Model Predictive Control to Get Desired Output with Infinite Predictive Horizon for Bilinear Continuous Systems

Huong T. M. Nguyen and Thai T. Mai
Faculty of Electronics, Thai Nguyen University of Technology, Thai Nguyen City, Viet Nam
Email: maihuongnguyen79@gmail.com, thaimt@tnu.edu.vn

Lai K. Lai
Thai Nguyen University, Thai Nguyen City, Viet Nam
Email: laikhaclai@gmail.com

***Abstract*—Nowadays, although Model Predictive Control has been interested in stabilizing systems, it mainly considered terminal equality constraints method by adding a terminal – state penalty into the cost function (i.e finite predictive horizon). But up to now there is no any law to find a terminal – state penalty. The Twin Rotor MIMO systems (TRMS) is a nonlinear object with a complex dynamic. This paper offers a method which differs one based on the Bellman's dynamic programming for stably tracking of bilinear continuous systems with infinite predictive horizon instead of controlling stably and uses continuous model directly instead of non-continuous approximate model. This method is built basing on variation of optimal control and apply to control TRMS.**

***Index Terms*—Twin Rotor MIMO System (TRMS), stability, dynamic programming, variation method, stably tracking model predictive control**

I. Introduction

Optimization of the model predictive control is a problem that is researching by many scientists. Until now, it was mainly used line search methods with finite predictive horizon for solving to optimize the model predictive control [1], [2], because these methods are quite favorable for contrainted optimal problems. Moreover, there have few other optimization methods as Levenberg-Marquardt or trust region. However, all above methods were only used for finite predictive horizons. Therefore, these do not ensure the global optimization. So, the system is hard to be stable [3].

The dynamic programming method is an effective one for solving optimal problems in multivariable with ensuring the global of the optimal solution. Currently, this method is just applied to solve the optimal problem for linear systems with constant parameters or parameters changing over time. In [4], we applied the dynamic programming method to solve the optimal problem for systems with state-dependent state-space model as the TRMS. However, in this article, we studied only the stable control problem, and did not consider the stably tracking control problem. Based on the stability control method for a bilinear system built on the Bellman dynamic programming [4], in this paper, we will present the tracking model predictive control method to get desired output and an infinite predictive horizon for bilinear continuous systems by using the variation method.

II. Control Algorithm

Consider a bilinear system MIMO with the same input and output signals, presented by continuous model:

$$\begin{cases} \dot{\boldsymbol{x}} = A(\boldsymbol{x})\boldsymbol{x} + B(\boldsymbol{x})\boldsymbol{u} \\ \boldsymbol{y} = C(\boldsymbol{x})\boldsymbol{x} \end{cases} \quad (1)$$

where $\boldsymbol{u} \in \mathbb{R}^m$ are vector of m the input signals, $\boldsymbol{y} \in \mathbb{R}^m$ are vector of m the output signals, and $\boldsymbol{x} \in \mathbb{R}^n$ are vector of n the state variables. $A(\boldsymbol{x})$, $B(\boldsymbol{x})$ and $C(\boldsymbol{x})$ are state dependent matrices. In general, the model (1) has $n \geq m$.

Assuming that the system is controlled by the model predictive controller with the interval $T_k = t_{k+1} - t_k$ of predictive horizon which is also the sample time signal. If T_k is small enough, the matrices $A(\boldsymbol{x})$, $B(\boldsymbol{x})$, $C(\boldsymbol{x})$ can be approximated by constant matrices:

$$A(\boldsymbol{x}) \approx A_k,\ B(\boldsymbol{x}) \approx B_k,\ C(\boldsymbol{x}) \approx C_k \quad (2)$$

when $t_k \leq t < t_{k+1}$. An in this case, the system can be approximated by a linear model with constant parameters:

$$\begin{cases} \dot{\boldsymbol{x}} = A_k\boldsymbol{x} + B_k\boldsymbol{u} \\ \boldsymbol{y} = C_k\boldsymbol{x} \end{cases} \quad (3)$$

