Design and Control of Small Quadcopter System with Motor Closed Loop Speed Control

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***Abstract* —Most of the motors used on the small Unmanned Aerial Vehicles (UAVs) are three-phase brushless direct current (BLDC) motors. Owing to the size of the UAVs and the shape of the motors, most of motors are running without feedbacked control as there are no suitable feedback devices. As the system dynamics of the UAVs are closely related to the rotation speed of the motors, open loop control may result in deviations in terms of thrust and thus cause instability of the flight attitudes. A sensorless speed detection scheme of the motors with a controller is implemented to achieve the closed loop speed control. Hierarchical control structure with the motor speed control and the attitude control is used on a quadcopter. The method of adaption of attitude control to motor speed control is presented. Experimental results show the improvements of the stability of attitude control and the tolerance on motors with different performance.**

***Index Terms*—BLDC motor, quadcopter, rotation speed, closed loop motor control, hierarchical control**

I. Introduction

Unmanned Aerial Vehicles (UAVs) have been a hot topic in research field for several years. There are many aspects that can enhance the flight stability, for examples, sensor fusion models and control strategies. In fact, there is one issue always being neglected by people and it causes incorrect actuation of the UAVs, the rotation speed of the motors.

Coherence of rotation speed is defined as how close is the actual rotation speed to the desired rotation speed of motors. The flight dynamics of a quadcopter are closely related to the sum of thrusts generated by propellers driven by motors given that the magnitude of thrust is directly proportional to the square of the rotation speed of the propeller. The resultant thrust will be distorted even if one thrust is distorted. In the reality, deviations exist in many aspects of motors even they are with same specification such as internal resistance, internal inductance, magnetic flux density of permanent magnets, etc. All of the physical deviations mentioned above eventually cause deviations in rotation. That means the coherence of rotation speed is not high enough such that the actual actuation will never equal to the desired one. Unexpected errors in both the magnitude and direction of the thrusts generated build up and lead to a wrong body attitude and then result in drifting problem.

Analysis on phase voltage of motor to evaluate the rotation speed is proposed and added to the quadcopter to deal with the feedback issue. Closed loop speed controller is implemented for each motor in order to guarantee the rotation speed of the motors and propellers and thus the thrust generated. Attitude controller is built with the presence of guaranteed speed control of motors.

There are few numbers of similar works of achieving closed loop speed control of BLDC motors. Current sensors were used to estimate the disturbance torque and compensate for the speed of the motors in [1] and [2]. The results shown that speed of motor could be well-regulated. Current sensing techniques had also been adopted on small UAVs. Current sensors were introduced in [3] to measure the current through each motor and approximate the magnitude of thrust. The other measure for obtaining the real time rotation speed of motor is to use an additional rotation speed for each motor [4]. Both two methods demonstrated improvement of hovering stability.

However there is room for improvement for the above solutions mentioned. The accuracy of current sensing scheme is not high enough for the solution proposed in [3], while the speed sensors proposed in [4] are large in size, which are not easy to install on the small platforms.

This paper is organized as follows: Section II briefly introduces the idea of implementing a feedback device for obtaining the rotation speed of the BLDC motors by analyzing the phase voltage signals as well as the closed loop speed control. The mathematical modeling of the system dynamics of the quadcopter is presented in Section III. The hierarchical control structure is introduced with the related coordination between two kinds of controls in Section IV. In Section V, it covers the result analyses of implementing proposed hierarchical controller. Last but not least, conclusion of this work is given in Section VI.

II. Feedback and Speed Control of Small BLDC Motors

The phase voltage of the motor can used to calculate the rotation speed. Owing to the property of electronic commutation of BLDC motors, the switching process of phase voltage among three phases depends on the position of rotor only. On the other hand, the period of phase voltage reflects how fast the motor is spinning. After

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filtering the voltage spikes and converting to a readable square wave, the rotation speed of motor can be measured. The comprehensive idea and the implementation are presented in the previous work [5]. This method for obtaining the rotation speed feedback and constructing speed controller for small BLDC motors used on UAVs is continued in this work.

