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Research Paper

EFFECT OF SUSPENSION SYSTEM ON HANDLING BEHAVIOUR OF THREEWHEELED VEHICLE

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From the beginning of the industrial revolution to the concept of the car of the future, the three wheeled vehicles can hold their headlamps up with glory due to less fuel consumption, as the earth's energy resources get depleted. Regarding road safety and accident avoidance capabilities, handling characteristics of road vehicles are very important. Handling characteristics are rather confusing because these are affected by suspension and tyre design parameters as well as test driver's feelings. The vehicle model is represented by two rigid mass systems, and includes aerodynamic resistance, for analyzing the effect of suspension on three wheeled vehicle. The equations are developed for evaluating roll steer, roll camber, and roll moment distributions on front and rear axle. To find the handling behaviour, understeer coefficient has been evaluated and results are presented, considering the above parameters.

Keywords: Suspension system, Handling characteristics, Understeer coefficient, Three wheeled vehicles

INTRODUCTION

The three-wheeled vehicles are likely to see resurgence in their usage due to their advantages in being fuel efficient and relatively inexpensive. These are likely to be the most popular mode of public and private transport not only in India but in other countries as well. Three wheeled vehicle configurations are commonly employed for automatic guided vehicles, mobile robot and passenger transport units, popularly known as tempos and autorickshas.Its concept is as old as the automobile, but not popular as four wheeled and two wheeled vehicle due to its vile dynamics behaviour. The three wheeled motor vehicle likely to be most popular in India typically have a steering system like those of motorcycle and scooters with a single wheel in the front and the two wheels in the rear, with the differential and suspension system similar to those of automobile.

Steady state handling behaviour and stability of lateral motion of an automobile are described by Sharp R S (1973). Valkenburgh P G *et al*. (1982) concluded that the factors

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peculiar to three wheeled vehicle are longitudinal centre of gravity, roll rate distribution and single wheel camber properties. Johnson C W and Huston J C (1984) presented lateral stability of rider/cycle system and concluded that, if the centre of mass location is in front of the geometrical centre of the vehicle, the system is stable. Theoretical stability analyses of the cornering behaviour of three and four wheeled vehicles are described by Chiang-Nang Chang and Ding-Hwa Ding (1994). Goel V K (2003) investigated the lateral stability, rollover stability and handling behaviour of N-28 three wheeled vehicle.There are multiple factors in vehicle design that may influence the cornering forces developed in the presence of a lateral acceleration. The suspensions systems are one of the most important factors, which affect handling behaviour of a road vehicle. The angular positions of wheels and the vertical forces acting on them also depend upon the suspension system, which locates the wheel relative to the vehicle body. In analysing the effect of suspension, the main focus is the geometry in order to find the change in wheel camber and steer angles, and in the distribution of roll moments on the front and rear axles, as these will affect the handling behaviour of automobiles.

STABILITY AND HANDLING EVALUATION

When a vehicle is negotiating a turn at moderate or higher speeds, the effect of the centrifugal force acting at the centre of gravity can no longer be neglected. To balance the centrifugal forces, the tyre must develop appropriate cornering forces.

The understeer coefficient is defined as the steering angle needed (in radians) to bring about an extra centripetal acceleration of 1 g. mathematically, under steer coefficient (K_{us}) is defined as,

$$K_{us} = \alpha_f - \alpha_r \quad ...(1)$$

Depending on the value of the under steer coefficient, the steady-state handling characteristics may be classified into three categories: neutral steer, under steer and over steer.

If, understeer condition

If, neutralsteer condition

If, oversteer condition

Oversteer condition is more dangerous, so it is always desired to avoid this condition by changing configuration of vehicle parameters to achieve understeer behaviour.

For the standard Bajaj-RE three wheeled vehicle, travelling at various speeds on a curved path of 100 m, roll moment distributions on front and rear axle using Equations (10) and (11) and understeer coefficient due to load transfer using Equation (29) are given in Table 1. Roll steer angle for front and rear suspension can be evaluated using Equations (43) and (37) and camber angle for front and rear suspension can be found using Equations (39) and (42). For the standard Bajaj-RE three wheeled vehicle roll steer angle and camber angle are given in Table 2. Understeer coefficient due to roll steer, roll camber, load transfer and nominal cornering stiffness of tyre can be evaluated using Equations (46), (48), (29) and (28). Overall understeer coefficient is algebraic sum of understeer coefficients due to roll steer, roll camber, load transfer and nominal cornering stiffness of tyre given by using Equation (49), shown in Table 3.

RESULTS AND DISCUSSION

Handling evaluation is one of the most attractive themes in automobile engineering. Suspension and tyre designers want to know how their design parameters contribute to handling characteristics and test drivers want to know where their feelings are derived.

Table 1 shows more roll moment distribution on rear axle compare to front axle and understeer coefficient due to load transfer is negative for all speeds as shown in Figure 1, (i.e., vehicle is in oversteer condition for all speeds), which verifies the observations given by Gillespie T D (1992).

Table 2 shows roll steer angle and roll camber angle for rear suspension is very less compare to front suspension for Bajaj-RE three wheeled vehicle. So there is less effect of rear suspension on handling potential compare to front suspension.

From Table 3, it reveals that, the three wheeled vehicle is in oversteer condition for all speeds due to load transfer, as understeer coefficient is negative for all speeds, as shown in Figure 1. Understeer coefficient due to roll steer is almost constant (-0.22036 deg/g) for all speeds, i.e., vehicle is in oversteer condition for all speeds, while understeer coefficient due to roll camber is also constant (0.11002 deg/g), but positive for all speeds, i.e., vehicle is in undesteer condition for all speeds.

