International Journal of Mechanical Engineering and Robotics Research

ISSN 2278 – 0149 www.ijmerr.com
Special Issue, Vol. 1, No. 1, January 2014
National Conference on "Recent Advances in Mechanical Engineering" RAME – 2014

Research Paper

MODELING AND SIMULATIONS OF RAILWAY VEHICLE SYSTEM

Rakesh Chandmal Sharma[1]*

This paper presents a 37-degree of freedom coupled vertical-lateral model of Indian Railway vehicle formulated using Largrangian dynamics for the purpose of examining its dynamic behavior. The rail-vehicle for the present study is modeled as nine mass systems consisting of a carbody, two bolsters, two bogie frames and four wheel axle sets. The 37 coupled vertical-lateral motion equations are further utilized to investigate the ride behavior of Indian Railway General Sleeper and Rajdhani coach

Keywords: Lagrangian Dynamics, Degree of freedom, Track Roughness, Frequency response function

INTRODUCTION

The railway vehicle running along a track is one of the most complex dynamical systems in engineering. It has many degrees of freedom and the study of rail vehicle dynamics is a difficult task. The travel of a rail vehicle on track is always a coupled motion. There exists a coupling between vertical and lateral motions. The vertical irregularities of the track cause both vertical and lateral vibrations in the rail vehicle. In addition, the different rigid bodies, i.e., car body, bolsters, bogie frames and wheel axles set execute different angular motions, i.e., roll, pitch and yaw which influence the dynamic behavior of the rail vehicle system significantly. In developing the mathematical model to study vertical response, it would not be adequate to include bounce, pitch and roll degrees of freedom of the components. On the other hand, for the lateral response model, it would not be sufficient to use lateral, yaw and roll degrees of freedom of the components. There has been extensive work done various researchers on the lateral and vertical dynamics of the rail vehicle in order to study these motions separately. Coupled vertical-lateral motion of the rail vehicle has also been studied in past by Zhai *et al.* (1984) Goel *et al.* (2005) and Kalker (1979). The present research work is another effort in the same direction.

Coupled vertical-lateral dynamics of four wheeled road vehicle has been studied by Nathoo and Healey (1978) (Meachem and Ahlbeck, 1969) and of three-wheeled road vehicle has been studied by Ramji (2004) using Largrangian approach.

[1] M M University, Mullana, Ambala, Haryana, India.

In the present work a 37 degrees of freedom coupled vertical-lateral mathematical model of an Indian Railway vehicle is formulated using Largangian dynamics and its motion has been studied. Both vertical and lateral irregularities of railway track are incorporated in the analysis and are considered as random function of time. The simulated results are compared with the vertical and lateral acceleration data obtained through actual rail vehicle testing and the ride comfort is evaluated using ISO 2631-1 standards (1997).

MATHEMATICAL MODELING VEHICLE MODEL

The railway vehicle as shown in Figure 1 comprises of a carbody supported by two bogies one at each end. Bolsters are the intermediate member between the carbody and each bogie frame and is connected to carbody through side bearings. The bogie frame supports the weight of the carbody through a secondary suspension located between the carbody and the bogie frame. In passenger vehicles, each bogie usually consists of two wheel axle sets that are connected through the primary suspension to the bogie frame. In addition, the wheels are usually tapered or profiled to provide a self centering action as the axle traverses the track. The degrees of freedom assigned to the different rigid bodies of a railway vehicle are mentioned in Table 1. The inertial axes system is fixed with the track while the origins of the body fixed axes system are located at the centre of mass of different rigid bodies e.g. car body, bolster, wheel axle etc. The mathematical model is formulated with following assumptions.

