Decentralized Multi-Robot Cooperative Localization with Extended Information-Weighted Consensus Filter

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Abstract—This paper addresses the decentralized cooperative localization problem for multi-robot applications, where the robot estimates the state of the entire system by exchanging the information with nearby robots. At every time step, the robot measures the relative difference between two consecutive time instants with the odometry and relative ranges and bearings of other robots with the range sensor. Next, the robot combines the data sent from its own and its neighbors with the extended Information-weighted Consensus Filter, which guarantee the convergence to the optimal centralized performance. Finally, the proposed algorithm is demonstrated through Monte Carlo simulation.

Index Terms—cooperative localization, information-weighted consensus filter, relative measurements

I. INTRODUCTION

It is necessary for a robot to cooperatively estimate the configurations of itself and other robots for multi-robot applications. The cooperative localization introduced in [1] is to improve the robots’ ability by sensing one another and exchanging these measurements. If the information is fused by the average consensus mentioned in [2], the robot can compute the optimal estimate just by communicating with its neighbors. The Kalman Consensus Filter (KCF) [3] utilized the average consensus to combine the prior estimates under the assumption that they are equally reliable. Furthermore, the Generalized Kalman Consensus Filter (GKCF) [4] improved the robustness by weighting the prior estimates with their covariance matrices. However, these consensus-based filters did not guarantee the convergence to the optimal centralized performance. That was why the Information-Weighted Consensus Filter (ICF) [5] was suggested. In this paper, we propose a distributed cooperative localization framework using the extended ICF, where each robot combines the data from the odometries and the range sensors of itself and its neighbors. In addition, the proposed algorithm is validated by the Monte Carlo simulation.

II. PROBLEM DEFINITION

There are robots \( \mathcal{R}_i \), \( i = 1, \ldots, n \) in the workspace \( \mathcal{W} \subset \mathbb{R}^2 \), as shown in Fig. 1. Each robot’s configuration \( x_i \) is represented by its position \( p_i \) and bearing \( \theta_i \), expressed as \( x_i = [p_i^T \ \theta_i^T]^T \). The robot is equipped with an odometer to get a measurement \( \hat{\delta}_i(t) \) of the relative difference of \( \mathcal{R}_i \) between two consecutive time instants. Because the odometry motion model introduced in [6] is employed, the relative difference is decomposed into the initial rotation \( \hat{\delta}_{i,\text{rot}} \), the translation \( \hat{\delta}_{i,\text{trans}} \), and the second turn \( \hat{\delta}_{i,\text{rot2}} \), that is \( \hat{\delta}_i = [\hat{\delta}_{i,\text{rot}} \ \hat{\delta}_{i,\text{trans}} \ \hat{\delta}_{i,\text{rot2}}]^T \). In addition, the robot \( \mathcal{R}_i \) has a laser sensor that measures the range \( r_{ij} \) and the bearing \( \phi_{ij} \) of the robot \( \mathcal{R}_j \) in its detection region \( D_i \) relative to its local coordinate frame. Furthermore, the robot \( \mathcal{R}_i \) broadcasts its information to any other robots within its communication radius \( \rho \). In this paper, it is assumed that \( \mathcal{R}_i \) communicates with \( \mathcal{R}_j \) if the \( \mathcal{R}_i \) observes \( \mathcal{R}_j \) with its equipped sensor.

Figure 1. Decentralized multi-robot system for cooperative localization. Each of the robot communicates with one another and gets the measurement of the relative distance and bearing of other robots in its detection region.

The objective of the problem is to estimate the state of the entire system, specified by \( \mathbf{x} = [x_1^T \ldots \ x_n^T]^T \). The network can be sparsely connected because the communication radius is limited. Therefore, each of the robot estimates the system’s state vector in a distributed manner. Let \( \hat{\mathbf{x}} \) denote the state estimate by \( \mathcal{R}_i \) and \( \mathbf{J} \) be the associated information matrix. Our goal is to compute the
decentralized estimate \( \hat{\mathbf{x}}_i^k \), \( i = 1, \ldots, n \) as close as possible to that of the centralized scheme.

### III. SYSTEM MODELS

In this section, the robot’s model of motion, measurement, and communication is explained in detail.