Here, for Bajaj-RE three wheeled vehicle overall understeer coefficient varies from -0.233 deg/g to -0.853 deg/g i.e. negative for all speeds. Hence, vehicle is in oversteer condition for all speeds as shown in Figure 1.

From Table 4, it reveals that understeer coefficient due to nominal cornering stiffness of tyre varies from -0.12262 deg/g to -0.71396 deg/g, negative for all speeds, as shown in

Table 1: Roll Moment Distributions and Understeer Coefficient Due to Load Transfer

Speed V (km/h)	10	20	30	40	50	60
Steer angle θ (degree)	–1.053	–4.21	–9.47	–16.84	–26.32	–37.9
Roll angle Φ (degree)	0.05	0.20	0.46	0.83	1.3	1.8
Roll moment distribution on front axle $M_{\Phi f}$ (N-m)	1.3	5.2	11.7	20.9	32.2	47.1
Roll moment distribution on rear axle $M_{\Phi r}$ (N-m)	19.7	78.7	177.7	314.6	491.6	707.9
Understeer coefficient due to load transfer (degree/g)	–2.66E-05	–4.19E-04	–2.07E-03	–6.30E-03	–1.49E-02	–2.92E-02

Table 2: Roll Steer and Roll Camber

Speed V (km/h)	10	20	30	40	50	60
Steer angle θ (degree)	–1.053	–4.21	–9.47	–16.84	–26.32	–37.9
Roll angle Φ (degree)	0.05	0.20	0.46	0.83	1.3	1.8
Roll steer angle for front suspension (degree)	–1.0015	–4.00	–9.013	–16.02	–25.037	–36.05
Roll steer angle for rear right suspension (degree)	–0.000036	–0.00014	–0.00032	–0.00057	–0.0009	–0.0013
Roll steer angle for rear left suspension (degree)	–0.000036	–0.00014	–0.00032	–0.00057	–0.0009	–0.0013
Roll camber angle for front suspension (degree)	0.378	1.5124	3.402	6.049	9.4524	13.61
Roll camber angle for rear right suspension (degree)	0.00023	0.0009	0.0021	0.0037	0.0059	0.0084
Roll camber angle for rear left suspension (degree)	–0.00023	–0.0009	–0.0021	–0.0037	–0.0059	–0.0084

Table 3: Understeer Coefficient Due to Roll Steer, Roll Camber, Load Transfer, and Due to Nominal Cornering Stiffness of Tyre

Speed V (km/h)	10	20	30	40	50	60
Under steer coefficient due to nominal cornering stiffness of tyre (degree/g)	–0.12262	–0.16674	–0.24353	–0.35526	–0.50882	–0.71396
Understeer coefficient due to load transfer (degree/g)	–2.66E–05	–4.19E–04	–2.07E–03	–6.30E–03	–1.49E–02	–2.92E–02
Under steer coefficient due to roll steer (degree/g)	–0.2204	–0.2204	–0.2204	–0.2204	–0.2204	–0.2204
Under steer coefficient due to roll camber (degree/g)	0.11002	0.11002	0.11002	0.11002	0.11002	0.11002
Overall under steer coefficient (degree/g)	–2.33E–01	–2.77E–01	–3.56E–01	–4.72E–01	–6.34E–01	–8.53E–01

Figure 1: Understeer Coefficient for Bajaj RE Three Wheeled Vehicle

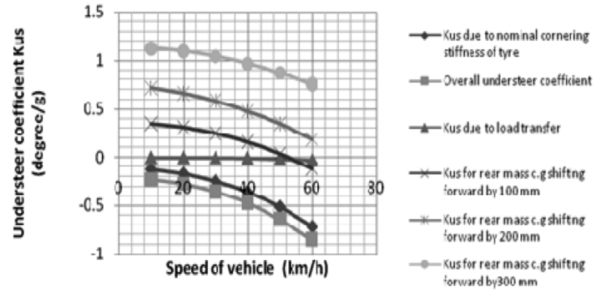

Figure 1, (i.e. vehicle is in oversteer condition for all speeds). But three wheeled vehicle is going to be undesteer condition when rear mass c.g. is shifted forward, as shown in figure 1 for shifting rear mass c.g. forward by 100 mm, 200 mm and 300 mm, i.e., for better handling behaviour vehicle rear mass c.g. should be near to L from front tyre contact point, Huston J C *et al.,* (1982).

Table 4: Understeer Coefficient Due to Nominal Cornering Stiffness of Tyre for Shifting Rear Mass cg Forward

Speed V (km/h)	10	20	30	40	50	60
Under steer coefficient due to nominal cornering stiffness of tyre Kus1 (standard vehicle) (degree/g)	-0.12262	-0.16674	-0.24353	-0.35526	-0.50882	-0.71396
Kus1 for rear mass c.g. shifting forward by 100mm (degree/g)	0.3458	0.3091	0.2469	0.1573	0.0372	-0.1185
Kus1 for rear mass c.g. shifting forward by 200mm (degree/g)	0.7171	0.6618	0.5831	0.4761	0.349	0.1867
Kus1 for rear mass c.g. shifting forward by 300mm (degree/g)	1.1286	1.0967	1.0435	0.9689	0.8724	0.753

CONCLUSION

Handling behaviour has been studied at different speeds for Bajaj RE three wheeled vehicle. Effect of front and rear suspension has been analysed. Therefore it is concluded on the basis of these limited studies that, vehicle handling potential decreases, with increase in speed. The forward shifting of the rear mass centre will turn the vehicle to understeer from the existing oversteerbehaviour. Driver effort on the steering will also be reduced due to the forward shifting of rear mass centre.