Figure 1: Rail Vehicle Model

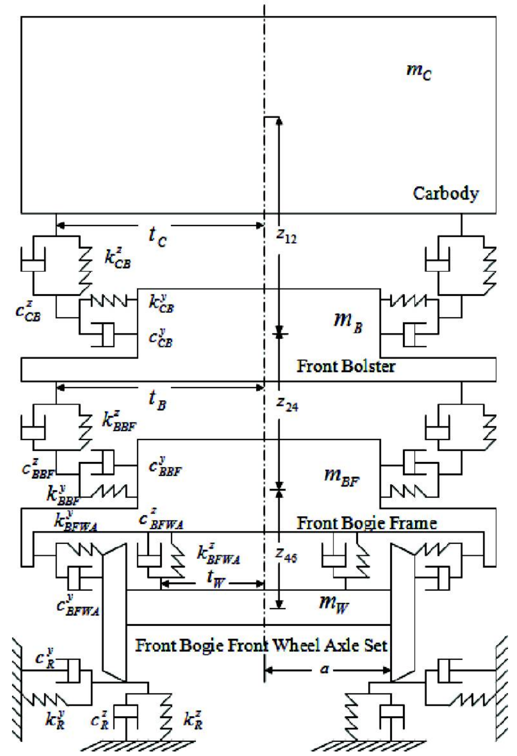


Table 1: Rigid Bodies and Their Degrees of Freedom

Components (Rigid Bodies)	Motion				
	Lateral	Vertical	Roll	Pitch	Yaw
Carbody	y_1	z_1	θ_1	ϕ_1	ψ_1
Front Bolster	y_2	z_2	θ_2		
Rear Bolster	y_3	z_3	θ_3		
Front Bogie Frame	y_4	z_4	θ_4	ϕ_4	ψ_4
Rear Bogie Frame	y_5	z_5	θ_5	ϕ_5	ψ_5
Front Bogie Front Wheel-Axle Set	y_6	z_6	θ_6		ψ_6
Front Bogie Real Wheel-Axle Set	y_7	z_7	θ_7		ψ_7
Rear Bogie Front Wheel-Axle Set	y_8	z_8	θ_8		ψ_8
Rear Bogie Rear Wheel-Axle Set	y_9	z_9	θ_9		ψ_9

- The rail vehicle possess longitudinal plane of symmetry (i.e., center of gravity of all masses lie in central plane).
- The vehicle is assumed to be traveling at constant speed therefore degree of freedom in longitudinal direction are not assigned to the rigid bodies.
- All springs and dampers are assumed to be linear
- Creep forces are assumed as linear function of creepages (linear Kalkar's theory (Kalker, 1979) is utilized to calculate the creep forces).
- Track is considered to be flexible.
- Car body is assumed to be rigid.
- Wheel and rail do not loose contact

The equations of motion describing coupled vertical-lateral dynamics of the rail-vehicle are obtained using the Lagrange's equations, which in general can be written as:

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}_i}\right]-\frac{\partial L}{\partial q_i}+\frac{\partial E_P}{\partial q_i}+\frac{\partial E_D}{\partial \dot{q}_i}=Q_i \qquad \text{... (1)}$$

Largrangian L is defined as $(T - V_g)$ where T is the kinetic energy and V_g is the potential energy due to gravity effect of the vehicle system, E_P is the energy stored in the system due to springs, E_D is the Rayliegh's dissipation function of the system and Q_i are the generalized forces corresponding to y_i the generalized coordinates.

The final equations of motion of rail vehicle are obtained in the following form

$$[M]\{\ddot{y}_i\}+[C]\{\dot{y}_i\}+[K]\{y_i\}=[F_r(\omega)] \qquad \text{...(2)}$$

[M], [K] and *[C]* are the 37×37 mass, stiffness and damping matrices respectively for rail vehicle. $[F_r(\omega)]$ is a 37×1 force matrix for displacement excitations at the eight wheel contact points.

i.e., $r = 1, 2....8$, due to vertical and lateral track irregularities.

TRACK MODEL

The track may be divided into a super structure and a sub structure. The super structure includes rails, rail fastenings, pads, sleepers and ballast (i.e. soil). The sub grade or subsoil is the sub structure of a track. The track in the present analysis is assumed to be flexible in both vertical and lateral directions. Its flexibility is accounted by considering wheel to be in series with sleeper, soil and subsoil (Figure 2) i.e.

$$\frac{1}{k_R^z}=\frac{1}{k_W^z}+\frac{1}{k_{SL}^z}+\frac{1}{k_S^z}+\frac{1}{k_{SS}^z} \qquad \text{...(3)}$$

and

$$\frac{1}{c_R^z}=\frac{1}{c_W^z}+\frac{1}{c_{SL}^z}+\frac{1}{c_S^z}+\frac{1}{c_{SS}^z} \qquad \text{...(4)}$$

similarly for lateral direction it can be assumed that

$$\frac{1}{k_R^y}=\frac{1}{k_W^y}+\frac{1}{k_{SL}^y}+\frac{1}{k_S^y}+\frac{1}{k_{SS}^y} \qquad \text{...(5)}$$

