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Research Paper

HUMAN INDUCED VIBRATION IN STRUCTURES

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Several structures are subjected to human loading, for example, floors, footbridges, stadium, etc. In fact, the aesthetic demand of human beings and recent advances in material and fabrication technologies have enabled the design and construction of stylish, light and slender long span structures such as bridges,stadiums, floors, etc. Consequently the modern structures have become flexible and prone to human induced vibrations. For example, recently built bridges and floors have shown to be sensitive to human induced vibration. Although there have been more cases of footbridges experiencing excessive vibration by pedestrians in the past, this problem attracted considerably greater public and professional attention only after the infamous swaying of the London Millennium Footbridge located across the Thames River in central London in 2000 at its opening day. The structure was closed and retrofitted with viscous and tuned mass dampers. Human apply static and dynamic loads on structures due to the various functions they perform, i.e., sitting, walking, running, jumping, bouncing, etc. These human activities on structures cause vibration in structures and once structure starts vibrating beyond certain limit, it results a serviceability problems. Passive humans (such as humans sitting or standing on the structure) influence the dynamic properties (mass, stiffness and damping) and modal characteristics of the structure carrying them and active humans (such as humans walking, jumping, bouncing or other rhythmic activities performs on structure) can bring the structure into vibration. Excessive vibrations may occur if the motion frequency of human coincides with a resonant frequency of the structural system. Human-structure interaction is applicable to the design of structures. Human walking possesses adaptive and feedback nature, inducing motion dependent human walking forces on structures. The excessive vibration caused by humans need to be mitigated and bring within acceptable limits. Moreover, passive and active dampers provide a reliable solution. However, a proper type of damper selection and design is a crucial part in the vibration mitigation of structures. This paper presents the structural problems of human induced vibrations, formulation of human loading on structures, and some proposed models for human-structure interaction along with some case studies. Moreover, some recent solutions of structural vibration mitigation applying dampers are discussed in this paper.

Keywords: Human loading, Structural vibration, Human-structure interaction, Vibration mitigation, Dampers

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INTRODUCTION

Several structures such as theatres, concert halls, dance floors, meeting rooms and footbridge, are designed to hold large numbers of human occupants. The aesthetic demands of humanbeings and recent

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advances in material and fabrication technologies have enabled the design and construction of stylish, light and slender long span structures. Consequently, the modern structures have become flexible and prone to human induced vibrations. Hence, vibration serviceability issues have arisen whereby the structures may be excited by occupying humans to discomforting or disturbing levels of vibration. For example, recently built structures have shown to be sensitive to human induced vibration. Correct design of structure has to face two different issues: the dynamic load induced by the motion of the occupants and the modification of the modal dynamic parameters of the structure due to the presence of the crowd. Those aspects are grouped in the so- called "human–structure interaction" (Sachse *et al.*, 2003). Of all human induced dynamic loadings on structures, jumping is generally considered the most severe (Racic and Pavic, 2009). The modern trend in structure design to realize more slender structures results in an increase of stands flexibility, i.e., in lowernatural frequencies of the structures, approaching the typical range of induced loads. The coupling of these two aspects can lead to dangerous vibrations of the stands, both in terms of people's comfort and structure serviceability.

Although there have been several cases of footbridges experiencing excessive vibration by pedestrians in the past, this problem attracted considerably greater public and professionalattention only after the infamous swaying of the Londonmillennium footbridge located across the Thames river in central London. The millennium bridge problem attracted more than 1000 press articles and over 150 broadcasts in the media around the world.

EARLY CASES OF EXCESSIVE VIBRATIONS IN STRUCTURE

Probably the oldest report of noticeable vibrations in footbridges was made by Stevenson (1821). In addition to this, he reported severe vibrations due to a marching regiment crossing over a bridge, indicating very early a need to considerhuman-induced dynamic loads in bridge design. It is interesting that 10 years after Stevenson's observations, as previously mentioned, a bridge collapse in Broughton was caused by marching soldiers. Tilden (1913) wrote an excellent article for that time primarily devoted to the crowd load. However, he also reported some experiments in which, although not having precise measurement devices, he tried to quantify the dynamic effect of a force generated by a single person due to different activities.