Let $\boldsymbol{y}_{ref}$ be the sample output signals that system must follow. Assume that the sample signals are constants (or

Manuscript received April 4, 2015; revised August 15, 2015.

doi: 10.18178/ijmerr.4.4.299-303

may be segment constants) as well as under the influence of constant signals, state feedback closed-loop system (3) will tend to the steady state, with steady state $\boldsymbol{x}_e$, i. e, constant signals $\dot{\boldsymbol{x}}_e = \mathbf{0}$, and input signals are also in steady state. Now, the established values of this system will satisfy:

$$\begin{cases} \mathbf{0}_n = A_k \boldsymbol{x}_e + B_k \boldsymbol{u}_e \\ \boldsymbol{y}_{ref} = C_k \boldsymbol{x}_e \end{cases} \quad (4)$$

Hence, we get a system of $n+m$ equations with unknowns $n+m$: $(\boldsymbol{x}_e, \boldsymbol{u}_e)$ as follows:

$$\begin{pmatrix} A_k \boldsymbol{x}_e + B_k \boldsymbol{u}_e \\ C_k \boldsymbol{x}_e \end{pmatrix} = \begin{pmatrix} \mathbf{0}_n \\ \boldsymbol{y}_{ref} \end{pmatrix} \Leftrightarrow \begin{pmatrix} A_k & B_k \\ C_k & \Theta \end{pmatrix} \begin{pmatrix} \boldsymbol{x}_e \\ \boldsymbol{u}_e \end{pmatrix} = \begin{pmatrix} \mathbf{0}_n \\ \boldsymbol{y}_{ref} \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} \boldsymbol{x}_e \\ \boldsymbol{u}_e \end{pmatrix} = \begin{pmatrix} A_k & B_k \\ C_k & \Theta \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{0}_n \\ \boldsymbol{y}_{ref} \end{pmatrix} \quad (5)$$

This system allow us to find the steady state values $(\boldsymbol{x}_e, \boldsymbol{u}_e)$ from the sample output signals $\boldsymbol{y}_{ref}$. Put $\boldsymbol{\delta} = \boldsymbol{x} - \boldsymbol{x}_e$ and $\boldsymbol{\rho} = \boldsymbol{u} - \boldsymbol{u}_e$. Since $(\boldsymbol{x}_e, \boldsymbol{u}_e)$ are constant vectors, and in (3) and (4), we have the equivalent model in transitional process as the following:

$$\dot{\boldsymbol{\delta}} = A_k \boldsymbol{\delta} + B_k \boldsymbol{\rho} \quad (6)$$

In order to design model predictive control for the continuous system (1) to get stable tracking, i.e., $\boldsymbol{y} \to \boldsymbol{y}_{ref}$, we will control the system (6) to achieve $\boldsymbol{\delta} = \boldsymbol{x} - \boldsymbol{x}_e \to \mathbf{0}$ and $\boldsymbol{\rho} = \boldsymbol{u} - \boldsymbol{u}_e \to \mathbf{0}$ by using the optimal control method LQR for the translated step of the $k-th$ predictive horizon with the infinite predictive horizon. That means, we minimize the objective function

$$J_k(\boldsymbol{\rho}) = \frac{1}{2} \int_{t_k}^{\infty} \left(\boldsymbol{\delta}^T Q_k \boldsymbol{\delta} + \boldsymbol{\rho}^T R_k \boldsymbol{\rho} \right) dt. \quad (7)$$

where Q_k, R_k are two arbitrary symmetric positive definite matrices, which can be changed at each translated step of the predictive horizon.