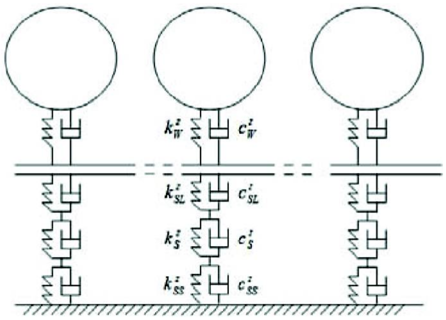
and

$$\frac{1}{c_R^y}=\frac{1}{c_W^y}+\frac{1}{c_{SL}^y}+\frac{1}{c_S^y}+\frac{1}{c_{SS}^y} \qquad \text{...(6)}$$

FREQUENCY RESPONSE FUNCTION

In this analysis spectral densities of the output variables are calculated directly from frequency response functions and rail roughness spectral

Figure 2: Track Model



densities. For computation of complex frequency response function, harmonic input is given at one wheel at a time while the inputs at the remaining wheels are kept zero. The Equation (2) may also be written as

$$([M]\ (-\omega^2)+[C](i\omega)+[K])y_i\, e^{i\omega t} = [F_r(\omega)]q_r\, e^{i\omega t} \quad ...(7)$$

The above equations may further be written as

$$[D_1]\ \ H_r(\omega)\ =\ F_r(\omega) \quad ...(8)$$

where $[D_1]$ is the dynamic stiffness matrix, and $H_r(\omega) = (y_i/q_r)$ is the complex frequency response function for r^{th} input. As the railway track possesses both vertical and lateral irregularities, therefore in the present analysis both vertical and lateral inputs are considered and it is considered that the final response of the system is a combined effect due to vertical and lateral inputs from the track. Using the principle of superposition, the resultant complex frequency response is obtained by combining the response matrices due to the eight inputs at the rail-wheel contact points.

REPRESENTATION OF TRACK ROUGHNESS

The irregularities in the railway track surface are random and represented by power spectral density functions. In the present analysis auto and cross-power spectral density functions of vertical and lateral irregularity for a straight track reported by Goel *et al* (2005) are used. Vertical and lateral irregularities, are of the type $S(\Omega) = C_{sp}\Omega^{-N}$.$C_{sp}$ is an empirical constant and N characterizes the rate at which amplitude decreases with frequency.

SYSTEM RESPONSE TO TRACK EXCITATION

The vehicle response to random excitations at the eight rail-wheel contact points has been obtained using statistical approach. For a multi-degree-of freedom lumped mass system, using the concept of random vibrations, the power spectral density for the response is related to the excitation through the complex frequency response function. Thus for a linear system subjected to random inputs, using input–output relationships for spectral densities, the auto and cross-spectral density matrix of the response may be written as

$$[S_{yy}(\omega)]_{37\times 37} = [H_r(\omega)]_{37\times 8}[S_r(\omega)]_{8\times 8}[H_r(\omega)]^T_{8\times 37} \quad ...(9)$$

The complex frequency response functions $[H_r(\omega)]_{37\times 8}$ can also be defined as the ratio of the response rate to unit harmonic input at a given point. The superscript T denotes transpose of matrix.

It may be noted here that above equation is used independently for vertical and lateral irregularities of the track.

In the evaluation of vehicle ride quality, the Power Spectral Density (PSD) for the acceleration of the carbody mass center as a function of frequency is of prime interest. The mean square acceleration response (MSAR) expressed as $(m/s^2)^2/Hz$, which is nothing but PSD of acceleration may be written as

$$MSAR = (2\pi f)^4 [H_r(\omega)]_{37\times 8} [S_r(\omega)]_{8\times 8} [H_r(\omega)]^T_{8\times 37} \quad ...(10)$$

The spectral density of output (or response) corresponding to each degree of freedom of carbody can be plotted against frequency with the help of the above equation. To determine root mean square acceleration response (RMSAR) at a center frequency f_c, power spectral density function is integrated over a one third octave band i.e.

$$RMSAR = sqrt \ [(2\pi)^4 \int_{0.89 fc}^{1.12 fc} [S_{YY}(f)](f)^4 df] \quad ...(11)$$

The root mean square acceleration values of acceleration both in vertical and lateral directions of carbody mass center at a series of center frequencies within the range of interest is obtained and ride comfort is evaluated and compared with the specified ISO 2631-1 standards (1997). In the present analysis principal frequency weightings with multiplication factors specified in ISO 2631-1 standards (1997) are applied to RMS acceleration values to obtain frequency-weighted acceleration for the evaluation of passenger comfort in sitting position.