One of the earliest reported incidents of excessive lateral vibrations induced by crowds are dated back to the late 1950s, one involving a road/railway bridge in China (the Wuhan Yangtze Bridge) in 1957 (Sun and Yuan, 2008) and another one involving a pedestrian suspension bridge in Kiev following its opening in 1958 (Blekherman, 2005). In fact, several large road bridges around the world have suffered from this problem during exceptional crowd events, such as opening day events (Wolmuth and Surtees, 2003), public demonstrations or festive events (Dallard *et al.*, 2001a and 2001b; Hurriyet Daily News & Economic Review). Even the 120 year old Brooklyn Bridge in New York City swayed remarkably when traversed by crowds of pedestrians during a power blackout, leading to several complaints from concerned citizens (Point of Collapse and Ye *et al.*, 2005). The first assessment of lateral crowd induced excitation was offered by Petersen (1972) (as reviewed by Bachmann and Ammann (1987), who observed strong lateral vibrations of a steel arch footbridge at Erlach in Germany, during crossing of about 300-400 pedestrians. The vibrations occurred on the 110 m long main span of the bridge at frequency around 1.1 Hz and were explained as a consequence of a lateral sway of the centre of gravity of the human body occurring at half the pacing frequency, resulting in resonant vibrations and a synchronisation of the step with the oscillation of the bridge (Bachmann, 1992). In this particular case, the vibration problem was solved by installing a horizontally acting Tuned Mass Damper (TMD). One of the most cited incidents of excessive lateral vibrations occurring in the last century is related to the Toda Park Bridge in Toda City, Japan (T-Bridge) (Fujino *et al.*, 1993; and Nakamura and Fujino, 2002). The bridge is a cable-stayed bridge with the overall length of 179 m divided into a main span (134 m) and a side span (45 m). The frequency of the fundamental vibration mode was reported in the range 0.9–1.0 Hz depending on the level of congestion.

PARIS PONT DE SOLFERINO AND LONDON MILLENNIUM BRIDGE

On December 15, 1999, the pont de Solferino footbridge (now called Passarelle Leopold-Sedar-Senghor), a 140 m long steelarch footbridge across the Seine in Paris (Figure 1), was opened to the public for crossing. On the opening day unexpected lateral oscillations were observed and the bridge was subsequentlyclosed to the public. A comprehensive test program was undertaken which involved modal testing of the structure, pedestrian crowd tests and installation of 14 TMDs followed by vibration testing and monitoring of the bridge. In November 2000, the bridge was reopened after almost a year of closure.

Figure 1: Pont de Solferino in Paris

The London Millennium Bridge (Figure 2), which connects St. Paul's Cathedral with the Tate Modern Gallery is the first entirely new bridge across the Thames in London since Tower Bridge was completed in 1894 [18]. The bridge is a shallow suspension bridge in three spans; a south span of 108 m, a central span of 144 m and a north span of 81 m. The bridge deck consists of aluminium box sections creating a very light superstructure (2 t/m) (Dallard *et al.*, 2001). On June 10, 2000, between 80,000 and 100,000 people gathered to cross the bridge on its opening day, with up to 2000 people on the deck at any one time. Large amplitude vibrations in four different vibration modes were reported; on the Southern span at a frequency of around0.8 Hz, at the central span in the first and second

Figure 2: The London Millennium Bridge

lateral vibration modes at 0.48 Hz and 0.95 Hz, respectively, and more rarely on the Northern span at a frequency of around 1 Hz. The amplitude of vibration was between 50 mm to 70 mm. On June12, 2000, it was decided to close the bridge while a retrofit solution could be developed and implemented. During the next18 months an extensive test program, similar to that in Paris was undertaken. One of the main observations made on theopening day was that the bridge exhibited an instability-type behavior. The bridge vibrated excessively when congested by alarge crowd of people, but if the number of pedestrians was reduced or if they stopped walking, the bridge vibration wouldreduce substantially (Dallard *et al.*, 2001; and Fitzpatrick *et al.*, 2001).

