Advanced Passive Suspension with Inerter Devices and Optimization Design for Vehicle Oscillation

Thanh-Tung Tran and Hiroshi Hasegawa
Graduate of Systems Engineering and Science, Shibaura Institute of Technology, Japan
Email: {nb12507, h-hase}@sic.shibaura-it.ac.jp

***Abstract*—In generally, a suspension system needs to be soft to insulate against road disturbances and hard to insulate against load disturbances. It cannot achieve with a traditional passive suspension that only considered to the stiffness and damper. In this study, the paper clarifies some issues related to suspension system with inerter to reduce sprung mass displacement and tire deflection in quarter-car model. In this paper, we integrate some kinds of suspension system with inerter on quarter-car models. We propose some new designs, which have some advantages for suspension system by improving vehicle oscillation. We optimize design of model based on the minimization of cost functions for displacement, tie deflection with constraint function of suspension deflection limitation and the energy consumed by the inerter. The advantage of research is integration a new mechanism, the inerter; this system can improve the vehicle oscillation on quarter-car model with different parameters. It shows the benefit of the inerter in proposal suspension system.**

***Index Terms*—inerter, suspension, optimization design, oscillation, formula car**

I. Introduction

Passive, semi-active and active suspension systems have been utilized to improve ride comfort of vehicles and their effectiveness has also been demonstrated. However, it is not easy to improve dynamics stability with passive suspension systems. To achieve it, several control methods have been proposed, but most of them relate the active suspension [1]-[3].

A. Comfort Specifications

The comfort characteristics are considered, it mainly related to displacement, chassis vibration, noise, etc. It has an impact on the driver reaction time, accuracy, situation evaluation and decision abilities, which makes this objective particularly active in the automotive community. According to these kind of models, they indicate some sensitive frequency zones related to the heart, the head, etc. having resonance or gain amplifications around some specific frequencies according to different disturbances such as the road irregularities [4].

In this study, the comfort is not directly discussed, but evaluated through the chassis analysis as sprung mass displacement. The comfort feeling analysis is performed by analyzing some specific frequencies of the vertical behavior of the quarter-vehicle model. We focus on the analysis of simpler variables behavior with respect to road unevenness, such as vertical acceleration $\ddot{Z}_s$ and displacement Z_s of the chassis. Then an improvement on these variables will imply comfort improvement with mathematical objective is:

$$\min \ddot{Z}_s(t) := \min Z_s(t) \qquad (1)$$

B. Road-Holding Specifications

Road-holding is a vehicle property which characterizes the ability of the vehicle to keep contact with the road and maximize wheel tracking to road unevenness. It is very simplified manner, the longitudinal (F_{tx}) lateral (F_{ty}) forces of each tire as follows:

$$F_{tx} = F_n\mu_x; \;\; F_{ty} = F_n\mu_y \qquad (2)$$

where μ_x and μ_y are the nonlinear functions, dependent on the slip ratio, the slip angle and the road roughness characteristics.

These forces are also affine functions of F_n, the normal load, defined as:

$$F_n = (M_s + M_u)g - k_t (Z_u - Z_r) \qquad (3)$$

where M_s and M_u are the sprung and un-sprung masses, k_t is the vertical tire stiffness characteristic and tire deflection :

$$Z_{t\text{-}def} = Z_u - Z_r \qquad (4)$$

Because $(M_s + M_u)g > 0$ and $k_t > 0$ then: F_n max when tire deflection $Z_{t\text{-}def}(t) = Z_u(t) - Z_r(t)$ go to minimization.

C. Suspension Limitations

When evaluating a suspension system and its associated control algorithm, to take into account the static suspension stroke limitations $Z_{sus} = Z_s - Z_u$ which should always remain in between the limitations defined by the technology, i.e. then:

$$\min Z_{sus} \leq Z_{sus} \leq \max Z_{sus}$$

where min Z_{sus} and max Z_{sus} are the suspensions deflection limits. This constraint is very important for practical applications, in order to preserve the mechanical suspension system.

To improve vehicle oscillation, this study proposes a design of passive suspension system taking with new component element named "inerter" into consideration the both sensitive of the sprung mass and tire deflection vehicle behavior when have road disturbance [5]. It can improve both displacement and tire deflection of vehicle proposed by optimizing the modal parameters of

Manuscript received March 1, 2015; revised September 8, 2015.

doi: 10.18178/ijmerr.4.4.354-360

suspension and tire. In order to verify the effectiveness of the proposed method, a quarter-car model that has variable stiffness, damping and inerter in suspension system is constructed and the numerical simulations are carried.