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APPENDIX 1

Nomenclature	
a	First co-efficient in cornering stiffness
a_f	Distance between c.g. of front system and rear system along X-axis
a_r	Distance between c.g. of rear system from rear wheel contact point
b	Second co-efficient in cornering stiffness
c	Roll centre of rear suspension
c_α	Cornering stiffness of one tyre
Fy	Lateral force
F_{yf}	Lateral force on front tyre
Fzo	Load on the outside wheel in the turn
Fzi	Load on inside wheel in the turn
F_{zf}	Normal load on front tyre
F_d	Aerodynamic drag
F_z	Normal load on the tyre
g	Acceleration due to gravity
hr	Roll centre height of rear part from ground level
h_d	Height at aerodynamic force acting
H_f	Height of c.g. of front system from roll axis
H'_f	Height of c.g. of front system from ground level
h_1	Height of c.g. of whole vehicle body from roll axis
H	Height of c.g. of whole vehicle body from ground level
H_r	Height of c.g. of rear system from roll axis

APPENDIX 1 (CONT.)

Nomenclature	
H'_r	Height of c.g. of rear system from ground level
k_{sf}	Spring stiffness of front suspension
$k_{\Phi f}$	Roll stiffness of the front suspension
k_{sr}	Spring stiffness of each spring for rear suspension
$k_{\Phi r}$	Roll stiffness of the rear suspension
L	Wheel base
l_1	Distance between c.g. of whole vehicle system from front wheel contact point
l_2	Distance between c.g. of whole vehicle system from rear wheel contact point
R	Radius of curvature
s_f	Lateral distance between steering axis and spring
s_r	Lateral separation between two springs for rear suspension
t	Track width of rear wheels
V	Speed of vehicle
W	Weight of whole vehicle
w_f	Weight of front part of vehicle
w_r	Weight of rear part of vehicle
Greek Symbols	
θ	Angular displacement of front wheel about steering axis
ε	Angle between ground and roll axis
α'	Rake angle
Φ	Roll angle of vehicle body
α	Slip angle
γ	Camber angle
δ_s	Roll steer angle
Subscripts	
i,j,k	Component in direction of X, Y & Z

APPENDIX 2

1. Roll Moment Distribution

Roll is geometrically equivalent to bump of one wheel and drop of the opposite one, relative to vehicle body. Suspension roll is defined (S.A.E.) as rotation of the vehicle sprung mass about a fore-aft axis with respect to a transverse line joining a pair of wheel centers. Suspension is characterized by a "roll centre", the point at which the lateral forces are transferred from axle to the sprung mass.

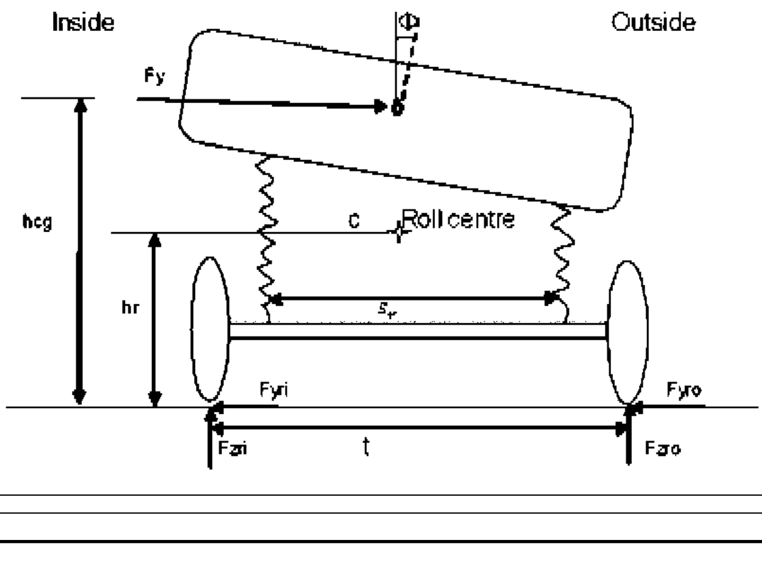
Asumptions

1.	All the three wheels of the vehicle are in contact with ground.
2.	Effects of flexibility of rubber bushes, used in suspension links are neglected.
3.	The road is considered to be rigid and plane surface.
4.	All suspensions are functionally equivalent to two linear spring for rear part and a single linear spring for front part of three wheeler.
5.	Effect of wind forces has been accounted for.
6.	Three wheeled vehicle as two mass system. Roll centre of front suspension is at front tyre contact patch and at some height 'h_r' from ground for rear suspension.

Rear Suspension of the Vehicle

The lateral separation of the springs causes them to develop a roll resisting moment proportional to the difference in roll angle between the body and axle.

Figure A1: Force Analysis of Rear Suspension of Three Wheeled Vehicle on Cornering



APPENDIX 2 (CONT.)

The roll moment may be found by assuming some angular position and calculating the moment. Roll stiffness is defined as rate of change of roll moment with respect to roll angle.