Using the variation method to find solution $\boldsymbol{\rho}^*$ of the optimal problem in transitional process, given by (6), (7). We will have [5]:

$$L_k B_k R_k^{-1} B_k^T L_k - L_k A_k - A_k^T L_k = Q_k \quad (8)$$

and

$$\boldsymbol{\rho}^* = R_k^{-1} B_k^T L_k \boldsymbol{\delta} = R_k^{-1} B_k^T L_k (\boldsymbol{x} - \boldsymbol{x}_e) \quad (9)$$

From these, we get optimal control signals:

$$\boldsymbol{u}^*(t) = \boldsymbol{\rho}^*(t) + \boldsymbol{u}_e \ \text{với} \ t_k \le t < t_{k+1} \quad (10)$$

for bilinear continuous systems (1) in the current predictive horizon.

Summarily, model predictive control with infinite predictive horizon to apply for bilinear continuous systems (1), will work well with an algorithm including iterative steps as follows:

Algorithm: The state feedback model predictive control so that the output signals track the reference for bilinear continuous systems with an infinite predictive horizon.

1. Choose the appreciate symmetric positive definite weight matrices Q_k, R_k. Take $t_0 = 0$ and $k = 0$.

2. Sample $\boldsymbol{x}_k$ and approximate $A(x), B(x), C(x)$ by A_k, B_k, C_k as (2).

3. Determine $(\boldsymbol{x}_e, \boldsymbol{u}_e)$ from $\boldsymbol{y}_{ref}$ by (5).

4. Find L_k that is a symmetric positive semidefinite solution of Riccati equation (8).

Find $\boldsymbol{\rho}^*$ corresponding L_k by (9), then find $\boldsymbol{u}^*$ from (10).

5. Choose t_{k+1} so that $\left\| e^{\hat{A}_k (t_{k+1} - t_k)} \right\| < 1$ with $\hat{A}_k = A_k - B_k R_k^{-1} B_k^T L_k$. Put $\boldsymbol{u}^*$ as the input of (1) in the interval $t_k \le t < t_{k+1}$ and assign $k := k+1$, then go back to step 2. Here, Q_k and R_k are the arbitrary weight matrices, which can be changed at each step of the translation of the predictive horizon, i. e they depend on k such that the solution of the optimal problem satisfies the bounded condition

$$|\boldsymbol{\rho}| \le \Omega \in \mathbb{R} \quad (11)$$

with Ω is the given upper bounded value whereas $\boldsymbol{\delta} = \boldsymbol{x} - \boldsymbol{x}_e \to \mathbf{0}$. In [6], we can find a law to change Q_k, R_k in which R_k decreases and the bounded conditions (11) are satisfied or Q_k increased with respect to k (i.e., with respect to the translation of the predictive horizon)

Moreover, to set up the algorithm, we have to use a method for solving the Riccati equation (8). We can find some methods to seek L_k effectively in [5].

A model predictive control working in the way of this algorithm was illustrated Fig. 1. This close-loop system works basing on the state feedback principles and is not a discrete system. In each control horizon with a infinite width $[k, \infty]$, $\boldsymbol{u}^*(t)$ are used to control the plant only in an sample time interval $t_k \le t < t_{k+1}$. Thus, we denote $\boldsymbol{u}^*(t)$ instead of $\boldsymbol{u}_k^*(t)$. In the whole control proccess, control signals $\boldsymbol{u}(t)$ will be a sequence of continuous signals $u_k^*(t), k = 0,1,...$ Therefore, this close-loop system is one of the sampled data systems [1].

Stability of a sampled data system are given in the following theorem.

Theorem: If $Q_k, R_k,\ k = 0,1,\ \ldots$ are symmetric positive definite matrices and sample time $t_0 = 0,\ t_1,\ t_2,\ \ldots$ are chosen, such that:

$$\begin{pmatrix} A_k & B_k \\ C_k & \Theta \end{pmatrix} \quad (12)$$

do not degrade $\forall k$ and

$$\left\| e^{\widehat{A}_k (t_{k+1}-t_k)} \right\| < 1 \qquad (13)$$

where $\widehat{A}_k = A_k - B_k R_k^{-1} B_k^T L_k$, then with the model predictive control is proposed in Fig. 1, we have $\boldsymbol{y} \to \boldsymbol{y}_{ref}$.

