# Absolute Stability of Mechatronic Module with Intelligent Controller Based on Associative Memory

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Abstract—In this paper the criteria of the absolute stability of the equilibrium and of the processes for mechatronic module with the controller on the basis of associative memory are considered. Based on the VM Popov criterion and its modifications for systems with multiple nonlinearities, two criteria and one consequence are formulated. The convenient graph-analytical techniques, which characterize these criteria, are given. Furthermore, these criteria provide settings the AM-controller, to guarantee stability of the system with a minimum amount of memory required. The required quality of control is provided if adjust the controller according to the method inverse dynamic problem and the examples and simulation results are shown to verify the effectiveness of the proposed methods.

*Index Terms*—associative memory technology, controller on the basis of associative memory, absolute stability, the VM Popov criterion, criteria of absolute stability.

### I. INTRODUCTION

The associative memory technology is one of the alternative approaches to the creation of quick-response intelligent control systems and very widely used in computer engineering today. On the one hand, this technology is based on processes of associative record and recovery information to enable access to the data with high speed. Usually, this aspect of the application is studied in the field of computer engineering. On the other hand, the technology of associative memory (AM) allows to classify the state of the system at a qualitative level, on the basis of associative relations. And at the same time, the AM-technology form the control, corresponding to the current state of the system and the specified control criterions. This aspect of the application of AM poorly studied, therefore it is investigated in this paper. The main advantage of AM is the simplicity both software and hardware implementation, which provides fast response time, determined by the access time to a memory register.

Information about the parameters of the system and about the control actions in the AM represented discretely. This leads to the fact that such systems are becoming a specific class of automatic control systems with discretely varying parameters. Therefore it is necessary to develop new approaches to the study of the stability and system performance.

# II. PROBLEM FORMULATION

This paper discusses the criteria of the absolute stability of the equilibrium and of the processes for mechatronic module with the controller on the basis of associative memory (AM controller). Generalized block diagram the system under consideration shown in Fig. 1. Classification of the control object state (transfer function is  $W_{CO}(s)$ ) is carried out by the identifier. Control actions are formed by using the controller with variable parameters and the structure (the transfer function is  $W_c(s)$ ). Identifier and the controller implemented on an associative memory (AM). Nonlinearity  $N_1$  and  $N_2$ show a discrete character of transformations in the AM. This is due to a discrete change of the controller parameters and to the digital character of the control [1]-[4].



Figure 1. Generalized block diagram of the control system with AM – controller

In the process of designing such systems, because of discrete representation of included knowledge, the question of choosing the value of discrete changes of the elements of the vector of adjustable controller parameters rises. Thus it is necessary to solve the problem of minimizing the required amount of associative memory on the one hand, and the task of ensuring the stability of the control system on the other.

# III. THE DESCRIPTION OF THE OBJECT OF STUDY

Mechatronic module (MM), as a basic element of a multilink mechatronic device, such as a manipulation robot, is

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considered as an object. In such objects DC motors, in particular, are used as actuators.

Work [1] shows that the variable parameters of multilink mechatronic device applied load to the shaft of the actuator in each control channel are the moment of inertia and its first derivative, the time-varying. This assertion does not depend on the type of the kinematic scheme. In systems with AM, the controller parameters remain constant on the sampling interval, tuned to  $J_0$  and  $J'_0$  (fixed value of inertia moment and its rate of change), while, the actuator parameters are constantly changing depending on the kinematic scheme of multilink device and movement speed of the working body by the path. At the same time, inertia moment values are positive and cannot be equal to zero:

$$0 < J_{\min i} < J_i(t) + J_{0i} \le J_{\max i}$$
(1)

where  $J_{\min i} = \inf_{t} (J_{0i} + J_i(t)), J_{\max i} = \sup_{t} (J_{0i} + J_i(t))$  are

minimum and maximum possible values of the changing moment of inertia, applied load to the shaft of the *i*-th e-motor;  $J_{0i}$  is moment of inertia, applied load to the shaft of the *i*-th engine, for which the controller parameters are configured;  $J_{i}(t)$  is variable part of moment of inertia relative to  $J_{0i}$ , applied load to the shaft of the *i*-th engine.

