitting and/or spalling [4].

The process of surface pitting can be visualized as formation of surface-breaking or subsurface initial cracks, which grow under repeated contact loading. Eventually the crack becomes large enough for unstable growth to occur, which results in a part of the surface material layer breaking away. The resulting void is a pit. The fatigue process leading to pitting is dependent on the material and operating conditions [5], [1].

It is very important to find which parameters of the system affect the Hertzian stress levels along the contact action line. The geometric profile of the tooth flank (module, number of teeth, pressure angle), the gear materials, the lubricant properties, the load transmitted and the kinematics of the movement are the main factors influencing Hertzian stress levels [6], [7].

The fluctuation in load is one of the parameters that are involved in surface pitting. In order to analyse contact fatigue, it is necessary to investigate the fluctuation in contact stress following the change in the transmitted load during power transmission [3].

D. Contact Ratio of Spur Gears

In order for spur gears to transmit motion continuously, surface contact between at least one pair of subsequent teeth should commence before the previous tooth-surface contact is complete. The contact ratio means the ratio that represents the average number of gear tooth pairs in contact for a pair of meshing gears. A greater contact ratio can create a smoother operation. The applied load on the pinion changes with the contact ratio. In general, the highest contact ratio leads to the least stress generation by distributing the load over the teeth [3]. Contact ratio between meshing gears and radii of curvature of mating teeth depends upon module and pressure angle of the gears.

E. Numerical Analysis

Finite Element Method (FEM) can be used to simulate rolling and sliding contact, and considerably precise stress results are expected. The worst load condition is often regarded as the condition wherein one pair of teeth carries the full load, and the position in the area around the pitch point of a single teeth pair engagement is simulated with the maximum value of contact pressure [4].

III. PRESENT WORK

Gears develop high stresses at contact region of pair of teeth, when it is subjected to external loads. The repeated stress that occurs on the contact region of spur gears pair tooth surface is practically found to be the deciding factor in fatigue failure of the gear tooth. The main objective of the present is to study the influence of design parameters like module and pressure angle on contact stress of spur gears for identical operating parameters of power, speed, gear ratio, face width and materials.

A. Methodology

In the present work, in order to study the influence of module and pressure angle on contact stresses in spur gears, operating parameters are selected as shown in Table I. Spur gear pairs of different modules varying from 3mm to 10mm in steps of 1mm and pressure angles of 14.5˚, 20˚ and 25˚are selected, maintaining the operating parameters constant, as shown in Table I.

TABLE I. GEAR SPECIFICATION

Sl. No.	Parameter	Pinion	Gear
1	Face width (L)	60mm	60mm
2	Centre distance (a)	325mm	325mm
3	Power (P)	120KW	120KW
4	Speed (N)	650 RPM	406.25 RPM

For the gear pairs of different modules and pressure angles, contact ratio is determined and thereby the load sharing pattern between mating teeth along the path of contact is analyzed.

The involute teeth geometry is also analyzed for the selected gear pairs and the radii of curvature of pinion and gear at different contact points along path of contact is determined.

Having determined the contact loads and radii of curvature of contacting surfaces at different contact points along the path of contact, contact stresses are evaluated at respective contact points along the path of contact. Fatigue life analysis is carried out based on strain life approach considering the maximum contact stress developed during the contact period between two mating teeth. Cyclic stress-total strain (Ramberg-Osgood) equation (2) and strain-life (Mason) equation (3) are used to estimate fatigue life, based on strain life approach.

$$\epsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{k'}\right)^{\frac{1}{n}} \quad (2)$$

$$\epsilon_a = \frac{\sigma_f'}{E}\left(2N_f\right)^b + \epsilon_f'\left(2N_f\right)^c \quad (3)$$

Stress analysis and fatigue life estimation has also been carried out by FEM approach using commercial software ANSYS workbench and compared with the results obtained from theoretical calculations. Table II shows the material properties selected for the analysis.

TABLE II. MATERIAL PROPERTIES

Material properties	Pinion	Gear
Materials	Cast Steel 8630	Cast Iron
Modulus of elasticity	206 GPa	166 GPa
Poisson's ratio	0.3	0.3
Density	7850 kg/m3	7250 kg/m3
Ultimate stress (σ_{ut})	1141 MPa	240 MPa
Fatigue strength coefficient (σ^1_f)	1936 MPa	920 MPa
Fatigue strength exponent (b)	-0.121	-0.106
Fatigue ductility co-efficient (ε^1_f)	0.420	0.213
Fatigue ductility exponent (c)	-0.693	-0.43
Cyclic strength co-efficient (K^1)	1502 MPa	712 MPa
Cyclic strain hardening exponent (n)	0.122	0.102