HUMANS AS VIBRATION SOURCE FOR STRUCTURES

During walking, a pedestrian produces a dynamic time varying force which has components in all three directions: vertical, horizontal-lateral and horizontal-longitudinal (Bachmann and Ammann, 1987). This single pedestrian walking force, which is due to accelerating and decelerating of the mass of their body, has been studied for many years. In particular, the vertical component of the force has been most investigated. It is regarded as the most important of the three forces because it has the highest magnitude. Other types of human-induced forces important for footbridges are due to running and some forms of deliberate vandal loading (jumping, bouncing or horizontal body swaying). Some of these types of human-induced forces have been studied not only for a single person, but also for small groups of people.

In walking, foot has continuous contact with the ground with frequency range 1.6 to 2.4 Hz. In Running, foot hasdiscontinuous contact with frequency range 2 to 3.5.

Conventionally, for normal walking (unhampered), frequency may be described by a Gaussian distribution with 2 Hz average and about 0.20 Hz standard deviation (from 0.175 to 0.22). Recent studies and conclusions drawn from recent testing have revealed even lower mean frequencies, around 1.8 Hz-1.9 Hz. The periodic function *F*(*t*), may therefore be resolved into a Fourier series, that is a constant part increased by an infinite sum of harmonic forces. The sum of all unitary contributions of the terms of this sum returns the total effect of the periodic action (Setra Technical Guide, 2006).

$$F(t) = G_0 + G_1 sin2\pi f_m t + \sum_{i=2}^{n} G_i \sin\,(sin2\pi i f_m t - \varphi_i)$$

with

G_0: static force (pedestrian weight for the vertical component);

G_1: first harmonic amplitude;

G_i: i^{th} harmonic amplitude;

f_m: walking frequency;

ϕ_i: phase angle of the ithharmonic in relation to thefirst one;

n: number of harmonics taken into account.

The mean value of 700 N may be taken for G_0, weight of one pedestrian. At mean frequency, around 2 Hz (f_m = 2 Hz) for vertical action, the coefficient values of the Fourier decomposition of $F(t)$ are the following (limited to the first three terms, that is $n = 3$, the coefficients of the higher of the terms being less than 0.1 G_0):

$G_1 = 0.4\ G_0;\ G_2 = G_3 \approx 0.1\ G_0;$

$\phi_2 = \phi_3 \approx \pi/2$

By resolving the force into three components, that is, a vertical component and two horizontal components (one in the longitudinal directionof the displacement and one perpendicular to the transverse or lateral displacement), thefollowing values of such components may be selected for dimensioning (in practice limited to the first harmonic):

Vertical component of one-pedestrian load:

$$F_v(t) = G_0 + 0.4G_0 \sin(2\pi f_m t)$$

Transverse horizontal component of one-pedestrian load:

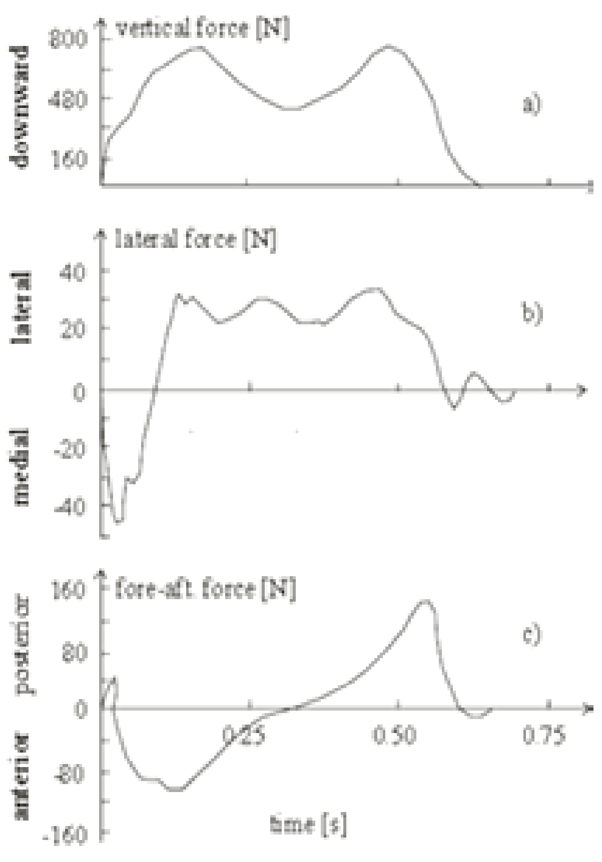
$$F_{ht}(t) = 0.05Gsin\left(2\pi \frac{f_m}{2}\right)t$$