The values of the parameters of a loaded General Sleeper and Rajdhani coach of Indian Railways and of Indian railway track for present analysis are obtained from Research Designs and Standards Organisation, Lucknow (India). The values of creep coefficients for the wheel-track interaction have been obtained from Yuping and McPhee (2005). These values are mentioned in Tables 2, 3 and 4.

Table 2: Values of Track Parameters and Creep Coefficients

Parameter	Parameter Value	Parameter	Parameter Value
k^z_{SL}	65 MN/m	k^y_{SS}	50 MN/m
k^z_S	20 MN/m	c^y_{SL}	10 kN-sec/m
k^z_{SS}	35 MN/m	c^y_S	50 kN-sec/m
c^z_{SL}	30 kN-sec/m	c^y_{SS}	70 kN-sec/m
c^z_S	40 kN-sec/m	f_{11}	7.5848 x 10^6 N
c^z_{SS}	50 kN-sec/m	f_{12}	1.5334 x 10^4 N
k^y_{SL}	50 MN/m	f_{23}	61.56 N.m^2
k^y_S	30 MN/m	f_{33}	8.6605 x 10^6 N

Table 3: Values of Rail Vehicle Parameters of Indian Railway General Sleeper ICF Coach

Parameter	Parameter Value	Parameter	Parameter Value	Parameter	Parameter Value	Parameter	Parameter Value
m_c	37960 kg	I_{BF}^X	1546 kg m^2	k_{BBF}^z	0.42375 MN/m	k_W^y	250 MN/m
m_B	400 kg	I_{BF}^Y	2893 kg m^2	k_{BBf}^y	0.2324 MN/m	c_W^y	4 MN-sec/m
m_{BF}	2346 kg	I_{BF}^Z	4298 kg m^2	c_{BBF}^z	0.0589 MN-sec/m	t_C	0.8 m
m_w	1487 kg	I_W^X	1181 kg m^2	c_{BBF}^y	1 MN-sec/m	t_B	1.127 m
I_C^X	63950 kg m^2	I_W^Y	108.5 kg m^2	k_{BFWA}^z	0.26935 MN/m	t_W	1.079 m
I_C^Y	1470750 kg m^2	I_W^Z	1181 kg m^2	k_{BFWA}^y	11.5 MN/m	z_{12}	1.3275 m
I_C^Z	1473430 kg m^2	k_{CB}^z	35 MN/m	c_{BFWA}^z	0.0206 MN-sec/m	z_{24}	0.1435 m
I_B^X	307 kg m^2	k_{CB}^y	17.5 MN/m	c_{BFWA}^y	0.5 MN-sec/m	z_{46}	0.194 m
I_B^Y	00	c_{CB}^z	0.035 MN-sec/m	k_W^z	1000 MN/m	a	0.871 m
I_B^Z	336.5 kg m^2	c_{CB}^y	0.0175 MN-sec/m	c_W^z	0.5 MN-sec/m	δ	0.15 rad

Table 4: Values of Rail Vehicle Parameters of Indian Railway Rajdhani Coach

Parameter	Parameter Value	Parameter	Parameter Value	Parameter	Parameter Value	Parameter	Parameter Value
m_c	33740 kg	I_{BF}^X	1713 kg m^2	k_{BBF}^z	0.42375 MN/m	k_W^y	250 MN/m
m_B	400 kg	I_{BF}^Y	3206 kg m^2	k_{BBf}^y	0.2324 MN/m	c_W^y	4 MN-sec/m
m_{BF}	2600 kg	I_{BF}^Z	4763 kg m^2	c_{BBF}^z	0.0589 MN-sec/m	t_C	0.8 m
m_w	1600 kg	I_W^X	1271 kg m^2	c_{BBF}^y	1 MN-sec/m	t_B	1.127 m
I_C^X	56932 kg m^2	I_W^Y	117 kg m^2	k_{BFWA}^z	0.26935 MN/m	t_W	1.079 m
I_C^Y	1307220 kg m^2	I_W^Z	1271 kg m^2	k_{BFWA}^y	11.5 MN/m	z_{12}	1.3275 m
I_C^Z	1309744 kg m^2	k_{CB}^z	35 MN/m	c_{BFWA}^z	0.0206 MN-sec/m	z_{24}	0.1435 m
I_B^X	307 kg m^2	k_{CB}^y	17.5 MN/m	c_{BFWA}^y	0.5 MN-sec/m	z_{46}	0.194 m
I_B^Y	00	c_{CB}^z	0.0311 MN-sec/m	k_W^z	1000 MN/m	a	0.871 m
I_B^Z	336.4 kg m^2	c_{CB}^y	0.0175 MN-sec/m	c_W^z	0.5 MN-sec/m	δ	0.15 rad