The change rate of the inertia moment can take both positive and negative values in the range from its minimum possible to the maximum possible values

$$\dot{J}_{\min i} \le \dot{J}_i(t) \le \dot{J}_{\max i} \tag{2}$$

where  $\dot{J}_{\min i} = \inf_{t} (dJ_i(t)/dt), \\ \dot{J}_{\max i} = \sup_{t} (dJ_i(t)/dt)$  are

minimum and maximum possible values of the rate of change of inertia moment.

Based on the generalized scheme (Fig. 1), the block diagram of the system with the AM-controller and executive DC motor is shown in Figure 2. Here  $K_e$ ,  $K_m$  are coefficients of the rotating electromotive force and moment of motor,  $R_{aw}$  is active resistance of the armature winding,  $T_e$  is electrical time constant [1]. Such a system will be called mechatronic module with the AM-controller.

Nonlinear element  $N_1$  quantizes by the level the values of inertia moment, coming from the identifier. Nonlinear element  $N_2$  quantizes by the level the values of the rate of change of the moment. Thus, it shows the discrete character of knowledge representation in the AM.

The actuator model (from Fig. 2) is constructed on the basis of information about the rate of change of the values of the inertia moment J and of its inverse values  $J^{I}$ . This is inconvenient in the case of study, when the value of  $J_{min}$  is close to zero. Therefore, we construct another model taking into account the limitations (2). The dynamic equations of the actuators of multilink control object will take the form:

$$\begin{split} \frac{1}{2} \, \dot{J}_{\min i} \, \omega_i &= M_{id} + M_i - \frac{1}{2} \left( \frac{dJ_i(t)}{dt} - \dot{J}_{\min i} \right) \omega_i - J_i(t) \frac{d\omega_i}{dt} - J_{0i}(t) \frac{d\omega_i}{dt} \\ u_i &= k_{e\,i} \, \omega_i + R_{awi} I_i + L_{awi} \frac{dI_i}{dt} \,, \end{split}$$

$$M_{id} = k_{mi}I_i, \quad i = 1, 2, ..., n$$
 (3)

#### IV. CRITERIA FORMULATING

We formulate criteria for the absolute stability of the equilibrium and processes for mechatronic module with the AM-controller (Fig. 2).

Note, that the time-varying values of the inertia moment and its derivative can be represented as nonlinearities, because they are the coefficients of the equations of dynamics (3). However, these nonlinearity should be transformed to a class of nonlinear time-dependent functions  $\varphi(\sigma, t)$ , satisfying to the sector limitations [5, 6]. In this case it is possible to apply research methods of absolute stability.



Figure 2. Block diagram of the mechatronic module with the AMcontroller

 $0 \le \frac{\varphi(\sigma, t)}{\pi} \le K$ , if  $\sigma \ne 0$  and  $\varphi(0) = 0$ 

(4)

or

$$(K\sigma - \varphi(\sigma, t))\varphi(\sigma, t) \ge 0$$

After a series of transformations of the system of equations (3), we obtain the dynamics equations of a DC motor with separate excitation and with variable moment of inertia, on the motor shaft for each i-th actuator:

$$K_{J} \omega(t) = M_{d}(t) - M_{1}(t) - J_{\min} \dot{\omega}(t) - \varphi_{1}(\dot{\omega}(t), t) - \varphi_{2}(\omega(t), t),$$

$$M_{d}(t) = k_{m}I(t)$$

$$u(t) = k_{e} \omega(t) + R_{aw}I(t) + L_{aw} \frac{dI(t)}{dt}$$
(5)

where,  $\varphi_1(\dot{\omega}(t), t) = (J_0 - J_{\min} + J(t))\dot{\omega}(t)$  is timedependent nonlinearity, characterizes deviations of the inertia moment in the model of controller  $J_0$  and the real inertia

moment 
$$J_0 + J(t)$$
 ,  $K_J = \inf_t \left( \frac{dJ(t)}{2 dt} \right)$ 

$$p_2(\omega(t), t) = \left(\frac{dJ(t)}{2 dt} - K_J\right)\omega(t)$$
 is time-dependent

(

nonlinearity, characterizes the rate of change of the inertia moment (these nonlinearities satisfy the sector criterion [5]).  $M_{d}(t), M_{i} = -M_{i}(t)$  are moments of the engine and static load. To simplification notations, we omit the index *i*, but further will be considered.