III. Mathematical Modeling of System Dynamics

There are two common types of configuration for quadcopters, Cross-Configuration (CC) and Plus-Configuration (PC). They are different in the definition of orientation of the quadcopter as well as the actuation of the roll and pitch rotation. For PC, two motors opposing each other control the roll rotation while the other two control the pitch rotation. For CC, all the four motors are required for both roll and pitch rotation. It will be easier to control in PC while the load of motors is more evenly distributed in CC. In this paper, CC is adopted.

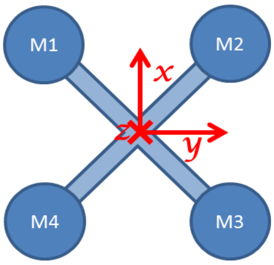


Figure 1. Simplified schematic diagram of a quadcopter in Cross-Configuration

A. Coordinate Frames

Original of body frame of the quadcopter locates at the center of mass position as shown in Fig. 1. The position of the quadcopter ($\boldsymbol{r}$) is defined in inertial frame.

$$\boldsymbol{r} = [r_x \quad r_y \quad r_z]^T \tag{1}$$

The orientation ($\boldsymbol{\eta}$) of the quadcopter is defined in the body frame as three kinds of rotation about three coordinate axes, roll, pitch and yaw respectively.

$$\boldsymbol{\eta} = [\varphi \quad \theta \quad \psi]^T \tag{2}$$

A rotation matrix ($\boldsymbol{R}$) performing rotation from body frame to the inertial frame in ZYX order is defined as [6]

$$\boldsymbol{R} = \begin{bmatrix} c_\theta c_\psi & c_\psi s_\varphi s_\theta - c_\varphi s_\psi & c_\varphi c_\psi s_\theta + s_\varphi s_\psi \\ c_\theta s_\psi & s_\varphi s_\theta s_\psi + c_\varphi c_\psi & c_\varphi s_\theta s_\psi - c_\psi s_\varphi \\ -s_\theta & c_\theta s_\varphi & c_\varphi c_\theta \end{bmatrix} \tag{3}$$

, where $c_\phi=\cos(\phi)$ and $s_\phi=\sin(\phi)$.

B. Linear Motion

There are many kinds of forces acting on the quadcopter. Besides the air resistance, thrusts and gravitational force are dominant.

Define $\boldsymbol{\omega_i}$ to be the rotation speed for the ith motors.

$$\boldsymbol{\omega_i} = [\omega_1 \quad \omega_2 \quad \omega_3 \quad \omega_4]^T \tag{4}$$

According to the Euler's first law of motion for rigid body dynamics, the equation of linear motion of the body can be modeled as

$$\frac{d^2\boldsymbol{r}}{dt^2} = g\begin{bmatrix}0\\0\\1\end{bmatrix} - \frac{\boldsymbol{R}K_T}{m}\sum_{i=1}^4 \boldsymbol{\omega_i^2}\begin{bmatrix}0\\0\\1\end{bmatrix} \tag{5}$$

where g, m and K_T denote the gravitational acceleration, mass of the quadcopter and thrust constant of propellers respectively.

By expanding and simplifying Equation (5), the accelerations of the quadcopter in three axes are

$$\frac{d^2r_x}{dt^2} = \left(-c_\varphi c_\psi s_\theta - s_\varphi s_\psi\right)\frac{K_T}{m}\sum_{i=1}^4 \boldsymbol{\omega_i^2} \tag{6}$$

$$\frac{d^2r_y}{dt^2} = \left(-c_\varphi s_\theta s_\psi + c_\psi s_\varphi\right)\frac{K_T}{m}\sum_{i=1}^4 \boldsymbol{\omega_i^2} \tag{7}$$

$$\frac{d^2r_z}{dt^2} = g - \left(c_\varphi c_\theta\right)\frac{K_T}{m}\sum_{i=1}^4 \boldsymbol{\omega_i^2} \tag{8}$$

It is obvious that the motions along x-axis and y-axis are independent of the yaw angle. Then Equations (6) to (8) can be further simplified by putting yaw to be zero by assuming there is no change in yaw. i.e.