RIDE BEHAVIOR FROM ACTUAL TESTING

The results from actual testings are obtained from Research Designs Standards Organisation, Lucknow. The rail vehicle is moved at a constant speed of 80 km/h (General Sleeper) and 130 km/h (Rajdhani) over straight track. The data acquisition is completed in two stages. In the first stage the record is obtained for 2 Km straight specimen run- down track and this record is verified covering a long run of about 25 Km in the second stage. A strain gauge accelerometer (Range: $\pm$1 g & $\pm$2 g; Frequency response: 25 Hz; Excitation: 5 V AC/DC; Sensitivity: 360 mV/V/g; Damping: silicon fluid) is placed at floor level near bogie pivot of the rail vehicle. The acceleration data is recorded in time domain with National Instruments cards (Sampling rate: 100 Samples/s, Resolution: 12 Bit) using Lab View software program and this record is converted in frequency domain using Fast Fourier Transformations (FFT).

Power spectral densities of accelerations of loaded General Sleeper coach obtained through actual testing in vertical and lateral directions are shown in Figures 3 and 4

Figure 3: PSD of Vertical Acceleration of Loaded GS Coach (Actual Testing)

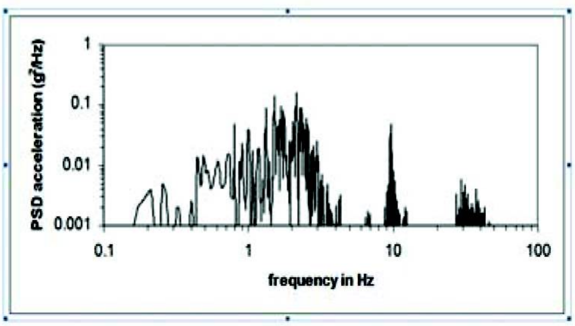
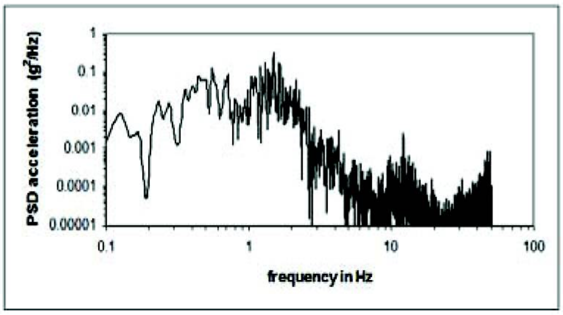


Figure 4: PSD of Lateral Acceleration of Loaded GS Coach (Actual Testing)



respectively and of loaded Rajdhani coach is shown in Figures 5 and 6 respectively.

Figure 5: PSD of Vertical Acceleration of Loaded Rajdhani Coach (Actual Testing)

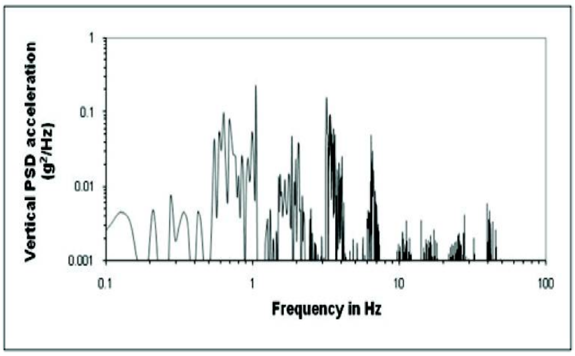
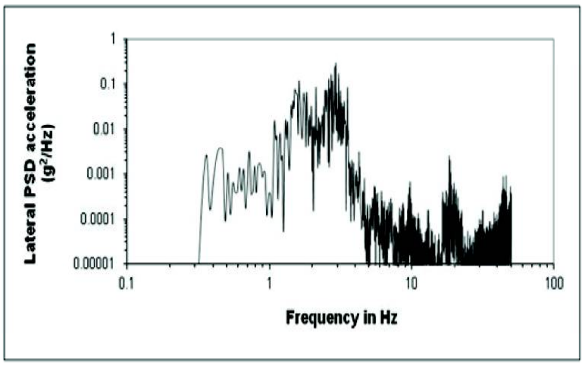