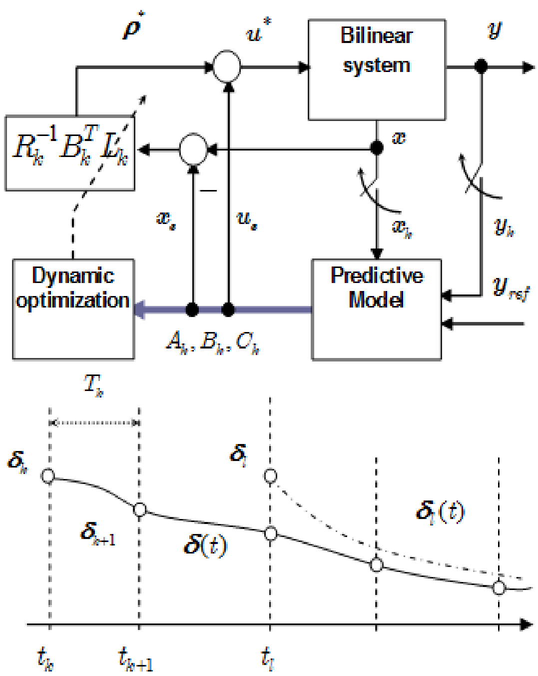

Figure 1. Model predictive control for bilinear continuous systems with infinite predictive horizon

Proof: With the moldel predictive control described in the algorithm, L_k is the symmetric positive semidefinite solution of the Riccati equation (8), which always is updated after each translated step of the predictive horizon. So, in fact, it will change with respect to k, and we denote L instead of L_k.

Evidently, with the supposition (12), we can show that $|\boldsymbol{\delta}(t)|$ is bounded and $\boldsymbol{\delta}(t) \to \mathbf{0}$, where $\boldsymbol{\delta}(t)$ is the state error trajectory in the whole time axis, which is obtained by connecting all sub-trajectories in each predictive horizon (figure 1).

Now, we consider the $k-th$ predictive horizon. Because Q_k, R_k are symmetric positive definite matrices, the close-loop system with the error model:

$$\dot{\boldsymbol{\delta}} = \left(A_k - B_k R_k^{-1} B_k^T L_k \right) \boldsymbol{\delta} = \widehat{A}_k \boldsymbol{\delta}$$

Subject to $kT \le t < (k+1)T$

We have: $\widehat{A}_k = A_k - B_k R_k^{-1} B_k^T L_k$ is a Hurwitz matrix [5]. Thus, in its respective state trajectory,

$$\boldsymbol{\delta}(t) = e^{\widehat{A}_k (t-t_k)} \boldsymbol{\delta}_k \text{ with } \boldsymbol{\delta}_k = \boldsymbol{\delta}(t_k)$$

there always exists t_{k+1} satisfying (13). So,

$$|\boldsymbol{\delta}(t_{k+1})| = \left| e^{\widehat{A}_k (t_{k+1}-t_k)} \boldsymbol{\delta}_k \right| < |\boldsymbol{\delta}_k| \quad \Rightarrow \quad |\boldsymbol{\delta}_{k+1}| < |\boldsymbol{\delta}_k|$$

