Modeling of Facility Location Problem for Reverse Logistics System with Uncertainty in Forward and Reverse Flows of Products

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***Abstract*—Facility location problem has been given lot of attention to efficiently manage the flow of new products and returned products from customer to remanufacturing facilities. However, little effort has been paid to describe its comprehensive model with considering uncertainty in the flow of products from producers to customers and returned products from customers to remanufacturing facilities simultaneously. A mixed integer model for facility location problem with uncertainty in demand and returning products, safety stock levels for new and returning products considering facilities fixed cost, holding costs, and shipping costs simultaneously is presented here. Current research is significant to solve simultaneous facility location and inventory levels for returning products and new products for reverse logistics research with uncertainty in demand and in the returning products, in future.**

***Index Terms*—facility location, reverse logistics, uncertain flow of products**

I. Introduction

The reverse logistics system is defined as a process of planning, implementation and controlling the efficient, cost effective flow of raw materials, in-process inventory, finished goods and related information from the point of consumption to the point of origin for the purpose of recapturing value or proper disposal [1]. In reverse logistics systems returned products can be directly reusable, repaired, recycled and remanufacturd [2]-[4]. Current research is focused on the remanufacturing, in which a product is reformed into its original shape by several operations which can include disassembly, overhauling and replacement operation. The network problem for facility location for remanufacturing is significant and it has been addressed by several researchers [5]-[10]. Melo *et al*. [11] indicated in their review that little research has considered comprehensive models for reverse logistics research with both characteristics of forward and reverse flow of material. Furthermore, inventory management problem in reverse logistics is also significant because holding costs in the facilities are also important. However, the classical inventory inventory decisions are considered as independent of the facility location [12]. In literature, a facility location and inventory management problem has been considered separately and a little literature has studied them together [13]. Furthermore, the demand of products is uncertain in real enviroment and is considered by some researchers [12], [14]-[16]. However, unicertanity has been considered only in the demand of products but in reverse logistics systems, there is uncertainty both in the demand of products and in the returning products. Moreover, in order to satisfy the customer demands of products as well as the returned products, they are desired to be stored in the production and remanufacturing facilities for better service level. This motivates to study the facility location problem with uncertainty in demand and returning products, safety stock levels for new and returning products and their holding costs simultaneously.

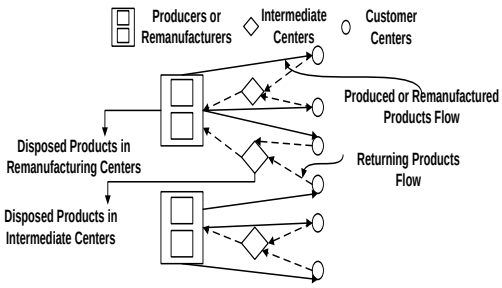

Figure 1. Reverse logistics system with facilities for remanufacturing

II. Facility Location Problem for Remanufacturing

In the remanufacturing system customer centers collects the used products from customers, intermediate centers clean, disassemble and check them and send to remanufacturing center. In remanufacturing centers the returned products are checked again and are

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remanufactured. The producers are used to made new products. There are two transportation roots called reverse flow forward flow. Fig. 1 illustrates the reverse logistics system considered in current study.

A. Notations Used

$C = \{1,2,...,i,....,n_C\}$ represents different locations of customer centers

$I = \{1,2,....,j,....,n_I\}$ indicates several intermediate center

$P = \{1,2,..,k,.....,n_R\}$ shows different producers or remanufacturing centers at location k

f_k^P indicates fixed cost to setup a producer center at location k

f_k^R indicates fixed cost to setup a remanufacturing center at location k

f_j^I indicates fixed cost to setup an intermediate center at location j

f_i^C indicates fixed cost to setup customer center at location i

$d_i \sim N(d_i^\mu, d_i^\sigma)$ represents normal distributed demand of the product at customer center i

$d_i^r \sim N(d_i^{\mu r}, d_i^{\sigma r})$ represents normal distributed returned products at customer center i

d_i^μ represents mean value of demand of the product at customer center i

d_i^σ represents variance value of demand of the product at customer center i

b_{im} represents the weight for the factor m for customer center i which causes variance in the demand

$V_i = \{v_{i1}, v_{i2}, ..., v_{iM}\}$ indicates set of different factors which may cause variance in demand at customer center i

$d_i^{\mu r}$ represents mean value of quantity of the returned product at customer center i