For modeling of an inerter, it was defined to be a mechanical two-terminal, one-port device with the property that the equal and opposite force applied at the nodes is proportional to the relative acceleration between the nodes. For example, the gear type inerter mechanism including a gear set and rack, it has two terminals at the rack and the base body. The dynamic equation of an inerter is derived as F=b*a, wherein F, b and a represent the applied force, the inerter coefficient (called inertance) and the relative acceleration of two terminals, respectively. The inertance is calculated from the radius and inertia of flywheel. Another objective is presented to reduce friction force and system energy dissipation is screw type inerter mechanism. It can be reduce conventional backlash general by gear transmission [6]. In other hands, hydraulic type inerter is mechanism which comprises a hydraulic cylinder. A hydraulic motor connected to hydraulic cylinder with an output shaft for converting the linear motion to rotary motion and an inerter body disposed on the output shaft. This system subjects to high external force loads and controllability [7].

Let focus attention first on the familiar two-terminal modeling elements: spring, damper and inerter. Each is an ideal modeling element, with a mathematical definition [8]-[10]. It is useful to discuss on mechanical networks, which give some hint toward the inerter idea, in order to highlight the new passive suspension system.

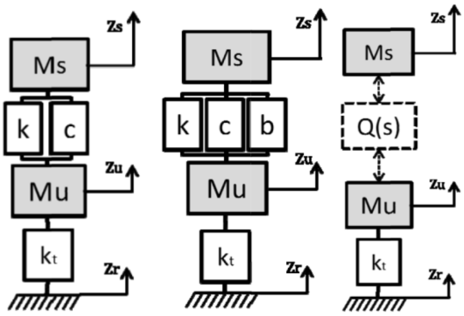


Figure 1. The base, parallel and quarter-car model with suspension function represented in laplace transformed respectively

II. Mathematical Model

A. Quarter-Car Model

Base on the conventional quarter-car model, we design new structure called quarter-car suspension parallel (Fig. 1). This model will change from normal passive suspension to new suspension with stiffness, damping and inerter in parallel. We study about the vertical displacement of sprung mass and tire deflection in some kinds of simulations.

For the quarter-car model, the suspension strut provides an equal and opposite force on the sprung and un-sprung masses by means of the positive real admittance function which relates the suspension force to the strut velocity through spring, damper and inerter. We define them as the following equations.

Base quarter-car model dynamics equation in time-domain:

$$M_s\ddot{Z}_s(t) = F_k(t) + F_c(t) \tag{5}$$

$$M_u\ddot{Z}_u(t) = F_{kt}(t) - (F_k(t) + F_c(t))$$

where:

$$\begin{gathered} F_k(t) = k(Z_u(t) - Z_s(t)) \\ F_c(t) = c(\dot{Z}_u(t) - \dot{Z}_s(t)) \\ F_{kt}(t) = k_t(Z_r(t) - Z_u(t)) \end{gathered} \tag{6}$$

State-space representation:

$$\begin{bmatrix} \ddot{Z}_s \\ \dot{Z}_s \\ \ddot{Z}_u \\ \dot{Z}_u \end{bmatrix} = \begin{bmatrix} \frac{-c}{M_s} & \frac{-k}{M_s} & \frac{c}{M_s} & \frac{k}{M_s} \\ 1 & 0 & 0 & 0 \\ \frac{c}{M_u} & \frac{k}{M_u} & \frac{-c}{M_u} & \frac{-k-k_t}{M_u} \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} \dot{Z}_s \\ Z_s \\ \dot{Z}_u \\ Z_u \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{k_t}{M_u} \\ 0 \end{bmatrix} Z_r \tag{7}$$

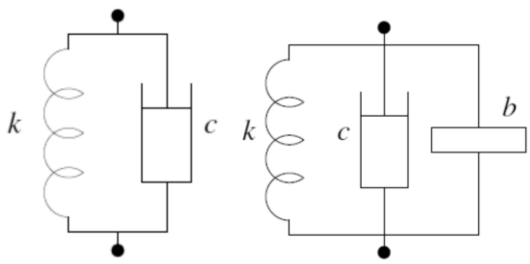


Figure 2. The conventional and parallel suspension with inerter

We summarize the approach of the suspension design problem was formulated as an optimal modal parameter to improve vertical displacement and tire deflection. The solution of the optimization problem made use structure of new quarter-car model that improve from traditional passive suspension system in adding inerter elements (Fig. 2). In some previous researching, the good and simple structure was able to come up with new network topologies involving inerter [11], [12]. To solving these problems, we use Laplace transform function as the ways to represent for suspension system in cases study by Q(s).