$\forall k$ Thus, $|\boldsymbol{\delta}(t)|$ is bounded decreasing for $0 \le t \le \infty$. By Cauchy's theorem of the convergence,

$$\lim_{t \to \infty} |\boldsymbol{\delta}(t)| = \delta_\infty \ge 0$$

We will prove that $\delta_\infty = 0$. Indeed, if we suppose that $\delta_\infty > 0$. We have:

$$a = \min_k \left\| \widehat{A}_k \right\| \ne 0$$

Choose a predictive horizon l and $\boldsymbol{\delta}_l(t)$ is the solution of:

$$\dot{\boldsymbol{\delta}}_l = \left(A_l - B_l R_l^{-1} B_l^T L_l \right) \boldsymbol{\delta}_l = \widehat{A}_l \boldsymbol{\delta}_l$$

$$\Leftrightarrow \boldsymbol{\delta}_l(t) = e^{\widehat{A}_l (t-t_l)} \boldsymbol{\delta}_l(t_l)$$

Then, we have:

$$0 < \lim_{t \to \infty} a |\boldsymbol{\delta}(t)| \le \lim_{t \to \infty} |\boldsymbol{\delta}_l(t)| \qquad (14)$$

However, since $\widehat{A}_l$ is Hurwitz, i.e., $\boldsymbol{\delta}_l(t) \to 0$, (14) is meaningless. Thus, the above assumtion is incorrect. In other words, we must have:

$$\lim_{t \to \infty} |\boldsymbol{\delta}(t)| = 0$$

. This completes the proof.

III. Twin Rotor MIMO System (TRMS)

The TRMS was given in Fig. 2.

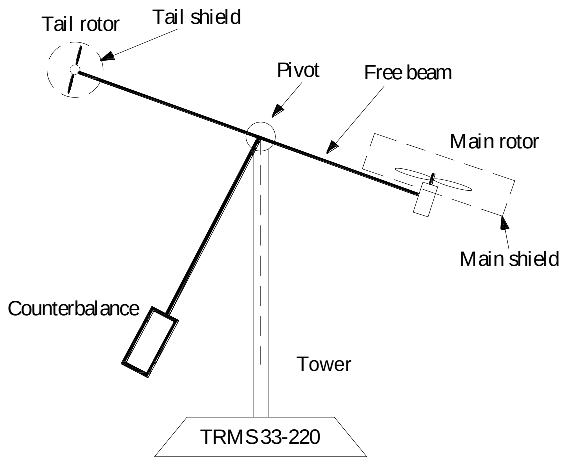

Figure 2. The TRMS

The TRMS is a bilinear system with two inputs and two outputs. It can be described by the continuous model (1). State variables, inputs, outputs, respectively, are:

$$x(t) = [i_{ah}(k), \omega_h(k), S_h(k), \alpha_h(k), i_{av}(k), \omega_v(k), S_v(k), \alpha_v(k)]^T$$

$$u(k) = \left[U_h(k) \quad U_v(k) \right]^T ; \; y(k) = \left[\alpha_h(k) \quad \alpha_v(k) \right]^T \qquad (15)$$

where:

i_{ah}: Armature current of the tail motor (*A*)

ω_h: Rotational velocity of the tail rotor (*rad/s*)

S_h :Angular velocity of TRMS beam in the horizontal plane without affect of the main rotor (*rad/s*)
i_{av}: Armature current of the main motor (*A*)
ω_v: Rotational velocity of main rotor(*rad/s*)
S_v: Angular velocity of TRMS beam in the vertical plane without affect of the tail rotor (*rad/s*)
α_v: Pitch angle of the TRMS beam (*rad*)
U_h: Input voltage signal of the tail motor (*V*)
U_v: Input voltage signal of the main motor (*V*)