#### A. Odometry Motion Model

The odometry measurement \( \mathbf{u}_i = [\mathbf{\bar{x}}_i(t-1)^T \mathbf{\bar{x}}_i(t)^T ]^T \) is used to compute the robot’s motion over time interval \([t-1, t]\), where \( \mathbf{\bar{x}}_i(t) \) is the output of the odometer at time \( t \). The vector \( \hat{\delta}_i \) is calculated as

\[
\hat{\delta}_{i,\text{rot}} = \arctan2(u_{i,5} - u_{i,2}, u_{i,4} - u_{i,1}) - u_{i,3}
\]  \hspace{1cm} (1)

\[
\hat{\delta}_{i,\text{trans}} = \sqrt{(u_{i,5} - u_{i,2})^2 + (u_{i,4} - u_{i,1})^2}
\]  \hspace{1cm} (2)

\[
\hat{\delta}_{i,\text{rot2}} = u_{i,6} - u_{i,3} - \hat{\delta}_{i,\text{rot1}}
\]  \hspace{1cm} (3)

where \( u_{i,j} \) is the \( j \)-th element of \( \mathbf{u}_i \). However, the true value \( \delta_i \) differ from \( \hat{\delta}_i \) due to the measurement uncertainty, described by

\[
\delta_i = \hat{\delta}_i - \mathcal{N}(0, \mathbf{M}_i)
\]  \hspace{1cm} (4)

where \( \mathcal{N}(\mu, \Sigma) \) is a multivariate random variable with mean vector \( \mu \) and covariance matrix \( \Sigma \). Here, \( \mathbf{M}_i \) is denoted as

\[
\mathbf{M}_i = \text{diag}\left( \begin{bmatrix} a_1 \hat{\delta}_{i,\text{rot1}} + a_2 \hat{\delta}_{i,\text{trans}} \\
 a_3 \hat{\delta}_{i,\text{trans}} + a_4 \hat{\delta}_{i,\text{rot1}} + \hat{\delta}_{i,\text{rot2}} \\
 a_2 \hat{\delta}_{i,\text{rot2}} + a_3 \hat{\delta}_{i,\text{trans}} \end{bmatrix} \right)
\]  \hspace{1cm} (5)

where \( a_j, \ j = 1, \ldots, 4 \) are error parameters related with the noise in robot motion [6]. The above equation presents that the faster the robot moves, the higher motion uncertainty is.

As a result, the true configuration, \( \mathbf{x}_i(t) \) is obtained from \( \mathbf{x}_i(t-1) \) and \( \delta_i : \mathbf{x}_i(t) = g(\hat{\delta}_i, \mathbf{x}_i(t-1)) \). Because \( \delta_i \) cannot be exactly known, the state transition model is given by

\[
\mathbf{x}_i(t) = g(\hat{\delta}_i, \mathbf{x}_i(t-1)) + \mathcal{N}(0, \mathbf{R}_i)
\]  \hspace{1cm} (6)

where

\[
g(\hat{\delta}, \mathbf{x}) = \begin{bmatrix} \delta_{\text{trans}} \cos(\theta + \hat{\delta}_{\text{rot1}}) \\
 \delta_{\text{trans}} \sin(\theta + \hat{\delta}_{\text{rot1}}) \\
 \hat{\delta}_{\text{rot1}} + \hat{\delta}_{\text{rot2}} \end{bmatrix}
\]  \hspace{1cm} (7)

\[
\mathbf{R}_i = \mathbf{F}_i(t) \mathbf{M}_i \mathbf{F}_i^T(t),
\]  \hspace{1cm} (8)

#### B. Laser Sensor Model

If \( \mathcal{R}_j \) is contained in the detection region \( D_i \) of \( \mathcal{R}_i \), the laser sensor returns the relative range \( r_{ij} \) and bearing measurement \( \phi_{ij} \) according to

\[
z_{ij} = h_{ij}(\mathbf{x}(t)) + \mathcal{N}(0, \mathbf{Q}_i)
\]  \hspace{1cm} (9)

where

\[
h_{ij}(\mathbf{x}) = \begin{bmatrix} \|\mathbf{x}_j - \mathbf{x}_i\| \\
 \text{atan2}(y_j - y_i, x_j - x_i) - \theta_i \end{bmatrix}
\]  \hspace{1cm} (10)

The detection region \( D_i \) is defined as

\[
D_i = \left\{ [x \ y]^T \left| (x-x_i)^2 + (y-y_i)^2 \leq r_{\max}^2, \text{atan2}(y-y_i, x-x_i) - \theta_i \leq \theta_{\max} \right| \right\}
\]  \hspace{1cm} (11)

where \( r_{\max} \) is the maximum measurable range and \( \theta_{\max} \) is the maximum measurable bearing. Since the detection region is not a circle, \( \mathcal{R}_j \) cannot detect the \( \mathcal{R}_i \) even though \( \mathcal{R}_j \) observes \( \mathcal{R}_j \).