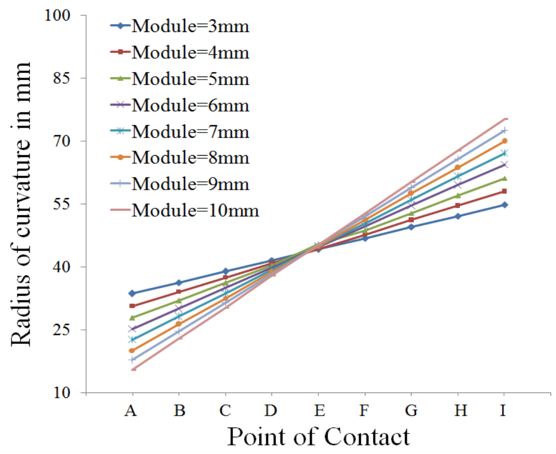

Figure 2. Variation of radius of curvature of pinion tooth along path of contact [Ø=14.5˚]

IV. RESULTS

A. Radius of Curvature

Fig. 2 and Fig. 3 shows the variation of radius of curvature of contacting pinion and gear teeth along the line of contact for gear pair with pressure angle of 14.5^0. A to E represents approach period and E to I represents recess period. It can be observed from the Figs, that during approach period, radius of curvature decrease with increase in module for pinion whereas, it increases with increase in module for gear. During recess period, the trend is opposite to that observed during approach period. Similar trend is observed for gear pairs with pressure angles of 20^0 and 25^0.

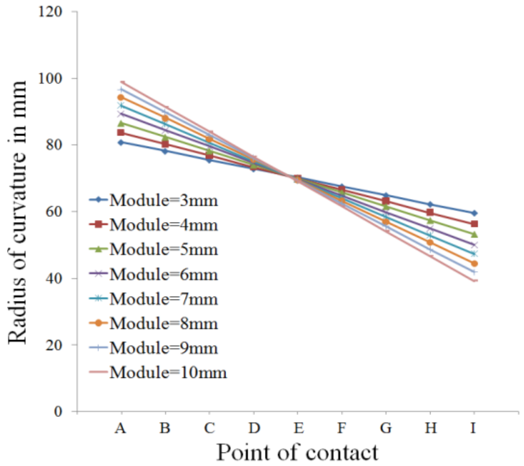

Figure 3. Variation of radius of curvature of gear tooth along path of contact [pressure angle 14.5˚]

B. Contact Ratio

Fig. 4 shows the influence of module and pressure angle on contact ratio of the spur gears. The contact ratio is the average number of teeth that are in contact when the gears are meshed and revolved. With increase in module, number of teeth will decrease resulting in reduced contact ratio. Also with increase in pressure angle, the length of arc of contact decreases resulting in reduced contact ratio.

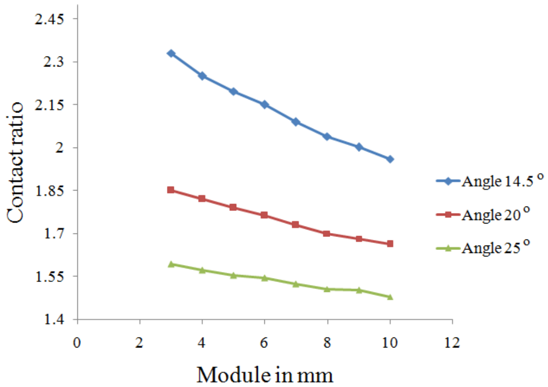

Figure 4. Contact ratio V/s Module for different pressure angles

C. Contact Stress

Fig. 5 shows the variation of maximum contact stress with module for different pressure angles. It can be observed that there is marginal increase in maximum contact stress with module. It can also be observed that maximum contact stress increases with increase in pressure angle. These variations of contact stress with respect to module and pressure angle are due to the variation in tooth geometry as well as the contact ratio with module and pressure angle.

D. Numerical Analysis

In the present work, the stress analysis and fatigue analysis are carried out by using FEA tool ANSYS workbench. 3D model of the gears is created using CATIA and imported to ANSYS workbench for analysis. Fig. 6 shows the meshed model of the mating gear pair. Fine mesh is used at the area of contact (location where stresses are critical) and coarse mesh at the remaining areas.