The nonlinear continuous state space equations of the TRMS are expressed in [7]-[9]:

$$x(k) = \left[\omega_h(k) \quad S_h(k) \quad \alpha_h(k) \quad \omega_v(k) \quad S_v(k) \quad \alpha_v(k)\right]^T \tag{16}$$

$$\frac{d}{dt}\begin{bmatrix}\omega_h \\ S_h \\ \alpha_h \\ \omega_v \\ S_v \\ \alpha_v\end{bmatrix} = \begin{bmatrix} -\frac{(k_{ah}\varphi_h)^2}{J_{tr}R_{ah}}\omega_h - \frac{B_{tr}}{J_{tr}}\omega_h - \frac{f_1(\omega_h)}{J_{tr}} + \frac{k_{ah}\varphi_h}{J_{tr}R_{ah}} f_6(U_h) \\ \frac{l_t f_2(\omega_h)\cos\alpha_v - f_7(\Omega_h) - f_3(\alpha_h)}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ S_h + \frac{k_m\omega_v\cos\alpha_v}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \\ -\frac{(k_{av}\varphi_v)^2}{J_{mr}R_{mr}}\omega_v - \frac{B_{mr}}{J_{mr}}\omega_v - \frac{f_4(\omega_v)}{J_{mr}} + \frac{k_{av}\varphi_v}{J_{mr}R_{av}} f_8(U_v) \\ \frac{f_5(\omega_v)(l_m + k_g\Omega_h\cos\alpha_v) - f_9(\Omega_v)}{J_v} + \frac{g\left[(A-B)\cos\alpha_v - C\sin\alpha_v\right] - 0.5\Omega_h^2 H\sin 2\alpha_v}{J_v} \\ S_v + \frac{k_t}{J_v}\omega_h \end{bmatrix} \tag{17}$$

where:

$$R_{ah}, L_{ah}, k_{ah}\varphi_h, J_{tr}, B_{tr}, l_t, D, E, F, k_m, R_{av}, L_{av}, k_{av}\varphi_v, J_{mr}, B_{mr}, l_m, k_g, g, A, B, C, H, J_v, k_t$$

are positive constants, Ω_h and Ω_v is defined by

$$\Omega_h = S_h + \frac{k_m\omega_v\cos\alpha_v}{D\cos^2\omega_v + E\sin^2\alpha_v + F} \tag{18}$$

$$\Omega_v = S_v + \frac{k_t\omega_h}{J_v} \tag{19}$$

IV. Simulations

Structure diagrams of state feedback model predictive control was given in Fig. 3.

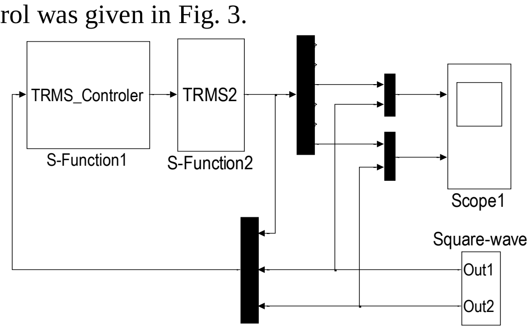

Figure 3. Structure diagrams of state feedback model predictive control to get desired outputs for TRMS.

Use the above algorithm and install on TRMS as structure diagrams of state feedback model predictive control in Fig. 3, we get the simulation results as in the figures from 4 to 9.

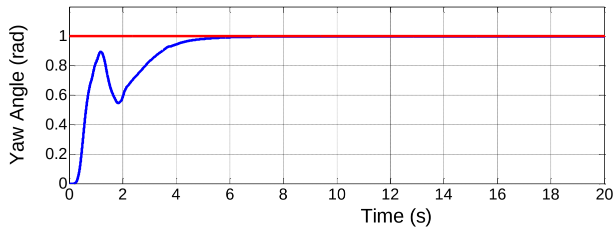

Figure 4. The response of the Yaw angle control loop with respect to a constant (t=20s).