Figure 6: PSD of Lateral Acceleration of Loaded Rajdhani Coach (Actual Testing)



RIDE BEHAVIOR FROM SIMULATION

Power spectral densities of accelerations of loaded carbody obtained through simulation in vertical and lateral directions are shown in Figures 7 and 8 (GS Coach) and Figures 9 and 10 (Rajdhani coach). The theoretical and actual results compare reasonably well except that the peak values are obtained at slight different frequencies The actual and simulated results not being exactly similar is due to the following facts:

Figure 7: PSD of Vertical Acceleration of Loaded GS Coach (Simulation)

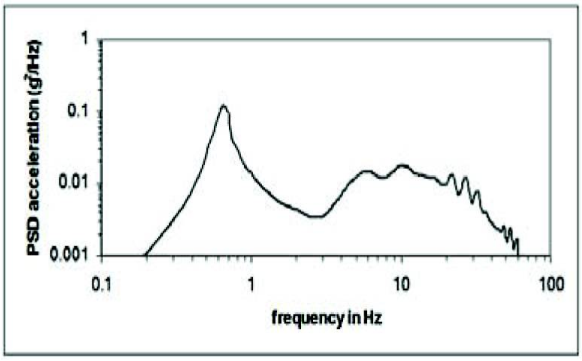


Figure 8: PSD of Lateral Acceleration of Loaded GS Coach (Simulation)

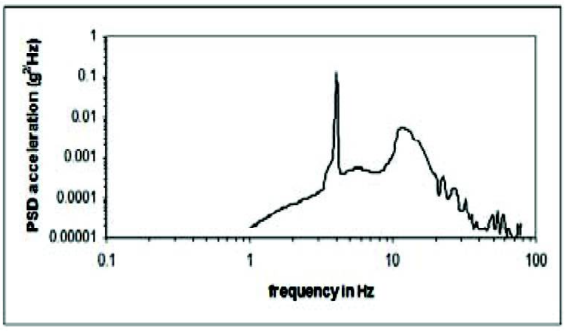
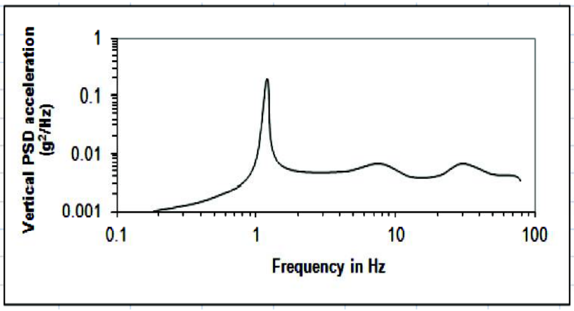
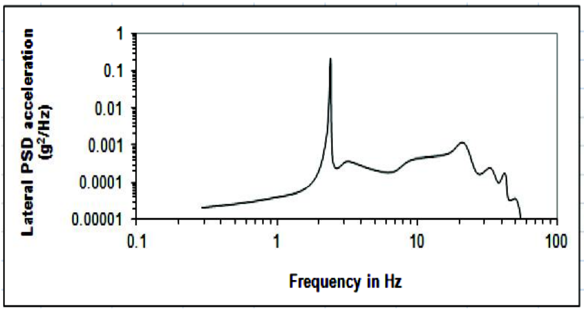


Figure 9: PSD of Vertical Acceleration of Loaded Rajadhani Coach (Simulation)



- The carbody in modeling is assumed as a rigid body but in practice it is a flexible structure. In the present analysis the other units such as braking system, lower plank,

Figure 10: PSD of Lateral Acceleration of Loaded Rajadhani Coach (Simulation)



etc., could not be individually incorporated in the analysis.

- In the present analysis only vertical and lateral irregularities of the track have been considered, whereas in actual practice other irregularities are also present in the track.
- The mass of the track is not considered in the present analysis and it is not assigned any degree of freedom in order to consider the inputs from the track itself.
- In actual testing the sensor for acceleration measurements is placed at the floor level of bogie pivot. This point is not exactly where the c.g. of the carbody is concentrated. In simulations the acceleration is determined at c.g. of the carbody i.e middle of the coach.
- In the present analysis creep forces are considered as linear function of creepage i.e. wheel axle set displacements and wheel axle set velocities. In actual the creep forces are non-linear functions of wheel-axle set displacements and wheel-axle set velocities.
- The effect of wind drag forces are not considered in the present analysis. In actual the wind forces from longitudinal and lateral

directions significantly affect the dynamics of rail vehicle system.