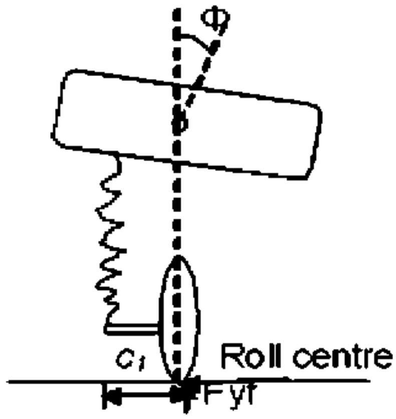
Roll stiffness of rear suspension is given by,

$$K_{\Phi r} = \frac{k_{sr}.\frac{s_r.\Phi s_r}{2}}{\Phi} = 0.5 k_{sr} s_r^2 \quad ...(1)$$

Front suspension of the vehicle,

Roll centre is defined at the midpoint of contact patch of tyre and road surface. The lateral separation between the spring and steering axis causes them to develop a roll resisting moment proportional to the difference i n roll angle between the body and axle.

Figure A2: Force Analysis of Front Suspension of Three Wheeled Vehicle on Cornering



Similar to the rear suspension, the roll stiffness of front suspension is given by,

$$K_{\Phi f} = \frac{k_{sf}.\frac{s_f.\Phi s_f}{2}}{\Phi} = 0.5 k_{sf} s_f^2 \quad ...(2)$$

Hence,

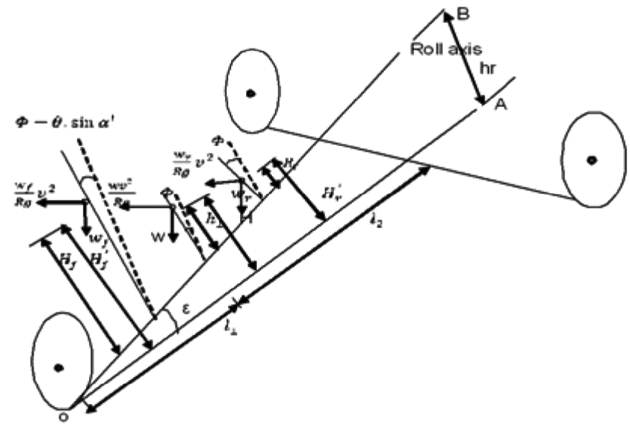
Roll stiffness of vehicle suspension system is given by,

$$k_{\Phi} = k_{\Phi f} + k_{\Phi r} \quad ...(3)$$

Roll axis is defined as the line connecting the roll centers of the front and rear suspensions, as shown in Figure 3.

APPENDIX 2 (CONT.)

Figure A3: Force Analysis for roll of Three Wheeled Vehicle



The vehicle is considered as two mass systems: the front system and the rear system. The rear system comprises the vehicle cabin, the engine, the driver, co-passengers and the rear suspensions. The front system consist of the steering system along with the front suspension, i.e. all the masses attached to the steering system and rotating with the steering wheel about the steering axis.

now,

Taking moment about roll axis for whole body,

For small roll angle,

$\sin\Phi \approx \Phi$ and $\cos\Phi \approx 1$

$$M_{\Phi} = Wh_1.[\Phi + \frac{v^2}{Rg}]\cos\cos\varepsilon \quad ...(4)$$

where, $W = w_f + w_r$

but, $M_{\Phi} = M_{\Phi f} + M_{\Phi r} = k_{\Phi f}\left(\Phi - \theta.\sin\sin\alpha'\right) + k_{\Phi r}.\Phi$ ...(5)

now,

moment on front part of vehicle,

$$k_{\varphi f}\left(\varphi - \theta.\sin\ \sin\alpha'\right) = -w_f H_f\left(\varphi - \theta.\sin\sin\alpha'\right) + \frac{wl_2}{RgL}v^2.h_1 \quad ...(6)$$

and, moment on rear part of vehicle,

APPENDIX 2 (CONT.)

$$k_{\Phi r}.\Phi = w_r.H_r\Phi + \frac{Wl_1}{RgL}v^2.h_1 \qquad \text{...(7)}$$

Using Equations (5), (6), (7) and (8), roll angle is given by

$$\phi = \left. Wh_1.\frac{\mathrm{v}^2}{\mathrm{Rg}}\cos Wh_1.\frac{\mathrm{v}^2}{\mathrm{Rg}}\cos\varepsilon + k_{\Phi f}\,\frac{\dfrac{Wl_1}{RgL}v^2.h_1}{k_{\Phi r} - w_r.H_r} - k_{\Phi f}\,\frac{\dfrac{Wl_2}{RgL}v^2.h_1}{w_f H_{f+k_{\Phi f}}} \middle/ k_{\Phi f} + k_{\Phi r} - Wh_1.\cos\cos\varepsilon \right. \qquad \text{...(8)}$$

now, Roll rate is defined as derivative of roll angle with respect to lateral acceleration.

So, using Equation (8), roll rate is given by,

$$R_\Phi = \frac{d\Phi}{da_y} = \frac{d\Phi}{d\,\dfrac{v^2}{Rg}}$$

$$= \left. Wh_1.\cos Wh_1.\cos\varepsilon + k_{\Phi f}\,\frac{\dfrac{Wl_1}{L}.h_1}{k_{\Phi r} - w_r.H_r} + k_{\Phi f}\,\frac{\dfrac{Wl_2}{L}.h_1}{w_f H_f + k_{\Phi f}} \middle/ k_{\Phi f} + k_{\Phi r} - Wh_1.\cos\cos\varepsilon \right. \qquad \text{...(9)}$$

now, Roll moment distribution on front axle,

$$M_{\Phi f} = k_{\Phi f}\left[\left\{ \left. Wh_1.\frac{\mathrm{v}^2}{\mathrm{Rg}}\cos Wh_1.\frac{\mathrm{v}^2}{\mathrm{Rg}}\cos\varepsilon + k_{\Phi f}\,\frac{\dfrac{Wl_1}{RgL}v^2.h_1}{k_{\Phi r} - w_r.H_r} - k_{\Phi f}\,\frac{\dfrac{Wl_2}{RgL}v^2.h_1}{w_f H_{f+k_{\Phi f}}} \middle/ k_{\Phi f} + k_{\Phi r} - Wh_1.\cos\cos\varepsilon \right. \right\}\right.$$

$$\left. - \left\{ \frac{\dfrac{Wl_1}{RgL}v^2.h_1}{k_{\Phi r} - w_r.H_r} - \frac{\dfrac{Wl_2}{RgL}v^2.h_1}{w_f H_f + k_{\Phi f}} \right\} \right] \qquad \text{...(10)}$$