The equations of motion in the Laplace transformed domains are:

$$M_s s^2\widehat{Z_s} = -sQ(s)(\widehat{Z_s} - \widehat{Z_u})$$

$$M_u s^2\widehat{Z_u} = sQ(s)\left(\widehat{Z_s} - \widehat{Z_u}\right) + k_t\left(\widehat{Z_r} - \widehat{Z_u}\right) \tag{8}$$

We can compute the relevant transfer functions as follows:

The transfer functions from the road disturbance Z_r to the displacement of the sprung mass Z_s is:

$$T_{Z_s} = \frac{k_t Q(s)}{M_s s(M_u s^2 + k_t) + ((M_u + M_s)s^2 + k_t)Q(s)} \tag{9}$$

The transfer functions from the road disturbance Z_r to tire deflection Z_{t-def} is:

$$T_{Z_{t-def}} = \frac{-s^2[M_s M_u s+(M_s+M_u)Q(s)]}{M_s s(M_u s^2+k_t)+((M_u+M_s)s^2+k_t)Q(s)} \quad (10)$$

The conventional and parallel suspension function represented in Laplace transformed respectively:

$$Q(s) = Y_k + Y_c = \frac{k}{s} + c \quad (11)$$

$$Q(s) = Y_k + Y_c + Y_b = \frac{k}{s} + c + bs \quad (12)$$

B. *Performance Specifications*

Considering the previous literature review and considering the quarter-car model given above, the following signals are considered for performance analysis and characterization of a suspension system:

To evaluate the comfort, the vertical displacement Z_s of the chassis is analyzed.

To evaluate the road-holding, the tire deflection Z_{t-def} is analyzed.

The suspension deflection limits ($\min Z_{sus}$ and $\max Z_{sus}$) could be analyzed as the constraint function.

Frequency responses of both main transfer functions under discussion in this paper are and then we define the sprung mass displacement and tire deflection performances. We have looked at suspension in regard to vehicle vibration characteristics affecting vehicle equipment [13]. Fig. 3 shows related vehicle vibration and noise classified according to source input and source frequency.

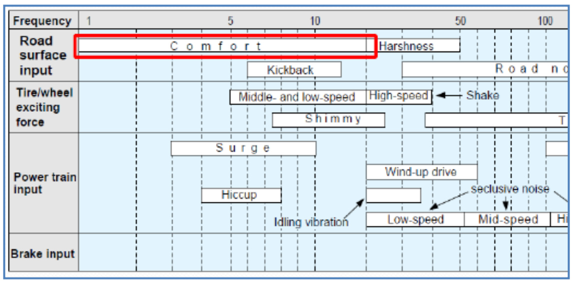


Figure 3. Vibration sources and input to the suspension

In this study, we focus to analyze the effect from road disturbance to kickback and comfort via displacement and acceleration of sprung mass. The frequency domain to estimate from 1-30 Hz, it is the main reason that we not mention about other disturbances.

For sprung mass displacement, the vibration isolation between [0; 20] Hz is evaluated by the transfer function.

For tire deflection, it is evaluated using the transfer function over the frequency range of [0; 30] Hz. A good road-holding, the tire deflection should be attenuated for low frequencies, and filtered around the resonance frequency of the wheel and over. Moreover, especially for high frequencies, the wheel should always remain linked to the road.

Suspension deflection is signal that should remain in the linear zone of suspension system to avoid limitation points. In this case, frequency limit is 30 Hz, this is the fact that high frequencies are filtered by vehicle mechanical elements.

C. *Parameters Specification*

For the evaluation of sprung mass displacement and tire deflection, we use a bump road profile and there is no load disturbances applied on the sprung mass. Base on previous study [14], we have modal parameters for passive suspension system as:

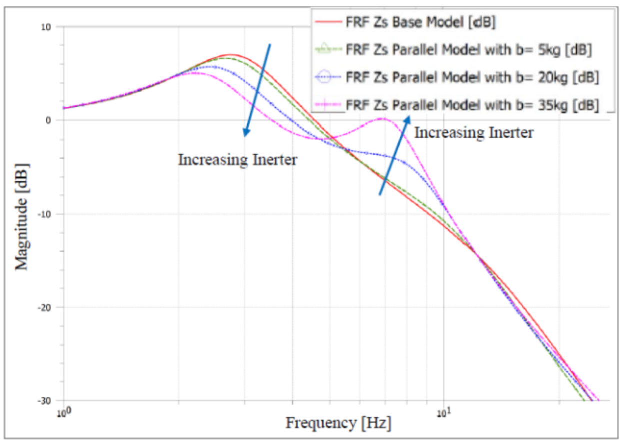
TABLE I. THE SPECIFICATION OF FORMULA SAE QUARTER-CAR MODEL

Symbols	Parameters	Values
Ms	Mass of body	63 [kg]
Mu	Mass of tire	12 [kg]
k	Stiffness coefficient	24000 [Nm-1]
c	Damping coefficient	1200 [Nsm-1]
b	Mass of inertance	20 [kg]
kt	Stiffness coefficient of tire	70000 [Nm-1]
H0	Bump road disturbance	0.05 m]