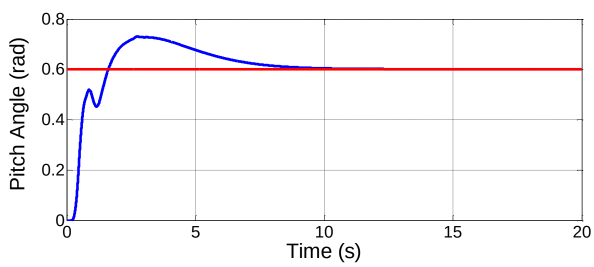

Figure 5. The response of the pitch angle control loop with respect to a constant (t=20s)

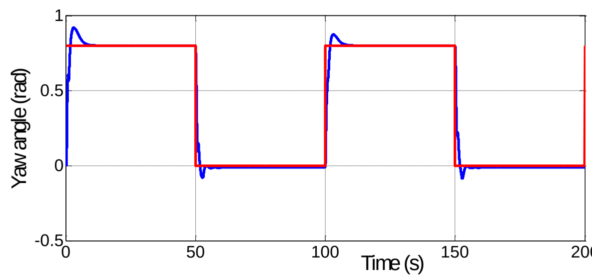

Figure 6. The response of the Yaw angle control loop with respect to a square – wave

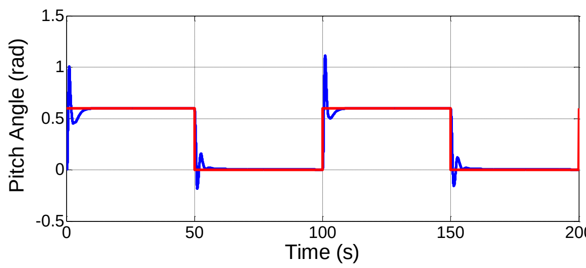

Figure 7. The response of the pitch angle control loop with respect to a square – wave

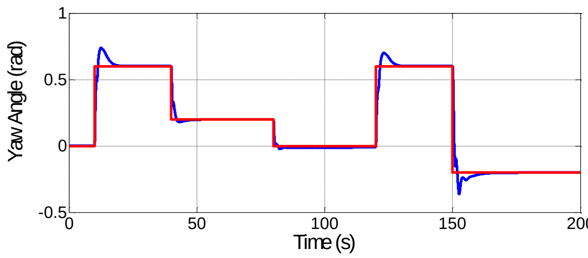

Figure 8. The response of the Yaw angle control loop with respect to a substep

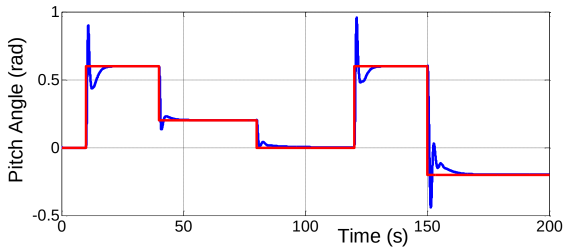

Figure 9. The response of the pitch angle control loop with respect to a substep

With simulation results given in Fig. 4 and Fig. 5, the responses of the yaw and pitch angles when reference is constant, we see that at sixth and eighth seconds, the responses of outputs track the reference. And if we change the inputs such as: square – wave or substep, then we get the same results in the figures from 6 to 9.

Especially, in the Fig. 6 and 7, when changing the values of the references then the responses are modified, this prove that TRMS has cross-coupling channels between the yaw and pitch angles. This is really reasonable because when the speed of the main rotor changes, the speed of the tail rotor also change, and vice versa. In the case where the speed of these both rotors change, this influence increases. This can be illustrated clearly in the Fig. 6 and Fig. 7 at the time of the simulation at the 0th second, the 50th second, the 100th second, and the 150th second. Similarly, in the figure 8 and 9, also shows the signal outputs always following the substep signals. In the seconds 10th, 120th and 150th when the signal of the yaw and pitch angles changes from 0 to 0.6 and from 0.6 to - 0.2, the cross – coupling is very clear. In the seconds 40th or 80th when the signal changes little, the influence between channels is small. From the above simulation results proved that the correctness of the proposed algorithm when applying the dynamic programming and variational methods to control stable tracking for continuous bilinear system with infinite predictive horizon.