APPENDIX 2 (CONT.)

and, Roll moment distribution on rear axle,

$$M_{\Phi r} = k_{\Phi r}\left\{ Wh_1.\frac{v^2}{Rg}\cos Wh_1.\frac{v^2}{Rg}\cos\varepsilon + k_{\Phi f}\frac{\frac{Wl_1}{RgL}v^2.h_1}{k_{\Phi r} - w_r.H_r} - k_{\Phi f}\frac{\frac{Wl_2}{RgL}v^2.h_1}{w_f H_{f+k_{\Phi f}}} \Big/ k_{\Phi f} + k_{\Phi r} - Wh_1.\cos\cos\varepsilon \right\} \quad ...(11)$$

Lateral force of one tyre is given by,

$$F_y = c_\alpha.\alpha \quad ...(12)$$

To represent load sensitivity effect, the two tyres (inside and outside) must be treated separately. The cornering stiffness of each tyre as a function of normal load can be represented by second order polynomial and the lateral force is given by,

$$F_y' = \left(aF_z - bF_z^2\right).\alpha \quad ...(13)$$

where,

$$c_\alpha = aF_z - bF_z^2$$

For vehicle cornering, the lateral force of both rear tyres F_y (α_{ro} and α_{ri} are assumed to be same) is given by,

$$F_y = \left(aF_{zo} - bF_{zo}^2 + aF_{zi} - bF_{zi}^2\right).\alpha \quad ...(14)$$

$$F_{zo} = F_z + \Delta F_z \quad ...(15)$$

$$F_{zi} = F_z - \Delta F_z \quad ...(16)$$

Then, using Equations (14), (15), and (16) we get,

$$F_y = [a(F_z + \Delta F_z) - b(F_z + \Delta F_z)^2 + a(F_z - \Delta F_z) - b(F_z - \Delta F_z)^2].\alpha$$

$$= \left[2aF_z - 2bF_z^2 - 2b\Delta F_z^2\right].\alpha \quad ...(17)$$

For rear suspension, using Equation (17), we get

APPENDIX 2 (CONT.)

$$\mathrm{F_{yr}} = \left[c_{\alpha r} - 2.b\Delta \mathrm{F_{zr}^2}\right]\alpha_{\mathrm{r}} = \frac{(\frac{Wl_1}{L})}{Rg}v^2 \quad ...(18)$$

The body roll acting through the springs imposes a torque on the axle proportional to the roll stiffness, times the roll angle. This results in an equation for the load difference from side to side of the form:

$$F_{zro} - F_{zri} = 2.\mathrm{F_{yr}}.\frac{\mathrm{h_r}}{\mathrm{t}} + 2.k_{\Phi r}.\frac{\Phi}{t} + 2.k_{\Phi f}.\frac{(\Phi + \theta.\sin\sin\alpha)}{t} = 2.\Delta\mathrm{F_{zr}} \quad ...(19)$$

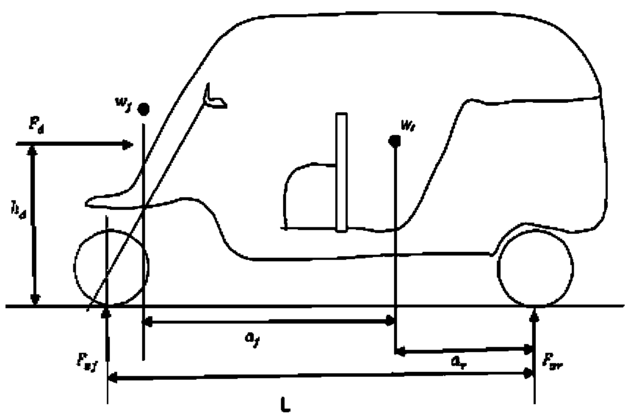
For front suspension, using Equation (13), we get

$$\mathrm{F_{yf}} = \left[a\mathrm{F_{zf}} - b\mathrm{F_{zf}^2}\right].\alpha$$

Putting the value of $c_{\alpha r}$ in the above equation, we get

$$= c_{\alpha f}.\alpha_f = \frac{(\frac{Wl_2}{L})}{Rg}v^2 \quad ...(20)$$

Figure A4: Two Mass System Model of Three Wheeled Vehicle



Assuming, C.G of front and rear mass system are in central plane. If 'F_d' is the aerodynamic drag and it acts at a height 'h_d', then normal loads on the front and rear tyres are given by,

$$F_{zf} = -w_f\frac{a_f + a_f}{L} - w_r\frac{a_r}{L} + F_d\frac{h_d}{L} \quad ...(21)$$

APPENDIX 2 (CONT.)

$$F_{zrr} = -w_f \frac{L-a_f-a_f}{2L} - w_r.\frac{L-a_r}{2L} - F_d \frac{h_d}{2L} \quad ...(22)$$

$$F_{zrl} = -w_f \frac{L-a_f-a_f}{2L} - w_r.\frac{L-a_r}{2L} - F_d \frac{h_d}{2L} \quad ...(23)$$