III. MATHEMATICAL MODAL ANALYSIS

The present numerical discussion consists in analyzing the previous model properties on a numerical example. The model used in the following simulations is the simple LTI passive one, as given in definition with the Formula SAE car parameters given in Table I. Since the main objective is to analyze passive suspensions, suspensions with inerter employed and changeable, the analysis carried in this section is sprung mass displacement and tire deflection of the passive system for varying inertance b values.

Fig. 4 illustrates the pole location of the passive quarter-car model for different models when we apply inerter in parallel model. First, according to this figure, it is notable that whatever the frequency smaller than 5 Hz, parallel model is better than base model in both displacement and tire deflection response frequency performance. The quarter-car system remains stable with parallel structure. However, from 5 Hz to 12 Hz, the parallel model is not good as base model.



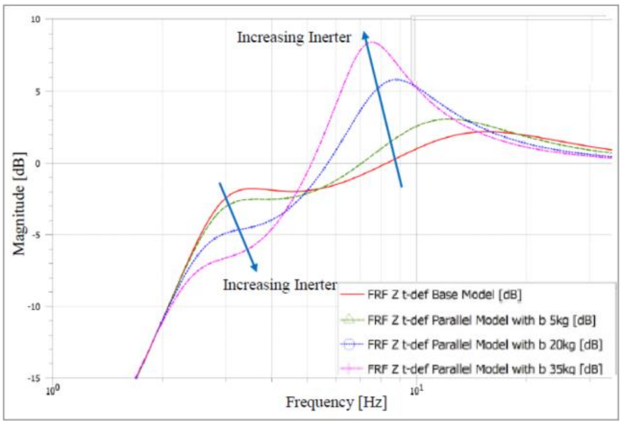


Figure 4. Frequency response function and of base and parallel quarter-car model with the change of inertance b value in respectively

Then in parallel structure, by increasing the b value, the displacement and tire deflection response magnitude poles module reduces.

When b = 5 kg, the poles are located on the imaginary axis and start to reduce in 1-5 Hz.

When b→35 kg, the poles tend to be located is continuous reduce in this frequency.

It is interesting to note the following points, as expected and accordingly to the pole location analysis provided above, increasing inertance b value reduce the resonance peaks, leading to an oscillatory behaviour. But, in other frequency phase, the system is not good. To improving suspension system in oscillation, we suggest using parallel structure to optimize quarter-car model.

IV. Optimization Modal Parameters

In this section we will focus to a single aspect of performance is related to the dynamics problem. We used the structure parameter which represented the following design variables: k, c, b, and kt, with lower and upper boundaries show in Table II. Simulation on mathematical quarter-car model was carried out to enable evaluation of vehicle performance measures. The resulted root-mean-squared (RMS) parameters, namely the RMS(Zs) sprung mass displacement and RMS(Zt-def) tire deflection were taken into consideration under constriant function of RMS(Zsus) suspension deflection limitation.

TABLE II. The Boundary of Design Variables

Lower	Parameter	Upper boundary	Unit
18000	k	30000	[Nm-1]
600	c	1800	[Nsm-1]
5	b	35	[kg]
50000	kt	90000	[Nm-1]

A. Optimal Displacement

The objective function is represented for root-mean-squared of sprung mass displacement is:

$$\mathrm{RMS}(Z_s) = f(k, c, b, k_t) \rightarrow \mathrm{Min} \tag{13}$$

Under the synthetic assumption, we can figure out this equation:

$$\mathrm{RMS}(Z_s) = -0.886 + 1.24e-04*k - 2.56e-09*k2 + 2.59e-03*c - 1.18e-06*c2 - 5.47e-03*b + 8.43e-05*b2 + 2.14e-07*kt - 1.94e-12*kt2 - 2.61e-07*k*c + 1.16e-10*k*c2 + 5.69e-12*k2*c - 2.53e-15*k2*c2 - 1.31e-07*k*b + 1.30e-09*k*b2 + 1.40e-12*k2*b - 3.73e-14*k2*b2 + 6.14e-06*c*b - 6.65e-08*c*b2 - 1.55e-09*c2*b + 1.58e-11*c2*b2 \tag{14}$$