The vertical and lateral RMS acceleration values of loaded GS coach obtained through simulation are shown in Figures 11 and 12 and of Rajdhani coach are shown in Figures 13 and 14.

Figure 11: Vertical RMS Acceleration of Loaded GS Coach (Simulation)

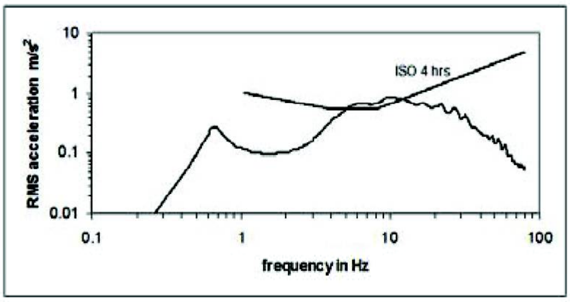


Figure 12: Lateral RMS Acceleration of Loaded GS Coach (Simulation)

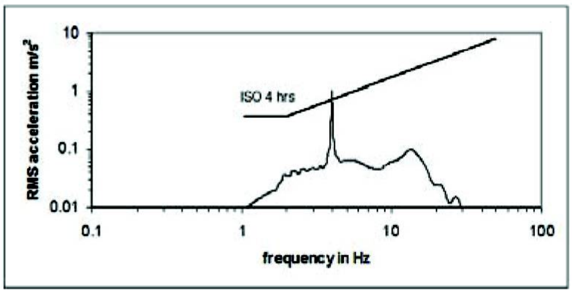


Figure 13: Vertical RMS Acceleration of Loaded Rajadhani Coach (Simulation)

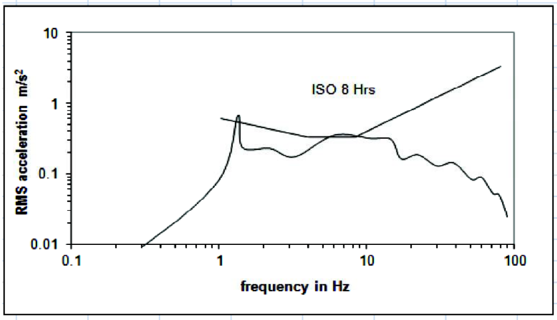
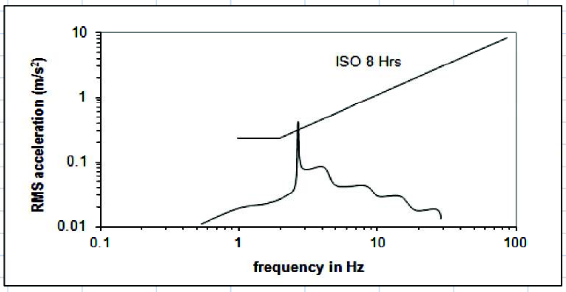


Figure 14: Lateral RMS Acceleration of Loaded Rajadhani Coach (Simulation)



CONCLUSION

The result of vertical root mean square (RMS) acceleration response (Figure 11) indicates that the response of loaded GS coach lies well within the ISO specifications (4 h comfort criteria) except for frequency range from 5 to 10.5 Hz. The result of lateral root mean square (RMS) acceleration response (Figure 12) indicates that the response of loaded GS coach lies well within the ISO specifications (4 h comfort criteria) except for frequency at nearly 4 Hz, where the peak value is obtained. The analysis indicates that discomfort frequency range belongs from 4 to 10.5 Hz and improvements in the rail vehicle design are required.

The result of vertical root mean square (RMS) acceleration response (Figure 13) indicates that the response of loaded Rajdhani coach lies well within the ISO specifications (8 Hrs comfort criteria) except for frequency 1.1 Hz and from frequency range from 5 to 10 Hz. The result of lateral root mean square (RMS) acceleration response (Figure 14) indicates that the response of loaded Rajdhani coach lies well within the ISO specifications (8 Hrs comfort criteria) except for frequency at nearly 2.5 Hz, where the peak value is obtained. The analysis indicates that discomfort frequency range belongs from 1 to 10 Hz and improvements in the rail vehicle design are required.