$d_i^{\sigma r}$ represents variance value of quantity of the returned product at customer center i

ed_i^μ represents estimated average value of demand of the product at customer center i

$ed_i^{\mu r}$ represents estimated average value of quantity of the returned product at customer center i

UC_k^P is the unit cost at producer center at location k

UC_k^R is the unit remanufacturing cost at remanufacturing center at location k

UC_j^I is the unit reprocessing cost at intermediate center at location j

SC_{ki} is the shipping cost from producer or remanufacturing center k to customer center i

SC_{ij} is the shipping cost from customer center i producer to intermediate center at location j

SC_{jk} is the shipping cost from intermediate center j to producer or remanufacturing center k

DC_j^I is the disposal cost at intermediate center j

DC_k^R is the disposal cost at remanufacturing center k

$YP_k = 1$ if the producer is located to site k and is zero, otherwise

$YR_k = 1$ if the remanufacturing center is located to site k and is zero, otherwise

$YI_j = 1$ if the intermediate center is located to site j and is zero, otherwise

$YC_i = 1$ if customer center is located to site i and is zero, otherwise

N_j^C is the maximum number of customers which are allowed to deliver their returned products to intermediate center j

N_k^I is the maximum number of intermediate centers which are allowed to deliver their returned products to remanufacturing or producer at location k

N_k^C is the maximum number of customers which are allowed to get delivery from products or remanufacturing center k

XP_{ik} is the fraction of demand from customer center i to remanufacturer or producer at location k

XR_{ijk} is the fraction of returned products at customer center i taken through intermediate center j to remanufacturer at location k

β is the percentage at which returned products can be disposed of at the intermediate centers (Suppose it is equal on all intermediate centers)

γ is the percentage at which checked returned products can be disposed of at the remanufacturing centers (Suppose it is equal on all remanufacturing centers)

S_k^P is the safety stock of the product at producer k

S_k^{rP} is the safety stock of the returned products at producer or remanufacturing center k

S_j^{rI} is the safety stock of the returned products at intermediate center i

$SCap_k^P$ is the storage capacity of products at producer or remanufacturing center k

$SCap_k^{rP}$ is the storage capacity of the returned products at producer k

δ_k^{rP} service level for the returned product at remanufacturing center or producer k

δ_k^P service level for the product at remanufacturing center or producer k

$SCap_j^{rI}$ is the storage capacity of the returned products at intermediate center j

H_k is the holding cost of returned and produced products at remanufacturing center or producer k

H_j^I is the holding cost of returned produced products at intermediate center j

III. Objectives

A. Fixed Cost.

Fixed cost is the sum of costs of producers, remanufacturers, intermediate centers and customer centers as shown below:

$$Fixed.cost = \sum_{k=1}^{n_R} f_k^P YP_k + \sum_{k=1}^{n_R} f_k^R YP_k + \sum_{j=1}^{n_I} f_j^I .YI_j + \sum_{i=1}^{n_C} f_i^C .YC_i$$

B. Shipping Cost.

Shipping cost is the summation of transporting the demand of products from remanufacturing center or producers to the required customer center, transportation cost of the returning products from different customer centers to the intermediate centers and transportation cost of the checked returning products from intermediate centers to the remanufacturing centers or producers.

$$Shipping.\cos t = \sum_{k=1}^{n_R}\sum_{i=1}^{n_C} SC_{ki}\left(ed_i^{\mu} + \sum_{m=1}^{M} b_{im} v_{im}\right) XP_{ik} \times \left(YC_i YP_k\right)$$

$$+ \sum_{k=1}^{n_R}\sum_{j=1}^{n_I}\sum_{i=1}^{n_C} SC_{ij}\left(ed_i^{\mu r} + \sum_{m=1}^{M} b_{im} v_{im}\right) XR_{ijk} \times \left(YC_i YI_j\right) + (1-\beta) SC_{jk}\left(ed_i^{\mu r} + \sum_{m=1}^{M} b_{im} v_{im}\right) XR_{ijk} \times \left(YI_j YP_k\right)$$

where, $d_i^{\mu} = \left(ed_i^{\mu} + \sum_{m=1}^{M} b_{im} v_{im}\right)$ represents mean demand of products at customer center i which is equal to the sum of the average value of estimated demand in certain customer center i and different factors causing change in demand.