Taking into account the load transfer due to centrifugal acceleration, the normal loads on the three wheels are given by,

F_{zf} = Same as given by Equation (21)

$$F_{zro} = -w_f \frac{L-a_f-a_f}{2L} - w_r.\frac{L-a_r}{2L} - F_d \frac{h_d}{2L} + \frac{W}{g}.\frac{v^2}{R}.\frac{h}{t} \quad ...(24)$$

$$F_{zri} = -w_f \frac{L-a_f-a_f}{2L} - w_r.\frac{L-a_r}{2L} - F_d \frac{h_d}{2L} - \frac{W}{g}.\frac{v^2}{R}.\frac{h}{t} \quad ...(25)$$

Here, aerodynamic drag 'F_d' is given by,

$$F_d = 0.5AC_D\rho v^2 \quad ...(26)$$

where,

A = frontal area of the vehicle (m^2)

C_D = coefficient of aerodynamic drag

ρ = Air density (kg/m^3)

v = Vehicle speed (m/s)

now,

Steer angle is given by,

$$\delta = 57.3\frac{L}{R} + \alpha_f - \alpha_r$$

Using equations (18) and (20), we get

$$= 57.3\frac{L}{R} + \left[\left(\frac{\frac{Wl_2}{L}}{c_{\alpha f}} - \frac{\frac{Wl_1}{L}}{c_{\alpha r}}\right) + \left(-\frac{\frac{Wl_1}{L}}{c_{\alpha r}}.2.b\frac{\Delta F_{zr}^2}{c_{\alpha r}}\right)\right].\frac{v^2}{Rg} \quad ...(27)$$

APPENDIX 2 (CONT.)

Hence,

Understeer co-efficient due to nominal cornering stiffness of tyre is given by,

$$\mathrm{K}_{us1} = \left(\frac{\frac{Wl_2}{L}}{c_{\alpha f}} - \frac{\frac{Wl_1}{L}}{c_{\alpha r}} \right) \quad ...(28)$$

and,

Understeer co-efficient due to lateral load transfer is given by,

$$K_{us2} = -\left(\frac{\frac{Wl_1}{L}}{c_{\alpha r}} .2.b \frac{\Delta \mathrm{F}_{zr}^2}{c_{\alpha r}} \right) \quad ...(29)$$

2. Roll Steer and Roll Camber

In a rolled position, suspension geometry is generally such that there are changes of wheel steer angles relative to body, called Roll steer. The term roll steer refers to change of steer of the pair of wheels, i.e., of the axle, when the body rolls. For a left turn of a three wheeled vehicle, which is counter clock wise, the vehicle body rolls in clock wise direction. Hence using the sign convention of angle rotations, angular displacement of front wheel about steering axis towards left is taken as negative and roll angle of vehicle body is taken as positive and vice-versa.

The steer angle directly affects handling as it alters the angle of the wheels with respect to the direction of travel. Roll steer co-efficient 'ε' on an axle is defined as "Degree steer/Degree roll". With independent suspensions the roll steer co-efficient must be evaluated from the kinematics of the suspension system.

The camber is defined as the inclination of a wheel with respect to vertical. The camber change is function of suspension geometry, roll angle, and track width. On independent wheel suspension systems, camber can play an important role in cornering. The camber thrust caused by camber angle can have significant effect on handling behavior because of camber change during suspension movement which will be different for front wheel and for inward and outward wheels of the rear suspension in three wheeled vehicle.

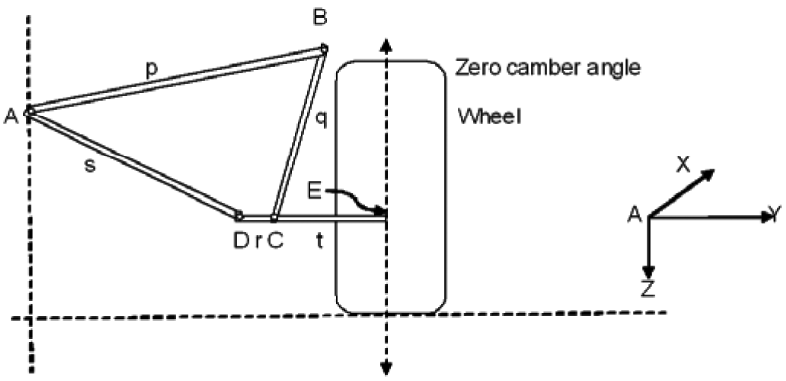
APPENDIX 2 (CONT.)

Rear Suspension of the Vehicle

A typical rear suspension of three wheeled vehicle is shown in Figure 5. For kinematic analysis the following assumptions are made.

1. Suspension links are mass less and rigid.
2. All links are treated as vectors.
3. Link AB is fixed with vehicle body.
4. All links are not coplanar.

Figure A5: Linkages Diagram of Rear Suspension of Three Wheeled Vehicle



Taking origin at point A, X-axis is along the longitudinal direction of vehicle body, Y-axis is along the lateral direction of vehicle body, and Z-axis is vertically downward. The axis system is taken so that *Y* and *Z* axes do not roll but remain parallel and perpendicular to the road surface respectively. p,q r & s are lengths of links AB, BC, CD, & DA respectively.