The constrained functions were represented by the suspension deflection:

$$\mathrm{Min}\{\mathrm{RMS}|Z_{sus}|\} \leq \mathrm{RMS}(Z_{sus}(k, c, b, k_t)) \leq \mathrm{Max}\{\mathrm{RMS}|Z_{sus}|\} \tag{15}$$

To calculating RMS(Zs) and RMS(Zsus), we used Orthogonal Arrays (OA) of Design of Experimental with L27 (313) then, we have:

$$\mathrm{Min}\{\mathrm{RMS}|Z_{sus}|\} = 0.0223\ [\mathrm{m}]$$
$$\mathrm{Max}\{\mathrm{RMS}|Z_{sus}|\} = 0.0474\ [\mathrm{m}] \tag{16}$$

We introduce the approximate optimization method, Sequential Quadratic Programming (SQP) with Response Surface Method (RSM). We made RSM using the OA to optimize modal parameters of the passive suspension system with inerter. The new modal parameters are represented for optimization suspension system called optimization model. In order to verify the effectiveness of the proposed optimal method, the numerical simulations were carried out. The vertical displacement optimization results were measured over various fixed structure suspensions. The optimization was performed for k_opt, c_opt, b_opt and kt_opt ranging from boundary conditions. The modal parameter results were obtained by fixed-structure is presented in Table III.

TABLE III. The Comparative Modal Parameters to Optimal Sprung Mass Displacement.

Parameter	k	c	b	kt
Base model	24000	1200	0	70000
Parallel model	24000	1200	20	70000
Optimization model	19091	1255	29.5	86363
Unit	[Nm-1]	[Nsm-1]	[kg]	[Nm-1]

B. Optimal Tire Deflection

The same method, we can optimal tire deflection in respectively. The objective function is represented for root-mean-squared of tire deflection is:

$$\mathrm{RMS}(Z_{t-def}) = f(k, c, b, k_t) \rightarrow \mathrm{Min} \tag{17}$$

Under the synthetic assumption, we can figure out this equation:

$$\mathrm{RMS}(Z_{t-def}) = -1.841 + 1.81e-04*k - 4.06e-09*k2 + 4.83e-03*c - 2.23e-06*c2 + 1.20e-02*b - 3.49e-05*b2 + 9.08e-07*kt - 8.68e-12*kt2 - 4.37e-07*k*c + 2.02e-10*k*c2 + 9.57e-12*k2*c - 4.36e-15*k2*c2 + 2.36e-07*k*b - 1.31e-09*k*b2 - 8.50e-12*k2*b + 6.73e-14*k2*b2 - 1.21e-05*c*b +$$

$$7.94e-08*c*b2+3.33e-09*c2*b-2.77e-11*c2*b2 \tag{18}$$

The constrained functions were represented by the suspension deflection:

$$\mathrm{Min}\{\mathrm{RMS}|Z_{sus}|\} \le \mathrm{RMS}(Z_{sus}(k, c, b, k_t)) \le \mathrm{Max}\{\mathrm{RMS}|Z_{sus}|\} \tag{19}$$

We used Orthogonal Arrays (OA) of Design of Experimental with L27 (313) then, we have:

$$\mathrm{Min}\{\mathrm{RMS}|Z_{sus}|\} = 0.0223\ [\mathrm{m}]$$
$$\mathrm{Max}\{\mathrm{RMS}|Z_{sus}|\} = 0.0474\ [\mathrm{m}] \tag{20}$$

The modal parameter results were obtained in Table IV.

TABLE IV. THE COMPARATIVE MODAL PARAMETERS TO OPTIMAL TIRE DEFLECTION.

Parameter	k	c	b	kt
Base model	24000	1200	0	70000
Parallel model	24000	1200	20	70000
Optimization model	24545	819	5	86363
Unit	[Nm-1]	[Nsm-1]	[kg]	[Nm-1]

C. *Optimal Both Displacement and Tire Deflection*

Optimal solutions for mixed performance of sprung mass displacement and tire deflection: Optimal performance solutions for RMS(Zs) and RMS(Zt-def) individually for parallel suspension network has been computed above. Furthermore, it is also important to consider combined optimal vehicle performance across different measures. Here we present the results for a mixed RMS(Zs) and RMS(Zt-def) measure:

$$\mathrm{RMS}(\overline{Z}) = (1\text{-}\alpha)\mathrm{RMS}(Z_s) + \alpha\mathrm{RMS}(Z_{t\text{-}def}) \tag{21}$$

where $\alpha \in [0,1]$ is a weighting between RMS(Zs) and RMS(Zt-def).