C. Production Cost.

Total production cost is composed of the unit production cost times the whole demand of products, unit cost for processing returning products in the intermediate centers, disposal cost of the returned products in the intermediate center, unit cost of the returning products in the remanufacturing centers, disposal cost of the returning products in the remanufacturing centers.

$$Prod.cost = \sum_{k=1}^{n_R}\sum_{i=1}^{n_C} UC_k^P \times \left(ed_i^{\mu} + \sum_{m=1}^{M} b_{im} v_{im}\right) XP_{ik} \times \left(YC_i YP_k\right) +$$

$$\sum_{k=1}^{n_R}\sum_{j=1}^{n_I}\sum_{i=1}^{n_C}\left[\begin{array}{l}\left\{(1-\beta)UC_j^I + \beta.DC_j^I\right\} \times YI_j + \\ \left\{(1-\beta)(1-\gamma)UC_k^R + \gamma(1-\beta)\times DC_k^R - (1-\beta)(1-\gamma)UC_k^P\right\} \times YP_k\end{array}\right] \times \left(ed_i^{\mu r} \sum_{m=1}^{M} b_{im} v_{im}\right) XR_{ijk} YC_i$$

D. Holding Cost.

Holding cost is incurred due to inventory of the returned products in the remanufacturing centers and the inventory in the producer centers. It is the sum of total holding cost of the new products in the producers, holding cost of returning products in the producers, holding cost of returning products in the remanufacturing centers and holding cost of the returning products in the intermediate centers.

$$Holding.\cos t = \sum_{k=1}^{n_R} h_k \left[\max\left(0, S_k^P - \sum_{i=1}^{n_C} d_i \times XP_{ik} YP_k\right)\right] + \sum_{k=1}^{n_R}\left[\max\left(0, S_k^{rP} - \sum_{i=1}^{n_C}(1-\beta) d_i^r \times XR_{ijk} YP_k\right)\right] +$$

$$\sum_{k=1}^{n_R}\left[\max\left(0, S_k^{rR} - \sum_{i=1}^{n_C}(1-\beta) d_i^r \times XR_{ijk} YR_k\right)\right] + \sum_{j=1}^{n_I}\left[\max\left(0, S_j^{rI} - \sum_{i=1}^{n_C} d_i^r \times XR_{ijk} YI_j\right)\right] = \sum_{k=1}^{n_R} h_k \left[Z_k^P \sqrt{\sum_{i=1}^{n_C}\left(d_i^{\sigma}\right)^2 \times \left(XP_{ik} YP_k\right)^2}\right] +$$

$$\sum_{k=1}^{n_R} h_k \left[Z_k^{rP} \sqrt{\sum_{i=1}^{n_C}\left((1-\beta) d_i^{\sigma r}\right)^2 \times \left(XR_{ijk} YP_k\right)^2}\right] + \sum_{k=1}^{n_R} h_k \left[Z_k^{rR} \sqrt{\sum_{i=1}^{n_C}\left((1-\beta) d_i^{\sigma r}\right)^2 \times \left(XR_{ijk} YR_k\right)^2}\right] + \sum_{j=1}^{n_I} h_j \left[Z_j^{rI} \sqrt{\sum_{i=1}^{n_C}\left(d_i^{\sigma r}\right)^2 \times \left(XR_{ijk} YI_j\right)^2}\right]$$

IV. Constraints

The new products and returning products are required to fully meet their demand.

$$\sum_{k=1}^{n_R} XP_{ik} = 1,\ \forall i\ ;\ \sum_{k=1}^{n_R}\sum_{j=1}^{n_I} XR_{ijk} = 1,\ \forall i$$

The difference of the safety stock of new products with demand of new products is greater than the difference of the safety stock of returned products with the quantity of returning products in the remanufacturing centers or producers

$$\sum_{i=1}^{n_C} S_k^P - \left(ed_i^{\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) X_{ik} YC_i \geq S_k^{rP} - (1-\gamma)(1-\beta) \sum_{j=1}^{n_I}\sum_{i=1}^{n_C} \left(ed_i^{r\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) XR_{ijk} YC_i YI_j, \forall k$$

The differnce between the net quantities of the new products required with the net quantity of the returning products can give the quantity of products that are needed to produc if there s a producer at the position *k*. The following three constraints are significnat to link the location and allocation variables in which three different type of facilities are related to three different location vaiables and forward and backward flows are linked to the three location variables. i.e., customer centers, intermediate centers and producers or remanufacturing centers.