Link AD can be written as,

$$\vec{AD} = (s\cos\alpha_s)\tilde{i} + (s\cos\beta_s)\tilde{j} + (s\cos\gamma_s)\tilde{k} \quad ...(30)$$

α_s, β_s and γ_s are direction cosines of link AD with positive X,Y and Z axis respectively.

So, coordinate of point 'D' before roll of vehicle body will be,

$$D \equiv [(s\cos\alpha_s),(s\cos\beta_s),(s\cos\gamma_s)] \quad ...(31)$$

Assuming an imaginary link AC of length 'u' and this can be written in vector form as,

$$\vec{AC} = (u\cos\alpha_u)\tilde{i} + (u\cos\beta_u)\tilde{j} + (u\cos\gamma_u)\tilde{k} \quad ...(32)$$

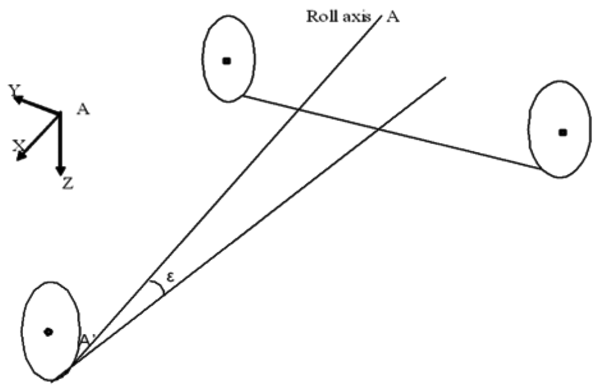
APPENDIX 2 (CONT.)

α_u, β_u and γ_u are direction cosines of imaginary link AC with positive X,Y and Z axis respectively and, coordinate of point 'C' before roll of vehicle body will be,

$$C \equiv [(u\cos\alpha_u),(u\cos\beta_u),(u\cos\gamma_u)] \quad \dots (33)$$

Let, vehicle body rolls with small angle 'Φ' about roll axis AA

Figure A6: Vehicle Roll About Roll Axis AA'



Direction cosines of AA' are given by: ($\cos\varepsilon$, 0, $\sin\varepsilon$)

So, transformation matrix due to rotation 'Φ' about roll axis AA' is given by, Bahar L Y et al [1].

$$R = \begin{bmatrix} \cos\Phi + (\cos\varepsilon)^2 - \cos\Phi(\cos\varepsilon)^2 & -\sin\varepsilon\sin\Phi & \sin\varepsilon\cos\varepsilon - \sin\varepsilon\cos\varepsilon\cos\Phi \\ \sin\varepsilon\sin\Phi & \cos\Phi & -\cos\varepsilon\sin\Phi \\ \sin\varepsilon\cos\varepsilon - \sin\varepsilon\cos\varepsilon\cos\Phi & \cos\varepsilon\sin\Phi & \cos\Phi + (\sin\varepsilon)^2 - (\sin\varepsilon)^2\cos\Phi \end{bmatrix} \quad \text{...(34)}$$

After rotation 'Φ' about roll axis AA', point D will change to D' so using transformation matrix, co-ordinate of point (D') after roll of vehicle body is given by,

$$\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = [R].\begin{bmatrix} s\cos\alpha_s \\ s\cos\beta_s \\ s\,c\cos\gamma_s \end{bmatrix}$$

After rotation 'Φ' about roll axis AA', point C will change to C' so using transformation matrix, co-ordinate of point (C') after roll of vehicle body is given by,

APPENDIX 2 (CONT.)

$$\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = [R].\begin{bmatrix} u\cos\alpha_u \\ u\cos\beta_u \\ u\cos\gamma_u \end{bmatrix}$$

Point 'E' divides the link CDE exterior in the ratio $\frac{r+t}{t}$,

So, coordinate of point 'E' before roll of vehicle will be given as,

$$E = \left[\left\{ \frac{(r+t).(u\cos\alpha_u) - t.(s\cos\alpha_s)}{r} \right\}, \right.$$

$$\left. \left\{ \frac{(r+t).(u\cos\beta_u) - t.(s\cos\beta_s)}{r} \right\}, \left\{ \frac{(r+t)(u\cos\gamma_u) - t(s\cos\gamma_s)}{r} \right\} \right] \quad ...(35)$$

After rotation 'Φ' about roll axis AA', point E will change to E', similarly co-ordinate of point (E') after roll of vehicle body is given by,

$$E' = \left[\left\{ \frac{(r+t).(x_2) - t.(x_1)}{r} \right\}, \left\{ \frac{(r+t).(y_2) - t.(y_1)}{r} \right\}, \left\{ \frac{(r+t)(z_2) - t.(z_1)}{r} \right\} \right] \quad ...(36)$$

Hence, roll steer angle δ_{sr} for rear suspension is given by,

Using x-coordinate of point D' and E'

$$\delta_{sr} = \left. \left\{ \frac{(r+t).(x_2) - t.(x_1)}{r} \right\} - \{x_1\} \middle/ (r+t) \right. \quad ...(37)$$

now,

Roll steer coefficient for rear suspension is given by,

$$\varepsilon_r = \frac{\delta_{sr}}{\Phi} \quad ...(38)$$

Camber angle γ_r for rear suspension will be given by,

Using z-coordinate of point D' and E'

APPENDIX 2 (CONT.)

$$\varepsilon_r = \frac{\delta_{sr}}{\Phi} \qquad \text{...(39)}$$

Equation (39) indicates how camber change depends upon suspension geometry. So it is a function of roll angle because the jounce on inside wheel and rebounce on the outside wheel relate directly to roll angle.