Equation (21) can be optimized with respect to the suspension parameters. The resulting optimal solutions are drawn for a particular Zs and Zt-def. We use the same method SQP with RSM, we optimize modal parameters of the passive suspension system with inerter. The modal parameter results to optimal both displacement and tire deflection are presented in Table V.

TABLE V. THE COMPARATIVE MODAL PARAMETERS TO OPTIMAL BOTH DISPLACEMENT AND TIRE DEFLECTION.

Parameter	k	c	b	kt
Base model	24000	1200	0	70000
Optimization model	21818	1037	5	86363
Unit	[Nm-1]	[Nsm-1]	[kg]	[Nm-1]

V. RESULTS AND DISCUSSIONS

Fig. 5 shows the results that the suspension system with inerter effects to the sprung mass displacement, the time histories of the displacement in respectively. From the results, it was verified that the displacement is reduced by comparison between base and parallel models. The optimization result is presented as circle symbol curve suggesting that the structure of the suspension optimize from the stiffness, damper and inerter get the best result for displacement. An encouraging feature of the optimization algorithm that it allows the change in the structure of suspension parameters varies in order to obtain the minimum vertical displacement values.

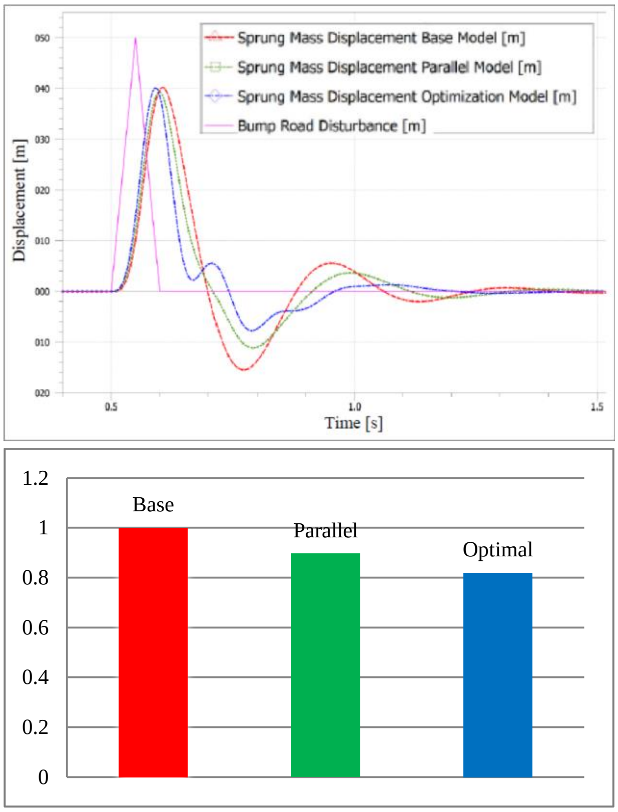


Figure 5. Comparative sprung mass displacement of base, parallel and optimal model in respectively

Comparing between base and optimization model, we found that correspondences between these RMS of displacement variables of sprung mass is summarized below. If we only apply inerter to base suspension system, we can get a better RMS displacement improve about 10 percent, while we modify the system with optimal parameters, this value reduce more than 18 percent. These results verified with new suspension system could be accomplished that the body displacement dynamics can be reduced with employment inerter component.

TABLE VI. COMPARISON THE RMS SPRUNG MASS DISPLACEMENT RESULTS

Model	RMS(Zs) [m]	Improvement
Base model	0.369	0%
Parallel model	0.331	10.32%
Optimization model	0.302	18.00%

Fig. 6 shows that sprung mass displacement is good at almost parameters of parallel model then the tire deflection is hard to get better RMS value. If we only apply inerter to base model under parallel suspension with no change of other parameters, the RMS value of tire deflection is increase more than 33 percent compare with base model. It means that tire deflection is not good for tire grip to contact the road. Otherwise, we optimal this value by changr the quarter ca parameter, the RMS(Zt-def) get better value by reduce around 17 percent. Therefore, inerter still have advanced effect to the tire deflection in this case study.