$$\frac{d^2r_x}{dt^2} = -c_\varphi s_\theta\frac{K_T}{m}\sum_{i=1}^4 \boldsymbol{\omega_i^2} \tag{9}$$

$$\frac{d^2r_y}{dt^2} = s_\varphi\frac{K_T}{m}\sum_{i=1}^4 \boldsymbol{\omega_i^2} \tag{10}$$

$$\frac{d^2r_z}{dt^2} = g - \left(c_\varphi c_\theta\right)\frac{K_T}{m}\sum_{i=1}^4 \boldsymbol{\omega_i^2} \tag{11}$$

C. Rotational Motion

There are three kinds of rotation of the quadcopter achieved by varying the speeds of different motors. The corresponding torque ($\boldsymbol{\tau_\eta}$) acting along three body axes is

$$\boldsymbol{\tau_\eta} = [\tau_\varphi \quad \tau_\theta \quad \tau_\psi]^T \tag{12}$$

Since torque is defined as the cross product of the lever arm distance (L) and the force, the torques result in change of roll and pitch angles are differences of torques acting along the same body axes [7], i.e.

$$\tau_\varphi = LK_T(\omega_1^2 - \omega_2^2 - \omega_3^2 + \omega_4^2) \tag{13}$$

$$\tau_\theta = LK_T(\omega_1^2 + \omega_2^2 - \omega_3^2 - \omega_4^2) \tag{14}$$

The torque acting along the z-axis is defined as the sum of torques due to drag force on the propellers, i.e.

$$\tau_\psi = K_D(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2) \tag{15}$$

where K_D denotes the drag coefficient of the drag torque. The angular velocity of the quadcopter in the body frame ($\boldsymbol{\omega_\eta}$) is defined as

$$\boldsymbol{\omega_\eta} = [\omega_\varphi \quad \omega_\theta \quad \omega_\psi]^T \tag{16}$$

According to Euler's equation for rigid body dynamics, the rotation motion of the quadcopter is [8]

$$\boldsymbol{J}\frac{d\boldsymbol{\omega_\eta}}{dt} = -\boldsymbol{\omega_\eta} \times \boldsymbol{J}\boldsymbol{\omega_\eta} + \boldsymbol{\tau_\eta} \tag{17}$$

given that the inertia matrix of the quadcopter ($\boldsymbol{J}$) is [9]

$$\boldsymbol{J} = \begin{bmatrix} J_{xx} & 0 & 0 \\ 0 & J_{yy} & 0 \\ 0 & 0 & J_{zz} \end{bmatrix} \tag{18}$$

Expanding and simplifying Equation (17) into three equations for three axes,

$$\frac{d\omega_\varphi}{dt} = \frac{J_{yy}-J_{zz}}{J_{xx}}\omega_\theta\omega_\psi + \frac{\tau_\varphi}{J_{xx}} \tag{19}$$

$$\frac{d\omega_\theta}{dt} = \frac{J_{zz}-J_{xx}}{J_{yy}}\omega_\varphi\omega_\psi + \frac{\tau_\theta}{J_{yy}} \tag{20}$$

$$\frac{d\omega_\psi}{dt} = \frac{J_{xx}-J_{yy}}{J_{zz}}\omega_\varphi\omega_\theta + \frac{\tau_\psi}{J_{zz}} \tag{21}$$

D. System Inputs

There are totally four kinds of kinematics for the quadcopter: lifting force along the z-axis, rotational moment along the xyz axes. These four kinds of kinematics result in the change in altitude, roll, pitch and yaw respectively. Define the input for the kinematic system ($\boldsymbol{u}$) accordingly. They are all related to the square of the rotation speed of the four motors [10].

$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} K_T & K_T & K_T & K_T \\ LK_T & -LK_T & -LK_T & LK_T \\ LK_T & LK_T & -LK_T & -LK_T \\ K_D & -K_D & K_D & -K_D \end{bmatrix} \begin{bmatrix} \omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2 \end{bmatrix} \tag{22}$$

IV. Hierarchical Control of the Quadcopter

There will be two types of controller of the whole system: motor speed controllers and attitude controllers. The motor speed controllers will evaluate the deviations of actual speed from the desired speed of motors and calculate the amount of output for correcting the errors. It helps to guarantee the coherence of rotation speeds.