Longitudinal horizontal component of one-pedestrian load:

$$F_{hl}(t) = 0.05G_0sin(2\pi f_m)t$$

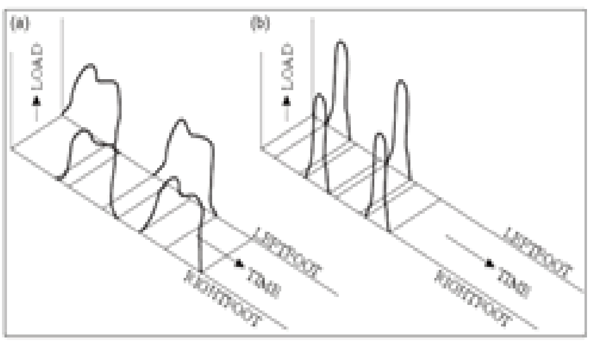
It should be noted that, for one same walk, the transverse load frequency is equal to half the frequency of the vertical and longitudinal load.

Figure 3: Typical Shapes of Walking Force in (a) Vertical, (b) Lateral and (c) Longitudinal Direction



Source: Andriacchi et al. (1977)

Figure 4: Typical Pattern of Walking (a) and Running (b) Forces



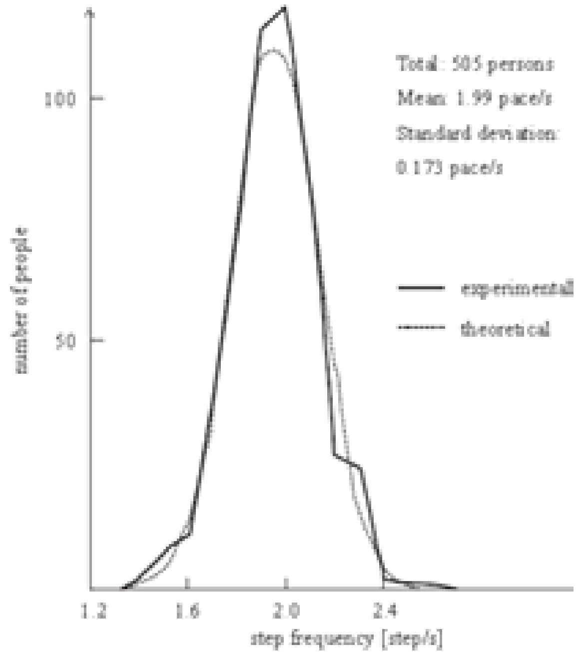
Source: Galbraith and Barton (1970)

Walking is generally characterized as regular predominantlyhorizontal human body motion where by at least one foot is in contact with the ground at all times, in the general frequency range of 1.4-2.2 Hz (Sahnaci and Kasperski, 2005). In addition to vertical loads generated by the raising and lowering of the body centre of mass, horizontal loads arise from friction and weight-shifting. Though these horizontal loads are rarely significant individually, on a large scale (e.g., crowds) or for an unbraced structure they could prove critical. Figure 4a illustrates the general form of walking loads in the time domain.