So,

$\frac{\partial \gamma_r}{\partial \Phi} = f_{rr}$ (suspension geometry, track width, roll angle)

now,

$$F_y = c_\alpha.\alpha + c_\gamma.\gamma \qquad \text{... (40)}$$

Using Equation (40), we can write slip angle in terms of lateral acceleration, (we know that, $F_y = \frac{wl_1}{L}.a_y$) by Gillespie T D [3].

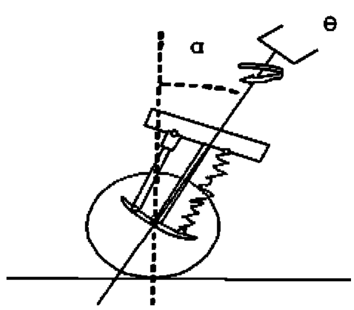
$$\alpha_r = \frac{\frac{Wl_1}{L}}{c_{\alpha r}} a_y - \frac{c_{\gamma r}}{c_{\alpha r}}.\frac{\partial \gamma_r}{\partial \Phi}\frac{\partial \Phi}{\partial a_y}.a_y \qquad \text{...(41)}$$

Front Suspension of the Vehicle

Assumptions

1. All links are mass less and rigid
2. Effect of flexibility of rubber bushes, used in suspension links are neglected.
3. The road is considered to be rigid and plane surface.

Figure A7: Front Suspension of Three Wheeled Vehicle



APPENDIX 2 (CONT.)

So,

Camber angle $\gamma_f = \Phi - \theta.\sin\alpha$ ...(42)

and,

Steer angle $\delta_{sf} = \theta.\cos\alpha$ ...(43)

now,

Roll steer coefficient for front suspension is given by,

$$\varepsilon_f = \frac{\delta_{sf}}{\Phi} \qquad ...(44)$$

Hence,

$$\frac{\partial \gamma_f}{\partial \Phi} = f_{\gamma f} \text{ (steering geometry)}$$

So, Using equation (40), we can write slip angle for front part of vehicle in terms of lateral acceleration, (we know that, $F_y = \frac{Wl_2}{L}.a_y$) by Gillespie T D [3].

$$\alpha_f = \frac{\frac{Wl_2}{L}}{c_{\alpha f}} a_y - \frac{c_{\gamma f}}{c_{\alpha f}}.\frac{\partial \gamma_f}{\partial \Phi}\frac{\partial \Phi}{\partial a_y}.a_y \qquad ...(45)$$

So, under steer gradient due to roll steer using Equations (9), (38), and (44) is given by,

$$K_{roll\ steer} = \left(\varepsilon_f - \varepsilon_r\right).\frac{\partial \Phi}{\partial a_y}$$

$$K_{roll\ steer} = \left(\frac{\delta_{sf}}{\Phi} - \frac{\delta_{sr}}{\Phi}\right)\left(\frac{Wh_1.\cos\varepsilon + k_{\Phi f}\dfrac{\frac{Wl_1}{L}.h_1}{k_{\Phi r} - w_r.H_r} + k_{\Phi f}\dfrac{\frac{Wl_2}{L}.h_1}{w_f H_f + k_{\Phi f}}}{k_{\Phi f} + k_{\Phi r} - Wh_1.\cos\varepsilon}\right) \qquad ...(46)$$

Value of δ_{sf}, δ_{sr} and Φ can be obtained from Equations (43), (37) and (8)

APPENDIX 2 (CONT.)

now,

Steering angle is given by,

$$\delta = 57.3\frac{L}{R} + \alpha_f - \alpha_r$$

Using Equations (41) and (45), we get

$$= 57.3\frac{L}{R} + \left[\left(\frac{\frac{Wl_2}{L}}{c_{\alpha f}} - \frac{\frac{Wl_1}{L}}{c_{\alpha r}}\right) + \left(\frac{c_{\gamma f}}{c_{\alpha f}}.\frac{\partial \gamma_f}{\partial \Phi}\frac{\partial \Phi}{\partial a_y} - \frac{c_{\gamma r}}{c_{\alpha r}}.\frac{\partial \gamma_r}{\partial \Phi}\frac{\partial \Phi}{\partial a_y}\right)\right].a_y \quad ...(47)$$

Hence,

Under steer coefficient due to camber change is given by,

$$K_{us3} = \frac{c_{\gamma f}}{c_{\alpha f}}.\frac{\partial \gamma_f}{\partial \Phi}\frac{\partial \Phi}{\partial a_y} - \frac{c_{\gamma r}}{c_{\alpha r}}.\frac{\partial \gamma_r}{\partial \Phi}\frac{\partial \Phi}{\partial a_y} \quad ...(48)$$

Overall understeer coefficient is given by, using Equations (28), (29), (46) and (48)

$$K_{us} = \left(\frac{\frac{Wl_2}{L}}{c_{\alpha f}} - \frac{\frac{Wl_1}{L}}{c_{\alpha r}}\right) - \left(\frac{\frac{Wl_1}{L}}{c_{\alpha r}}.2.b\frac{\Delta F_{zr}^2}{c_{\alpha r}}\right) + \frac{c_{\gamma f}}{c_{\alpha f}}.\frac{\partial \gamma_f}{\partial \Phi}\frac{\partial \Phi}{\partial a_y} - \frac{c_{\gamma r}}{c_{\alpha r}}.\frac{\partial \gamma_r}{\partial \Phi}\frac{\partial \Phi}{\partial a_y} + \left(\frac{\delta_{sf}}{\Phi} - \frac{\delta_{sr}}{\Phi}\right)\frac{\partial \Phi}{\partial a_y} \quad ...(49)$$