$$\left[\sum_{i=1}^{n_C} S_k^P - \left(ed_i^{\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) X_{ik} YC_i \right] - \left[S_k^{rP} - (1-\gamma)(1-\beta) \sum_{i=1}^{n_C}\sum_{j=1}^{n_I} \left(ed_i^{r\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) XR_{ijk} YC_i YI_j \right] \leq YP_k M, \forall k\ XR_{ijk} \leq YC_j, \forall i, j, k\ ;$$

$$(1-\beta) \left(ed_i^{r\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) XR_{ijk} \leq YR_k M^{'}, \forall i, j, k$$

The interation of constriants and non-negativelty constraints respectively are shown as under:

$$YP_k, YR_k, YI_j = 0,1\ \ \forall j, k;\ \ X_{ik}, XR_{ijk} \geq 0\ \ \forall i, j, k$$

There are some storage capacity constraints that can explain the storage capacity of returning products in intermediate centers, remanufacturing centers, and storage capacity constraint in the producers to store returning products and new products in it respectively.

$$\sum_{i=1}^{n_C} \left(ed_i^{r\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) XR_{ijk} \times YC_i \times YI_j + S_j^{rI} \leq SCap_j^{rI}, \forall j$$

$$\sum_{j=1}^{n_I} (1-\beta) \left(ed_i^{r\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) XR_{ijk} \times YI_j \times YR_k + S_k^{rP} \leq SCap_k^{rP}, \forall k$$

$$\sum_{j=1}^{n_I} \left((1-\beta) \left(ed_i^{r\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) XR_{ijk} \times YI_j \times YP_k \right) + S_k^{rP} + S_k^P - \sum_{j=1}^{n_I} \left(ed_i^{\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) X_{ik} \times YI_j \times YP_k \leq SCap_k^P, \forall k$$

First constraint from following constraints tells that at any demand of the product, there is a positive probability that the safety stock of new products is greater than the required demand. The second constraint tells that there is positive probability that the demand of products is greater than the sum of safety stock of the returned products in the producers or in the remanufacturing centers and the newly entered returned products. The third constraint explains that the safety stock of new products can have such a level that it can fulfill the remaining demand of products.

$$Prob_P \left\{ \left(\sum_{i=1}^{n_C} S_k^P - \left(ed_i^{\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) X_{ik} \right) \geq 0 \right\} \geq \delta_k^P, \forall k$$

$$Prob_{rP} \left\{ \left(\sum_{i=1}^{n_C} \left(ed_i^{\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) X_{ik} \right) - \left(S_k^{rP} + (1-\gamma)(1-\beta) \sum_{i=1}^{n_C}\sum_{j=1}^{n_I} \left(ed_i^{r\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) XR_{ijk} \right) \geq 0 \right\} \geq \delta_k^{rP}, \forall k$$

$$Prob_{rP} \left\{ \left(\sum_{i=1}^{n_C} \left(ed_i^{\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) X_{ik} \right) - \left(S_k^P + S_k^{rP} + (1-\gamma)(1-\beta) \sum_{i=1}^{n_C}\sum_{j=1}^{n_I} \left(ed_i^{r\mu} + \sum_{m=1}^{M} b_{im} v_{im} \right) XR_{ijk} \right) \geq 0 \right\} \geq \delta_k^{rP}, \forall k$$

Following constraints can satisfy the condition that each intermediate center can be assigned to some specific number of customer centers. Similarly, each remanufacturing center or producers is connected with some specific number of intermediate centers. Moreover, each producer or remanufacturing center is responsible to

deliver their products to some predefined number of customer centers.

$$\sum_{i=1}^{n_C} YC_i \times YI_j \le N_j^C \ , \ \forall j \ ; \ \sum_{j=1}^{n_I} YI_j \times YR_k \le N_k^I \ , \ \forall k \ ;$$

$$\sum_{j=1}^{n_I} YI_j \times YP_k \le N_k^I \ , \ \forall k \ ; \ \sum_{i=1}^{n_C} YR_k \times YC_i \le N_k^C \ , \ \forall k \ ;$$

$$\sum_{i=1}^{n_C} YP_k \times YC_i \le N_k^C \ , \forall k$$

V. Conclusion

Facility location problem in reverse logistics is significant and is therefore has got lot of attention. In literature a little research has described its comprehensive model with considering uncertainty in the flow of products from producers and returned products from customers. A mixed integer model for facility location problem with uncertainty in demand and returning products, safety stock levels for new and returning products considering the fixed cost, shipping cost and holding costs simultaneously is presented here. Current research is significant to solve simultaneous facility location and inventory levels for returning products and new products for reverse logistics research with uncertainty in demand and in the returning products simultaneously.

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