V. Conclusion

Using dynamic programming of Bellman and variation methods, the model predictive control are designed to get stable tracking for TRMS with infinite predictive horizon. Simulation results on Matlab demonstrate that the outputs of system track and keep stably to the reference. These results also prove that the accuracy of the methodology was built in the above algorithm. In the future, the authors will apply this method for other nonlinear objects to illustrate the strength of proposed algorithm.

References

[1] D. P. Nguyen, *Analysis and Control of Nonlinear Systems*, Publishing Technology, Ha Noi University of Sciences and Technology, 2012.

[2] J. Nocedal and S. J. Wright, *Numerical Optimization*, Springer-New York, 1996.

[3] L. Grüne and J. Pannek, *Nonlinear Model Predictive Control: Theory and Algorithms*, Springer, 2010.

[4] T. M. H. Nguyen, T. T. Mai, K. L. Lai, and T. T. Anh Do, "Stabilization for twin rotor Mimo system based on Bellman's dynamic programming method," *Journal of science and Technology of Thai Nguyen University*, vol. 128, no. 14, pp. 161-165, 2014.

[5] D. P. Nguyen, *Optimization of Control and Optimal Control*, Publishing Technology, Ha Noi University of Sciences and Technology (Forthcoming Publishing in 2015).

[6] T. T. Anh. Do, "Output feedback model predictive control folowing separation principle for nonlinear systems," Ph.D. dissertation, Ha Noi University of Sciences and Technology, 2014.

[7] A. Rahideh and M. H. Shaheed, "Constrained output feedback model predictive control for nonlinear systems," *Control Engineering Practive*, vol. 20, no. 5, pp. 431-443, 2012.

[8] T. M. H. Nguyen, T. T. Mai, H. C. Nguyen, and K. L. Lai., "Researching effects of state parametters in Twin Rotor Mimo System (TRMS)," *Journal of Science and Technology of Thai Nguyen University*, vol. 120, no. 6, pp. 87–92, 2014.

[9] A. Rahideh and M. H. Shaheed, "Mathematical dynamic modelling of a twin rotor multiple input–multiple output system," *Proc. I Meche, Part I. Journal of Systems and Control Engineering*, vol. 221, no. 1, pp. 89–101, 2007.

Huong T. M. Nguyen was born in Thai Nguyen City, Viet Nam, in 1979. She received the B.Eng. degree in Teachical of training from Thai Nguyen University of Technology, Viet Nam in 2001, and the Master degree in Automation from Thai Nguyen University in 2008. She is currently a PhD student. She teached in faculty of Teachical of training from 2002 to 2009. Then she worked in Research Development Institute of Advanced Industrial Technologies from 2009 to 2014. After that she teached in faculty of Teachical of training from 2014 to day. Her current research interests control and automation. Expecialy, she enjoys the Model Predictive Control for nonlinear systems. Her dissertation is researching for solve optimal algothirm in Model Predictive Control.

Thai Trung Mai was born in Thai Nguyen City, Viet Nam, in 1977. He received the B.Eng. degree in Automation from Thai Nguyen University of Technology, Viet Nam in 2000, and the Master degree in Automation from Thai Nguyen University in 2005. He is currently a PhD student.
He teached in faculty of Electronics from 2000 up to now. His current research interests Electronics, Control and Automation. Expecialy, he enjoys the Optimal Control for nonlinear systems. Now, his dissertation is researching for Optimal Control for Distributed Parameter Systems.

Assoc. Prof. Lai Khac Lai was born in Nam Dinh Province, Viet Nam, in 1956. He received the B.Eng. degree electrification of factories from Thai Nguyen University of Technology, Viet Nam in 1979, the Master degree in Control from Ha Noi University of science and Technology in 1998, and the Doctor in 2003. He became a Assoc. Prof in 2007. He was a excellent teachers in 2008. His current research interests Fuzzylogic, nerron network, Model Predictive Control… for control systems. He is teaching for students, Masters and driver PhD students