Attitude controllers are built on top of the motor speed controllers. With reference to the desired attitude, the attitude controller is responsible for finding out the errors and the required adjustment. With a given altitude, the four kinds of kinematic input of the system are known. The required speed of each motor can be obtained and assigned to the dedicated motor controllers. The block diagram for this hierarchical control structure is demonstrated on Fig. 2.

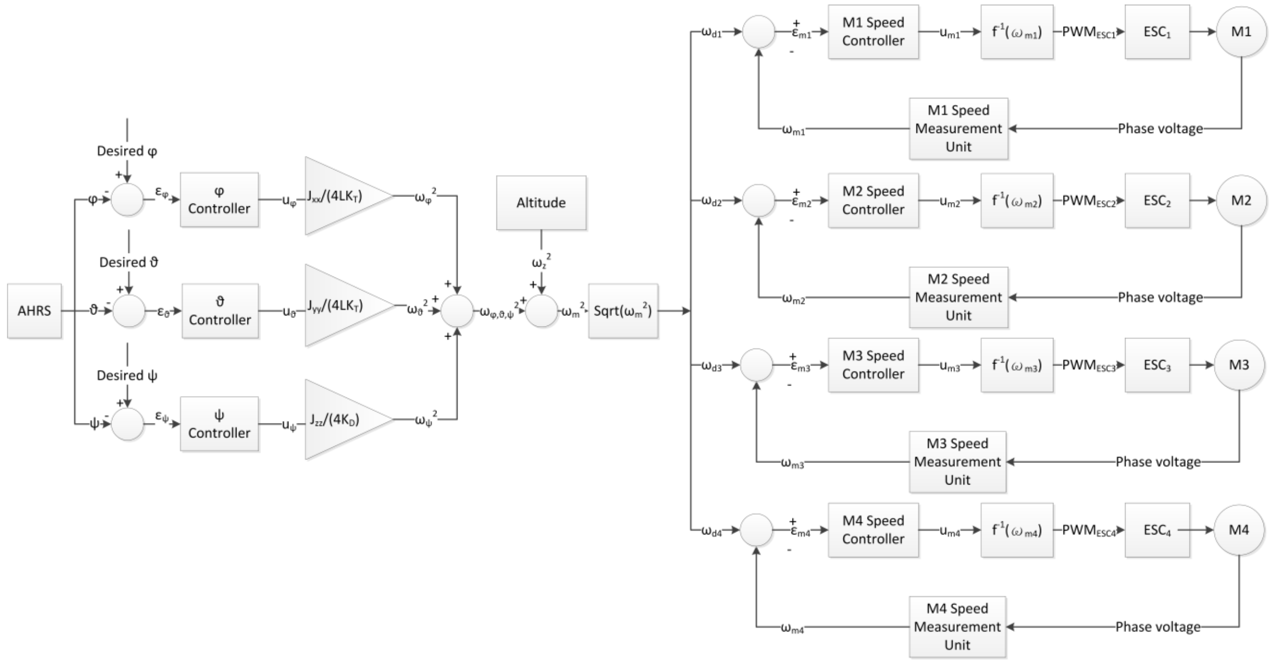


Figure 2. Block diagram of hierarchical control

A. Motor Speed Control

1) Design of controller

As shown in Fig. 2, there are four speed controllers for four motors respectively. Actual rotation speeds of four motor is feedbacked by the speed measurement units mentioned in [5] and errors ($\boldsymbol{\varepsilon_m}$) are evaluated by subtracting them by the desired speeds ($\boldsymbol{\omega_d}$).

$$\boldsymbol{\varepsilon_m(t) = \omega_d(t) - \omega_m(t)} \tag{23}$$

PI controller is adopted to calculate the amount of correction output ($\boldsymbol{u_m}$).

$$\boldsymbol{u_m(t) = K_{P_m}\varepsilon_m(t) + K_{I_m}\int_0^T \varepsilon_m(\tau)\,d\tau} \tag{24}$$

2) Mapping of control signals

The rotation speeds of motors are expressed with unit rad/s. The relationship between rotation speed of motor and duty cycle of control signal (d), Pulse Width Modulation (PWM) of the Electronic Speed Controller (ESC), has to be found out in advance. A mapping (f) is generated by recording the duty cycle of PWM applied to ESC and the rotation speed as shown in Fig. 3. This curve also defines the reference speed for particular control PWM under different circumstances, for examples, different loadings and different battery voltage levels.