Walking loads are usually characterized by two distinct peaks, heel-strike and toe-off, with a plateau between where the foot is 'rolled' across the surface being walked on. As the walking rate increases these peaks move closer together until theyeffectively merge and only one part of the foot makes solid contact with the surface. Running is a similar activity to walking, but performed at a faster rate which results in brief periods where neither foot is in contact with the floor. Due to the haste of the activity, the load histories for running do not necessarily show the plateau between impact and take-off that represents the load spreading or rolling of the feet found in walking. Instead, running may be modeled as a series of pulses featuring single distinct peaks, as shown in Figure 3b. Walking and running loads are usually only critical on the most light weight and flexible of structures such as footbridges.

A reliable statistical description of normal walking frequencies was first given by Matsumoto *et al.* (1972 and 1978) who investigated a sample of 505 persons. They concluded that the frequencies followed a normal distribution with a mean pacing rate of 2.0. Hz and standard deviation of 0.173 Hz.

Figure 5: Normal Distribution of Pacing Frequencies for Normal Walking



FORCE MODELLING

To successfully apply the measured dynamic forces in design it is necessary to model them analytically. Two types of suchmodels can be found in the literature. One is time-domainmodels and other is frequency domain models. Time-domain models are more common. However, mathematical modelling of human-induced dynamic forces is a complicated task because:

- There are many different types of human-induced forces and some of them change not only in time but also in space (e.g., walking and running);
- Dynamic force generated by a single person is essentially a narrow-band process which is not well understood and therefore difficult to mathematically model;

- The influence of the number of persons as well as their degree of synchronization/ correlation is difficult to generalize.

However, force models do exist and are used in contemporary design. They are based on some more or less justifiable assumptions which will be presented.

TIME-DOMAIN FORCE MODELS

Generally, two types of time-domain models have been found in the literature: deterministic and probabilistic. The first type intends to establish one general force model for each type of human activity, while the other takes into account the fact thatsome parameters which influence human force, such as the previously mentioned activity frequency, person's weight and so on are random variables whose statistical nature should be considered in terms of their probability distribution functions.

In any case, time-domain models for walking and running are based on an assumption that both human feet produce exactly the same force and that the force is periodic. The assumption of perfect repetition is also frequently used in modeling of vandal loading generated by a single person and small groups.

Deterministic force models: It is well-known that each periodic force $F_p(t)$ with a period T can be represented by a Fourier series (Bachmann *et al.*, 1995)

$$F_p(t) = G + \sum_{i=1}^{n} G\alpha_i \sin(2\pi i f_p t - \phi_i)$$

where

G is the person's weight (N);

α_i the Fourier's coefficientof the ithharmonic, i.e., Dynamic Load Factor (DLF);

f_p the activity rate (Hz);

ϕ_i the phase shift of the ith harmonic;

i theorder number of the harmonic;

n the total number of contributing harmonics.

Based on Fourier decomposition, many researchers have tried to quantify DLFs which are the basis for this most commonmodel of perfectly periodic human-induced force.

Probabilistic Force Models: A more detailed probabilistic approach to the walking force model is based on the fact that a person will never produce exactly the same force-time history during repeated experiments. In the case of two persons it is even more so (Saul *et al.*, 1985). For a single person force, which is still assumed to be periodic, randomness, can be taken into account by probability distributions of person's weight, pacing rate and so on. For several people, the probability distribution of time delay between people who perform a particular activity can be added. The main idea of this philosophy is to get a reliable estimate of the force from a group of people by combining forces from individuals. Naturally, for a reliable statistical description of human forces, a large database of measurements with a single person should be provided.

VIBRATION REDUCTIONTECHNIQUES FOR STRUCTURES

In order to suppress the human induced vibration in structures, the following damper systems are useful (Anusas *et al.*, 2007).

- Absorbing dampers;
- Mass dampers;
- Active tuned dampers;
- Structural or physical damping.