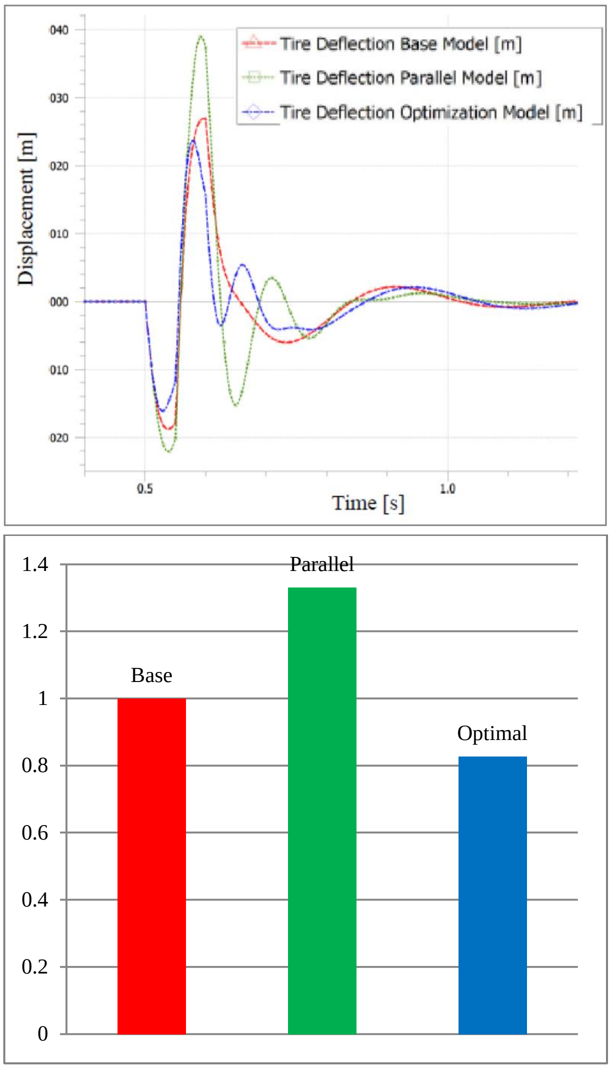


Figure 6. Comparative tire deflection of base, parallel and optimal model in respectively

TABLE VII. COMPARISON THE RMS TIRE DEFLECTION RESULTS

Model	RMS(Zt-def) [m]	Improvement
Base model	0.206	0%
Parallel model	0.274	-33.06%
Optimization model	0.170	17.29%

In general, it can be seen that suspension with inerter offer performance advantages over conventional suspension for both RMS(Zs) and RMS(Zt-def) combined. The Fig. 7 shows that not only sprung mass displacement improves but also tire deflection reduces. Although, both RMS displacement and tire deflection value are reduce slightly compare with individual optimization, near 8 percent in Table VIII compare to 18 percent in Table VI for sprung mass displacement; and 12 percent in Table VIII compare to 17 percent in Table VII of tire deflection improvement. All of that it shows the advanced results for vehicle oscillation when we apply inerter on suspension system.

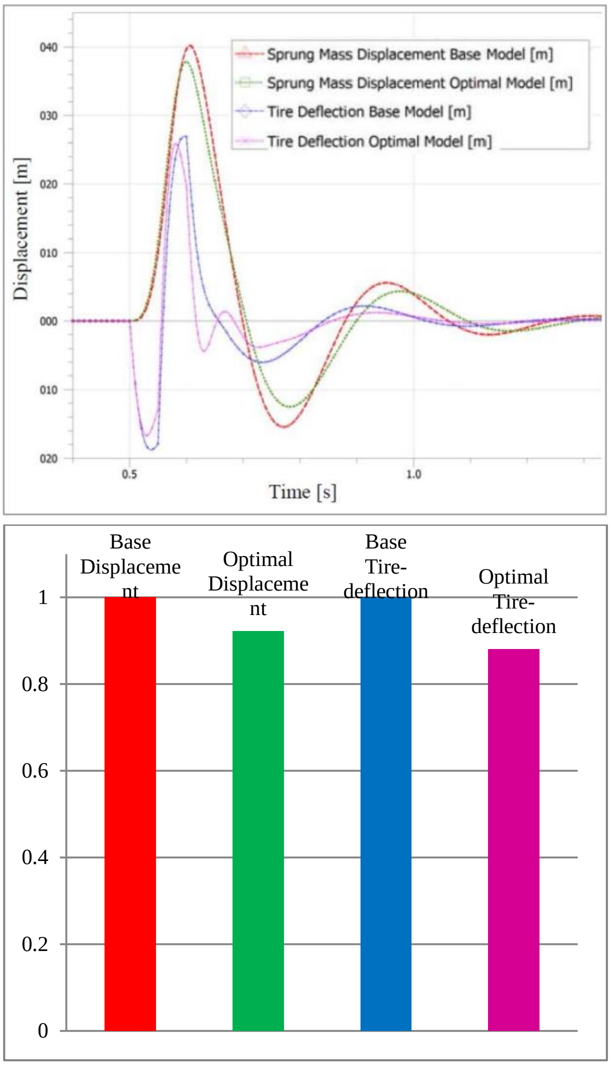


Figure 7. Comparative sprung mass displacement and tire deflection of base and optimal quarter-car model in respectively