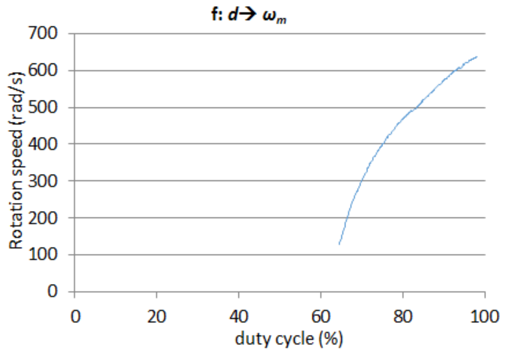


Figure. 3. Plotting rotation speed of motor against duty cycle of PWM of ESC

Inverse of the mapping is used to convert a certain amount of rotation speed to the required PWM signals for controlling the ESC in order to achieve to the desired speed.

B. Attitude Control

1) Design of controller

PID control is also used as the attitude controllers. The errors of attitude (ε_η) are defined as the difference of the desired angles ($\boldsymbol{\eta_d}$) and the actual angles

$$\boldsymbol{\varepsilon_\eta(t) = \eta_d(t) - \eta(t)} \tag{25}$$

The corresponding outputs of the controllers are

$$u_\eta(t) = K_{P_\eta}\varepsilon_\eta(t) + K_{I_\eta}\int_0^T \varepsilon_\eta(\tau_\eta)\, d\tau_\eta + K_{D_\eta}\frac{d\varepsilon_\eta(t)}{dt} \tag{26}$$

Define the correction torque applied to the quadcopter ($\boldsymbol{\tau_\eta}$) as

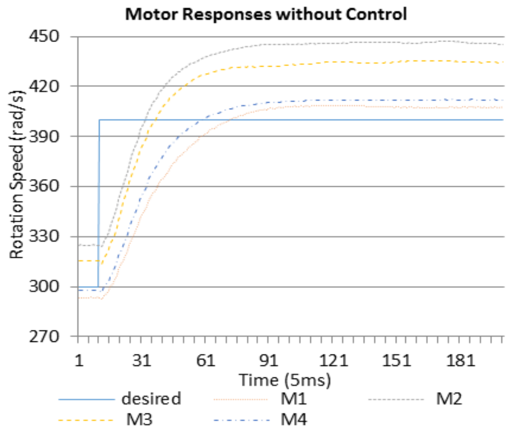
$$\begin{bmatrix}\tau_\varphi \\ \tau_\theta \\ \tau_\psi\end{bmatrix} = \begin{bmatrix} J_{xx}u_\varphi(t) \\ J_{yy}u_\theta(t) \\ J_{zz}u_\psi(t)\end{bmatrix} \tag{27}$$

2) Load distribution of motors

Having the required correction torque, the amount of torque acting on three axes is converted into amount of squared rotation speed of four motors according to Equations (13) to (15). The required lift force of the quadcopter is also converted into amount of squared rotation speed of four motors. Assuming that four motors share the same amount of load, the torque acting on each axis and the lift force will be evenly distributed. The required squared speed for each motor is

$$\begin{bmatrix}\omega_1^2 \\ \omega_2^2 \\ \omega_3^2 \\ \omega_4^2\end{bmatrix} = \begin{bmatrix} \frac{1}{4K_T c_\varphi c_\theta} & \frac{J_{xx}}{4LK_T} & \frac{J_{yy}}{4LK_T} & \frac{J_{zz}}{4K_D} \\ \frac{1}{4K_T c_\varphi c_\theta} & \frac{-J_{xx}}{4LK_T} & \frac{J_{yy}}{4LK_T} & \frac{-J_{zz}}{4K_D} \\ \frac{1}{4K_T c_\varphi c_\theta} & \frac{-J_{xx}}{4LK_T} & \frac{-J_{yy}}{4LK_T} & \frac{J_{zz}}{4K_D} \\ \frac{1}{4K_T c_\varphi c_\theta} & \frac{J_{xx}}{4LK_T} & \frac{-J_{yy}}{4LK_T} & \frac{-J_{zz}}{4K_D} \end{bmatrix} \begin{bmatrix} mg \\ u_\varphi(t) \\ u_\theta(t) \\ u_\psi(t)\end{bmatrix} \tag{28}$$

where the first column of the matrix in Equation (28) denotes the corrected vertical thrust given a deviation in roll and pitch of magnitude φ and θ respectively. Taking the square root of results by the above equation, the required speed for each motor for obtaining desired attitude can be obtained.