#### C. Communication Model

Let \( \mathcal{N}_i \) denote the set of the robots that directly communicate with \( \mathcal{R}_i \). It is assumed that two robots communicate each other if the distance between them is less than \( \rho > r_{\max} \). Hence, if a robot detects other robots, it can communicate with them.

The robot \( \mathcal{R}_i \) sends the robots \( \mathcal{R}_j \in \mathcal{N}_i \) the two kinds of information in turn: the odometry measurement \( \mathbf{u}_j \) and the consensus variables, which include the prior estimate of the system, \( \hat{\mathbf{x}}_{-i}^k \) and the relative measurements \( z_{ij} \) for \( \mathcal{R}_j \cap D_i \neq \emptyset \).

### IV. EXTENDED INFORMATION-WEIGHTED CONSENSUS FILTER

Information-weighted Consensus Filter is one of the distributed state estimate framework introduced in [5]. Because the ICF guarantees the convergence to the optimal centralized estimate opposed to KCF or GKCF, it is appropriate for the case that each robot estimate the state of the entire system in a distributed fashion. In this paper, the extended version of ICF is adopted owing to the non-linearity in the motion and sensor model. The extended
ICF consists of 3 stages: state prediction, average consensus, and state update.

A. State Prediction

Suppose the posterior state estimate for time $t-1$, $\hat{x}^i(t-1)$, and the information matrix, $J^i(t-1)$, are given. To predict the state by using the odometry motion model, the odometer measurements $\hat{\delta}_j$ is required. Since the robot $R_i$ can acquire the odometer measurements of itself and the robots in $N_i$ by communication, for the index $j$ such that $R_j \in R_i \cup N_i$,

$$\hat{x}^j_-(t) = g(\hat{\delta}_j, \hat{x}^i_+(t-1))$$

(13)

$$J^j_+(t) = \left( G(t)(J^i(t-1))^{-1} G^T(t) + E_i(t) M F_i(t) \right)^{-1}$$

(14)

where

$$G(t) = \frac{\partial g(\hat{\delta}_j, \hat{x}^i_+(t-1))}{\partial \hat{x}^i_+(t-1)}$$

$$= \begin{bmatrix} 1 & 0 & -\hat{\delta}_{\text{trans}} \sin(\theta_i + \hat{\delta}_{\text{rot}}) \\ 0 & 1 & \hat{\delta}_{\text{trans}} \cos(\theta_i + \hat{\delta}_{\text{rot}}) \\ 0 & 0 & 1 \end{bmatrix}$$

(15)

For the index $j$ such that $R_j \in R_i \cup N_i$, $\hat{x}^j_-(t) = \mathbf{0}_{3,1}$ and $J^j_+(t) = \mathbf{0}_{1,3}$. The prior state estimate $\hat{x}^-_i$ is the concatenation of the vectors $\hat{x}^-_j$ for $j = 1, \ldots, n$.

$$\hat{x}^- = \begin{bmatrix} \hat{x}^-_1 \\ \vdots \\ \hat{x}^-_n \end{bmatrix}$$

(16)

and associated information matrix $J^-_i$ is the block diagonal matrix whose main diagonal blocks are $J^-_j$ for $j = 1, \ldots, n$.

$$J^- = \begin{bmatrix} J^-_1 & 0 \\ 0 & J^-_n \end{bmatrix}$$

(17)

B. Average Consensus

After the robot measures the relative range and bearing of the other robots, the consensus variables, $v_i$ and $V_i$, are defined as follows:

$$V_i[0] = \frac{1}{n} J^- + \sum_{R_j \cap R_i \neq \emptyset} H_{ij}^T(t) Q_i^{-1} H_{ij}(t)$$

(18)

$$v_i[0] = \frac{1}{n} J^- \hat{x}^- + \sum_{R_j \cap R_i \neq \emptyset} H_{ij}^T(t) Q_i^{-1} z_{ij}(t)$$