TABLE VIII. COMPARISON BOTH RMS SPRUNG MASS DISPLACEMENT AND TIRE DEFLECTION RESULTS

Model	RMS(Zs) [m]	Improvement	RMS(Zt-def) [m]	Improvement
Base model	0.369	0%	0.206	0%
Optimization model	0.339	7.84%	0.181	11.94%

VI. CONCLUSIONS

This paper has described the background and application of a new element called inerter through the suspension synthesis in Formula SAE car. The passive suspension was considered that is possible application of the inerter. In frequency response function domain, the parallel suspension system is carried out and can be designed to improve vehicle dynamics in low frequency.

The results showed that suspension with inerter was not only have better displacement but also tire deflection. The conventional spring and damper always resulted in very normal vibration behavior, but the use of inerter can reduce the oscillation. In this studying, an optimal design to achieve variables stiffness, damping and inerter in suspension system with better results for both comfort and road holding.

Furthermore, base on these advanced optimization results; we verify that the suspension controlled under simple conditions to apply on normal car. We will integrate other types of inerter which can be controlled, and apply on suspension system for large dynamic stability. We should construct some physical modeling for suspension systems then it will be validated with mathematical modeling.

REFERENCES

[1] Bosch, *Automotive Handbook, SAE International*, 2011.

[2] M. Chiaberge, "New trends and developments in automotive system engineering," First Published, 2011.

[3] B. T. Kulakowski, J. F. Gardner, and J. L. Shearer, *Dynamic Modeling and Control of Engineering Systems*, 3rd ed., Cambridge University Press, 2007.

[4] S. M. Savaresi, C. Poussot-Vassal, C. Spelta, O. Sename, and L. Dugard, *Semi-Active Suspension Control Design for Vehicles*, Published by Elsevier Ltd ,2010.

[5] M. C. Smith, "Synthesis of mechanical networks: The inerter," *IEEE Transactions on Automatic Control*, vol. 47, no. 10, 2002.

[6] F. C. Wang, M. S. Hsu, W. J. Su, and T. C. Lin, "Screw type inerter mechanism," U.S. Patent 0108510, 2009.

[7] F. C. Wang and T. C. Lin, "Hydraulic inerter mechanism," U.S. Patent 0139225, 2009.

[8] M. C. Smith, "The inerter concept and its application," Plenary Lecture, Department of Engineering, University of Cambridge, U.K. 2003.

[9] M. C. Smith, "Passive network synthesis revisited," Semi-plenary Lecture, Department of Engineering, University of Cambridge, U.K. 2003.

[10] M. Z. Q. Chen, C. Papageorgiou, F. Scheibe, F. C. Wang, and M. C. Smith, "The missing mechanical circuit element," *IEEE Circuits and Systems Magazine*, 2009.

[11] M. C. Smith and F. C. Wang, "Performance benefits in passive vehicle suspensions employing inerters," in *Proc. 42nd IEEE, Conf. on Decision and Control*, Maui, Hawii USA, 2003.

[12] F. C. Wang and W. J. Su, "Inerter nonlinearities and the impact on suspension control," in *Proc. American Control Conference*, Seattle, Washington, USA, 2008.

[13] J. Izumi, "Combined environmental testing for equipment used on automobiles - overview and test approach," ESPEC Technology Report No. 6.

[14] T. Uemura, "Proposed suspension system that aims to achieve both ride comfort and maneuverability," Master thesis, Shibaura Institute of Technology, 2009.

Tran Thanh Tung received the B.E. (2006) and M.E. (2009) degrees in Mechanical Engineering from Hanoi University of Science and Technology. Now, he studies as a PhD candidate at Shibaura Institute of Technology, Japan.

He is a lecturer at Department of Automotive Engineering at HUST, and he is also a member of VSAE. His research interests include CAE, Optimal Design.

Professor Hiroshi Hasegawa received his B.E. (1992) and M.E. (1994) from Shibaura Institute of Technology, Japan. He received Dr. Eng. (1998) in Mechanical Engineering from Tokyo Institute of Technology, Japan. He has been working at Shibaura Institute of Technology, and currently is a Professor. He is member of JSME, ASME,JSCES and JSST. His research interests Computer Aided Exploration and Creativity of Design.