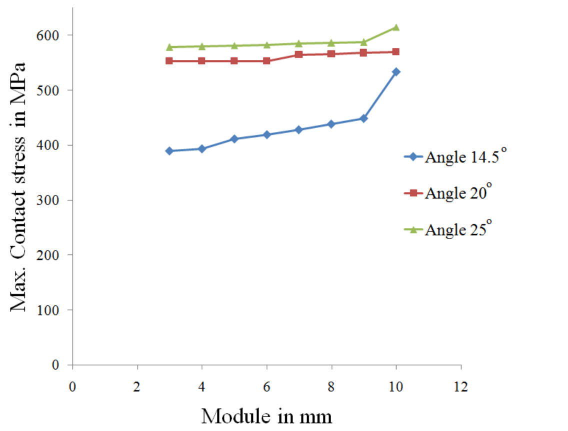

Figure 5. Max.Contact stress V/s Module for different pressure angles

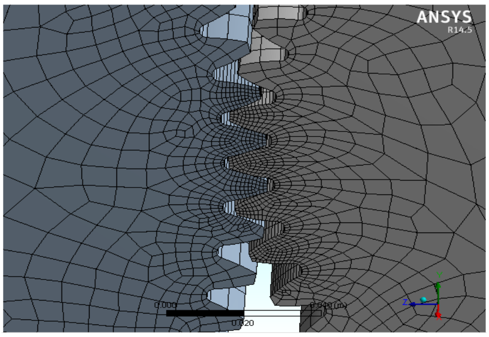

Figure 6. Hex-dominant meshing

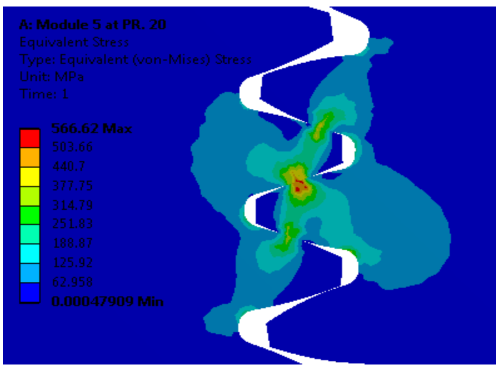

Figure 7. Von-mises stress for gear pair with module 5 and pressure angle 20˚

Following boundary conditions are applied for the present analysis. Frictionless support is applied at the pinion centre, which is free to rotate on its axis. Fixed support is applied at the gear centre, which offers resistance to the torque. Turning Moment is applied on the pinion centre as the loading condition. Fig. 7 shows the stress distribution at the contact zone between mating teeth for the gear pair with 5mm module and 20^0 pressure angle.

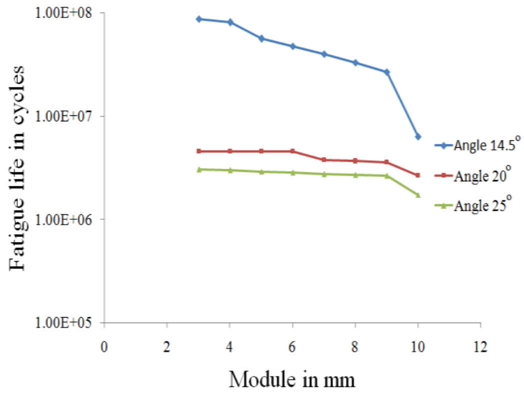

Figure 8. Fatigue life V/s Module for different pressure angles

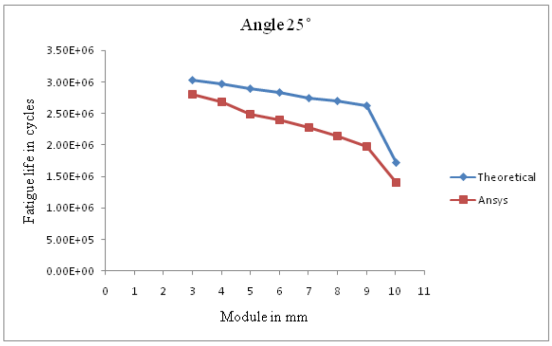

Figure 9. Fatigue life V/s Module at pressure angle 25˚

E. Fatigue Life

In the present work strain life approach is considered for evaluating of fatigue life since the contact stress exceeds the yield stress of the gear material. Fig. 8 shows the variation of the fatigue life of the gear with module, for different pressure angles. It can be observed from the Fig., that the fatigue life due to contact stress marginally decreases with increases in module. This is an interesting finding, since it has been found from the literature that fatigue life of the gears due to bending stress actually increases with module. Hence, selection of gear module needs to be optimized in order to achieve the desired fatigue life. It can also be observed from the figure that the fatigue life is higher for 14.5˚ pressure angle and decreases with increase in pressure angle. This can be attributed to the fact that contact stresses are least for 14.5˚ pressure angle gears, due to higher contact ratio.

Fig. 9 shows the variation of fatigue life with module obtained analytically and numerically. It can be observed that the trend of the results is identical and they are in close tolerance with each other.

V. CONCLUSIONS

Present investigation is a small attempt to study the influence of module and pressure angle on some of the design aspects of spur gear drive like contact stress and fatigue life. Following are some of the salient conclusions drawn from the present investigation.

The results of the investigation indicate that

- The contact ratio between spur gears is inversely proportional to the module and pressure angle. Increase in module results in reduction in number of teeth with corresponding increase in circular pitch of the gears, causing reduction in contact ratio. Increase in pressure angle causes reduction in length of path of contact, resulting in reduced contact ratio.
- Contact stress between the meshing gear teeth varies along the path of contact because of the varying radii of curvature of the tooth profile. The contact stress also depends upon the contact ratio between the mating gears. It is found that the maximum contact stress between spur gears is directly proportional to the module as well as pressure angle.
- Higher contact stress between meshing teeth results in reduced fatigue life. It is found that the fatigue life of spur gear is inversely proportional to the module as well as the pressure angle.

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