(19)

where

$$H_{ij} = \begin{bmatrix} \frac{\partial h_{ij}(\hat{x}^-(t))}{\partial \hat{x}^i_-(t)} & \frac{\partial h_{ij}(\hat{x}^-(t))}{\partial \hat{x}^j_+(t)} \end{bmatrix}$$

(20)

$$\frac{\partial h_{ij}(\mathbf{x})}{\partial x_i} = \begin{bmatrix} s_j \gamma_i - y_j \gamma_i \\ y_j \gamma_i - s_j \gamma_i \\ v_j \gamma_i - s_j \gamma_i \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

(21)

$$\frac{\partial h_{ij}(\mathbf{x})}{\partial x_j} = \frac{\partial h_{ij}(\mathbf{x})}{\partial x_i}$$

(22)

The consensus variables include the information of the prior state estimates and the relative measurements. Afterward, the robot $R_i$ iteratively communicates with the other robots in $N_i$ by performing the average consensus on the consensus variables:

$$V_i[m+1] = V_i[m] + \varepsilon \sum_{R_j \in N_i} (V_j[m] - V_i[m])$$

(23)

$$v_i[m+1] = v_i[m] + \varepsilon \sum_{R_j \in N_i} (v_j[m] - v_i[m])$$

(24)

where $\varepsilon \in (0, \frac{1}{n}]$. The number of iterations to update the variables is limited to $k$.

C. State Update

Based on the consensus variables, the robot $R_i$ calculates the posterior state estimate $\hat{x}^i(t)$ and its information matrix $J^i(t)$:

$$\hat{x}^i = (V_i[k])^{-1} v_i[k]$$

(25)

$$J^i = nV_i[k]$$

(26)

V. SIMULATION RESULTS

The algorithm was implemented in the MATLAB language and simulated on a PC Intel Core i7 3.40GHz. In the simulation, three robots moved along the predefined trajectories, as shown in Fig. 2(a). The black marks indicate the initial position of the robots, which are summarized in Table I.

| TABLE I. INITIAL POSE OF THREE ROBOTS IN THE SIMULATION |
|-----------------|-----------------|-----------------|
| $x_i$ (unit)    | $y_i$ (unit)    | $\theta_i$ (deg.) |
| Robot1          | 100             | 200             | 30      |
| Robot2          | 400             | 250             | -150    |
| Robot3          | 200             | 380             | -90     |

In addition, the range sensor had the maximum measurable range $r_{\text{max}} = 100$ unit and bearing $\phi_{\text{max}} = 120$ deg. It was chosen to have the communication radius $\rho = 120$ unit and the maximum number of communic-
tion $k = 10$. The performance of the algorithm was compared to that of the algorithms which utilize the other decentralized state estimate framework such as the KCF and the GKCF and was validated by Monte Carlo simulation with 1000 different initial estimates of the robots’ poses. The initial pose estimator has an uncertainty in the position of 10 unit and the bearing of 0.2 rad.

Fig. 2(b) and 2(c) present the simulation results, where the error bars show the mean and standard deviation of the position and bearing errors from the central scheme based on 1000 runs. In the position estimation, the mean error of the proposed algorithm is less than 40.59% that of the algorithm employing the KCF and 17.15% that of the algorithm using the GKCF. Although the proposed algorithm does not show the smallest error in the bearing estimation, the difference to the minimum is only 0.14 deg. On the basis of the one sigma errors, we see that the suggested algorithm is the most stable framework to estimate the configuration of robot system in the decentralized manner.

VI. CONCLUSIONS

In this paper, we have proposed the decentralized cooperative localization scheme when the robot’s motion is tracked by its odometer, the relative displacement between robots is measured by the equipped range sensor, and the measurements and the estimate data are shared by the communication. The extended ICF is exploited to fuse the information because the motion and measurement model have nonlinearity. Also, it is demonstrated that the proposed algorithm outperforms the other algorithms that utilize the KCF and the GKCF.

However, this algorithm needs to be extended to estimate relative configurations between robots without a common fixed frame of reference because the proposed algorithm requires the initial configurations of the robots. In addition, the data transmitted between the robots, which are the odometry measurements and the consensus variables, should be integrated to simplify the communication procedure in the further work.

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