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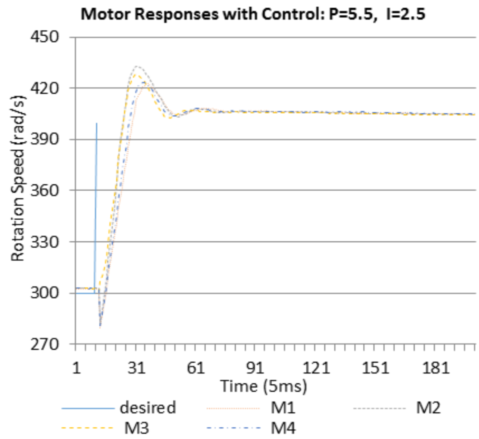


(b)

Figure. 4. Response of motors from 300 rad/s to 400 rad/s

V. Result Analyses

A. Motor Control

With the help of the closed loop speed control for the BLDC motors, the coherence of the motor performance can be guaranteed. Fig. 4 shows the responses of four motors changing the speed from 300 rad/s to 400 rad/s.

In this test, the deviations in speed and the generated thrusts are calculated. The below analysis of data was taken at t=500ms. Even there exist quite large differences among the four motors shown in Table I, the deviations approach to zero after applying the control as shown in Table II. From the experimental results, improved coherence for the fours can be observed.

TABLE I: Deviation in Thrust of Motors without Control

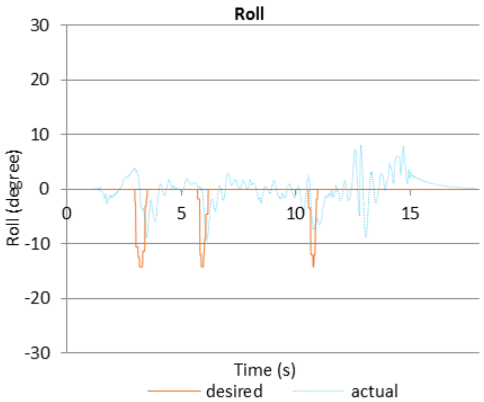
	M1	M2	M3	M4
Rotation Speed (rad/s)	407.632	445.511	432.445	411.194
Deviation in speed (rad/s)	7.632	45.511	32.445	11.194
Deviation in thrust (N)	0.088	0.552	0.387	0.130

TABLE II: Deviation in Thrust of Motors with Control

	M1	M2	M3	M4
Rotation Speed (rad/s)	406.261	406.312	405.923	406.533
Deviation in speed (rad/s)	6.261	6.312	5.923	6.533
Deviation in thrust (N)	0.072	0.073	0.068	0.076

B. Attitude Control

The attitudes in terms of roll pitch and yaw of the quadcopter during flight with attitude control only is shown in Fig. 5 while the attitudes in flight with cascaded motor speed control and attitude control is shown in Fig. 6 The readings for the desired attitudes and the actual attitudes are displayed. The three desired angles are controlled by joystick which the quadcopter will try to maintain at the desired attitudes. The data was sampled for every 20ms. It is obvious that there is a great improvement after enabling the motor speed controller. With the speed controller, the quadcopter is able to maintain its body attitudes very close to the desired ones and the delays are approximately zero.