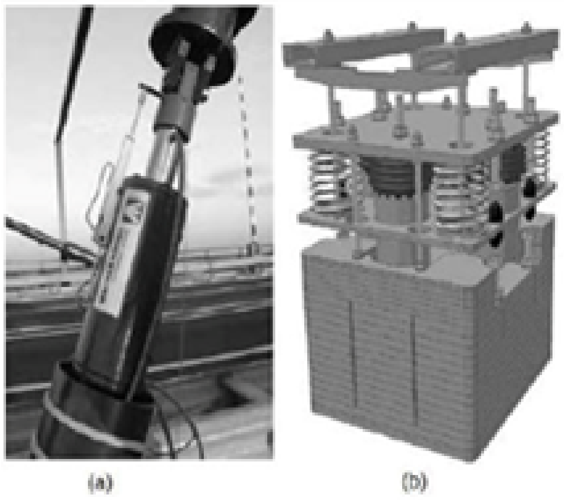
Absorbing Damper

Two types of dampers can be distinguished: mechanical dampers and hydraulic dampers. In mechanical dampers the structure's oscillation damping is achieved by springs or elastic pieces, in hydraulic dampers—y liquids, oil or gas. Hydraulic or viscous damper is made of hydraulic cylinder, surrounding a piston, the head of which forms two chambers. A relative motion applied to the actuator results in a pressure difference between the two chambers, creating a potential for flow and energy dissipation (see Figure 6a).

Tuned Mass Damper (TMD)

TMD is a device absorbing vibrations by vibration of the self- mass (see Figure 6b). A TMD consists of a mass-spring damper system which is attached to the structure and carefully tuned with the structure's vibration characteristics. When the counterweight of the damper moves in one direction as the structure moves in the other, the damping of the structure's oscillation is achieved. A limitation of the device, however, that it is only being effective over a narrow band of frequencies. However, TMD system might not be a good countermeasure for mitigating the buffeting response of a long-span bridge (Chang *et al.*, 2003).

Figure 6: (a) Absorbing Damper (b) Mass Damper



Active Tuned Damper

This type of vibration absorbers can suppress and tune the vibrations over a range of frequencies by incorporating a variable stiffness element that can be adjusted in real-time. Among others the Magneto-Rheological (MR) fluid dampers can be mentioned. MR fluids,which are typically, consist of micrometer-sized, magnetically polarisable particles in a water, oil or silicon. When a magnetic field is applied to the fluids, the fluids become semisolid and exhibit viscoplasticity adjusting their tuning to the dynamic parameters of the structure and help to reduce the structure vibration. The absorbers viscosity and elasticity are controlled by an external electric source. At present implementation of multiple absorbers activated when bridge motion exceeds admissible bounds is carried out.

Structural or Physical Damping

It consists of local or total structural modifications by changingmainly the stiffness or the mass of a structure. Such as local strengthening of members, introducing of additional beams in the cross-sections, installing a stiffer hand rail by truss or solidwall construction, intermediate piers, additional hangers, ties or cables can be mentioned. By these measures the natural frequencies could be changed, leading to frequency tuning of the bridge structures.

CONCLUSION

Human-structure interaction is an important aspect of human- induced vibrations. Human induced vibration in structure has serviceability problem rather than safety. Vibration induced in structure play a significant role in comfort of human present onit. It is found that the whole issue is very complex and under-researched. However, nowadays when dealing with vibration performance of footbridges generally consider three key aspects, the vibration source, path and receiver.

There are different types of human-induced forces acts on structures. Among them walking force due to a single pedestrian was established in the past as the most important load type because of its most frequent occurrence. Also, almostall existing force models for this type of load are developed with the assumption of perfect periodicity of the force and are based on force measurements conducted on rigid (i.e., high frequency) surfaces. Whereas, footbridges which exhibit vibration serviceability problems are low-frequency flexible structures with natural frequencies within the normal walking frequency range. So in such a situation, walking near to resonant frequency is expected to generate the highest level of response. However, the walking force is not perfectly periodic and it could be attenuated due to interaction between the structure and the pedestrian. These two facts required more attention in future force modeling. For a reliable estimate of vibration serviceability performanceofstructures, an appropriate modeling of its dynamic properties (mass, stiffness and damping) is very important. The reliable way to determine structural damping is to conduct the testing of the full-scale structure after it is built. As for evaluation of human-induced vibrations on it, i.e., their acceptability to human receiver, it is accepted that in the case of normal footbridges, the vibration level should be evaluated for a walking person.

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