REFERENCES

1. Garg V K and Dukkipatti V (1984), *Dynamics of Railway Vehicle Systems*, Academic Press, New York.
2. Goel V K, Manoj Thakur, Kusum Deep and Awasthi B P (2005), "Mathematical Model to Represent the Track Geometry Variation using PSD", *Indian Railway Technical Bulletin*, Vol. LXI, No. 312 – 313, pp. 1-10.
3. ISO 2631: 1997, *Mechanical Vibration and Shock Evaluation of Human Exposure to Whole Body Vibrations-Part 1: General Requirements*.
4. Kalker J (1979), "Survey of Wheel-Rail Contact Theory", *Vehicle System Dynamics*, No. 8, pp. 317-358.
5. Meachem H C and Ahlbeck D R (1969), "A Computer Study of Dynamic Loads caused by Vehicle Track Interaction", *J. of Engg. for Industry. Trans. of the ASME*, Ser. B, Vol. 91, No. 3, pp. 808-816.
6. Nathoo N S and Healey A J (1978), "Coupled Vertical-Lateral Dynamics of a Pneumati c Ti red Vehi cl e: part 1- A Mathematical Model", *Journal of Dy namic Syst em Measu rement and Control*, Vol. 100, pp. 311-318.
7. Nathoo N S and Healey A J (1978), "Coupled Vertical-Lateral Dynamics of a Pneumatic Tired Vehicle: part 2- Simulated verses Experimental Data", *Journal of Dynamic System Measurement and Control*, Vol. 100, pp. 319-325.
8. Ramji K (2004), *Coupled Vertical-Lateral Dynamics of Three-Wheeled Motor Vehicles*, Ph.D. Dissertation, Deptt of Mechanical and Industrial Engg., I I T Roorkee.
9. Yuping H and McPhee J (2005), "Optimization of Curving Performance of Rail Vehicles", *Vehicle System Dynamics,* Vol. 43 No. 12, pp. 895-923.
10. Zhai W M, Cai C B and Guo S Z (1996), "Coupling Model of vertical and lateral vehicle/track interactions", *Vehicle System Dynamics*, Vol. 26, No.1, pp. 61-79.
11. Zhai W M, Wang K and Cai C (2009), "Fundamentals of vehicle-track coupled dynamics", *Vehicle System Dynamics*, Vol. 47, No. 11, pp. 1349-1376.

APPENDIX

NOMENCLATURE

$m_{C,\ B}$	Mass of carbody and bolster respectively
$m_{BF,\ W}$	Mass of bogie frame and wheel axle respectively
$I_C^{x,y,z}$	Roll, pitch and yaw mass moment of inertia of carbody respectively
$I_B^{x,y,z}$	Roll, pitch and yaw mass moment of inertia of bolster respectively
$I_{BF}^{x,y,z}$	Roll, pitch and yaw mass moment of inertia of bogie frame respectively
$I_W^{x,y,z}$	Roll, pitch and yaw mass moment of inertia of wheel axle respectively
$k_{CB}^{z,\ y}$	Vertical (½ part) and lateral (½ part) stiffness between carbody and bolster respectively
$c_{CB}^{z,\ y}$	Vertical (½ part) and lateral (½ part) damping coefficient between carbody and bolster respectively
$k_{BBF}^{z,\ y}$	Vertical (¼ part) and lateral (½ part) stiffness between bolster and bogie frame respectively
$c_{BBF}^{z,\ y}$	Vertical and lateral damping coefficient between bolster and bogie frame respectively (½ part)
$k_{BFWA}^{z,y}$	Vertical (¼ part) and lateral (½ part) stiffness between bogie frame and corresponding wheel axle
$c_{BFWA}^{z,\ y}$	Vertical (¼ part) and Lateral (½ part) damping coefficient between bogie frame and corresponding wheel axle
$k_R^{z,\ y}$	Vertical and lateral stiffness of rail respectively
$c_R^{z,\ y}$	Vertical & lateral damping coefficient of rail respectively
t_W	Lateral distance from bogie frame c.g. to corresponding vertical suspension between bogie frame and wheel axle

APPENDIX (CONT.)

t_C	Lateral distance from carbody c.g. to side bearings
t_B	Lateral distance from bolster c.g. to vertical suspension between bolster and bogie frame
z_{12}	Vertical distance between carbody c.g. and bolster c.g.
z_{24}	Vertical distance between bolster c.g. and bogie frame c.g
z_{46}	Vertical distance between bogie frame c.g. and corresponding wheel axle c.g.
a	Half of wheel gauge