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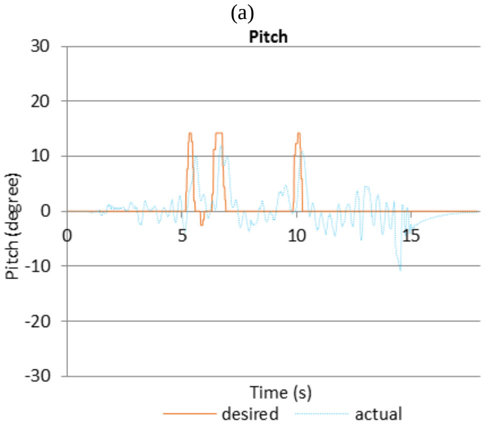

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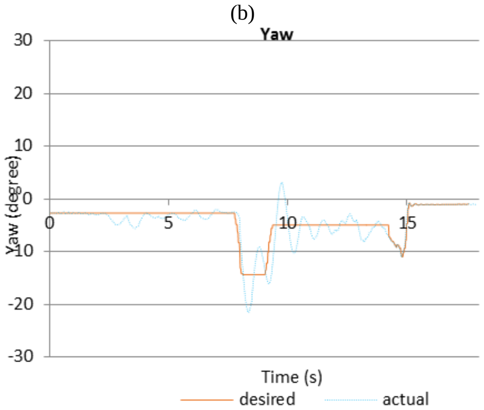
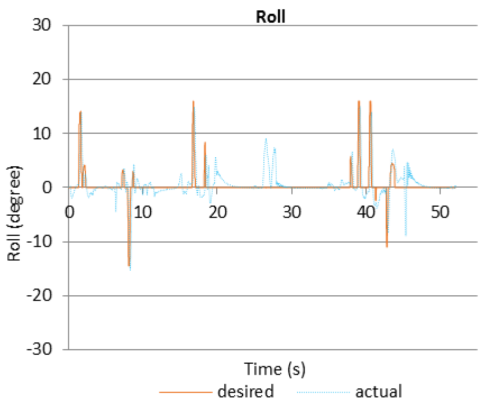


(c)

Figure 5. Attitudes in flights with attitude control only



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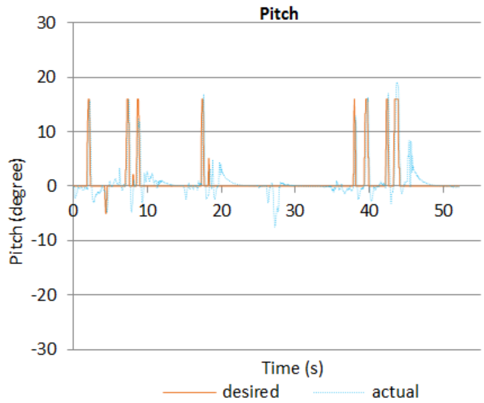

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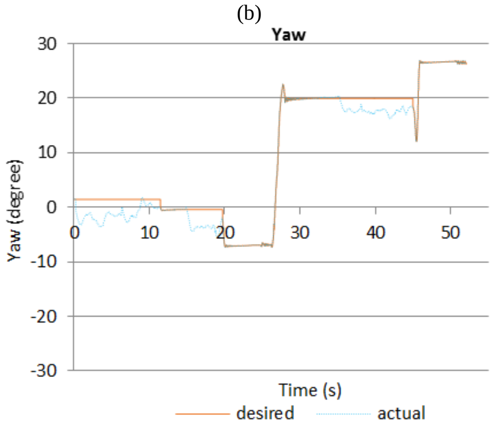

(c)

Figure 6. Attitudes in flights with cascaded motor control and attitude control

VI. Conclusion and Future Improvement

Without speed control of motors, positive deviation to the desired speed causes a larger thrust while negative deviation causes a smaller thrust. Both cases lead to inclination of the attitude angles. In the traditional scenario, the problem will be sensed by the attitude controller and the inputs of motors will then be adjusted. As a result, this creates overhead from sensing the errors in attitude till completing the adjustment. The arms of the quadcopter will actually experience up and down movement which contributes to the vibrations of the quadcopter.

Benefitted by the speed control for the motors, the attitude control will be more efficient because the motors will be spinning very close to the expected manner. Even for the slightly defected low quality motors with deviations in speeds, the presence of controllers is able to maintain a consistent speed. This makes a contrast to the traditional implementation in which there will be less unnecessary vibrations. This also reduces the drift issue of the quadcopter when it is hovering in the air.

This is one important step of improving the flight stability. There is still quite large room of improvement of the flight control, for examples, replacing the traditional PID attitude controller with a more advanced controller like adaptive neural controller, or using the Unscented Kalman filter for state estimation.

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