Equivalent block diagram of a nonlinear control system constructed in accordance with the equations (5) is shown in Fig. 3. Here, the non-linear elements  $\varphi_1$  and  $\varphi_2$  characterize the changing values of the actuator inertia moment and it first derivative at constant parameters of the controller on the sampling interval of AM (nonlinear elements  $N_1$  and  $N_2$  in Fig. 2). In this case, the transfer function of the controller has the following form

$$W_{\rm c}(s) = \frac{k_{\rm i} \left(\frac{R_{\rm aw}}{k_{\rm m}} (J_0 \, s + 0.5 \, \dot{J}_0) (T_{\rm e} \, s + 1) + k_e\right)}{s} \quad (6)$$

By developing the VM Popov criterion and its modifications for systems with multiple nonlinearities [5], [7], two criteria can be formulated.



Figure 3. Equivalent block diagram of a mechatronic module AMcontroller

Criterion 1. Mechatronic module with AM-controller is asymptotically stable in the large, if some  $v_1$  and  $v_2$ satisfy to the following conditions:

$$1 + \mu_{1} \operatorname{Re}(j \, \omega W(j \, \omega)) - \mathcal{G}_{1} \, \mu_{1} \, \omega^{2} \operatorname{Re}W(j \, \omega) +$$

$$+ \mu_{2} \operatorname{Re}W(j \, \omega) + \mathcal{G}_{2} \, \mu_{2} \operatorname{Re}(j \, \omega W(j \, \omega)) \neq 0,$$

$$-\infty < \omega < +\infty, \qquad (7)$$

$$1 + \mathcal{G}_{1} \, \mu_{1} \operatorname{Re}(\lim_{s \to \infty} s^{2} W(s)) > 0,$$

$$1 = \mathcal{G}_{1} - \mathcal{D}_{2} (W(s)) = 0,$$

$$1 + \mathcal{G}_2 \ \mu_2 \operatorname{Re}(\lim_{s \to \infty} sW(s)) > 0,$$

 $\mathcal{G}_1 \operatorname{Re}(\lim_{s \to \infty} s^2 W(s)) > 0, \quad \mathcal{G}_2 \operatorname{Re}(\lim_{s \to \infty} s W(s)) > 0$ 

where

$$\mu_{1} = \sup_{t} (J_{0} - J_{\min} + J(t)) \ \mu_{2} = \sup_{t} \left( \frac{dJ(t)}{dt} - K_{J} \right),$$
$$W(s) = \left( K_{J} + J_{\min} \ s + \frac{W_{c}(s) \ k_{m} + k_{e} \ k_{m}}{R_{aw} \ (T_{e} \ s + 1)} \right)^{-1}$$

Criterion 2. The processes in mechatronic module with AM-controller are absolutely stable if the following condition is satisfied

$$1 + \mu_1 \operatorname{Re}(j \,\omega W(j \,\omega)) + \mu_2 \operatorname{Re}(W(j \,\omega)) \neq 0,$$
  
$$-\infty < \omega < +\infty, \qquad (8)$$

where  $W(j\omega)$  from (7) by substituting  $s = j\omega$ .

For mechatronic module with tuned controller by the method of inverse dynamic problem [8, 9, 10] and with type of control

$$u(t)T_{c} = \frac{J_{0}L_{aw}}{k_{e}k_{m}}(\dot{\omega}_{3}(t) - \dot{\omega}(t)) + \frac{J_{0}R_{aw}}{k_{e}k_{m}}(\omega_{3}(t) - \omega(t)) + + \int_{0}^{t} (\omega_{3}(t) - \omega(t)) dt$$
(9)

it is possible to formulate a Consequence 1 from Criterion 2.

Consequence 1. The processes in mechatronic module with AM-controller, that implements control (9), are absolutely stable if the condition

$$1 + \mu_1 \operatorname{Re}(j \,\omega W(j \,\omega)) + \mu_2 \operatorname{Re}(W(j \,\omega)) \neq 0$$
$$-\infty < \omega < +\infty \tag{10}$$

where

$$W(s) = \frac{s(T_3 s + 1)}{s^3 J_{\min} T_3 + s^2 \left( J_{\min} + \frac{J_0 T_3}{T_p} + K_J T_3 \right) + s \left( \frac{k_m k_e}{R_g} + \frac{J_0}{T_p} + K_J \right) + \frac{k_m k_e}{R_g T_p}}$$
  
when  $s = i \rho_0$ .

# **EXAMPLE AND SIMULATION RESULTS**

As an example, we investigate the absolute stability on the basis of Consequence 1.

Fig. 4 shows the graphics functions

$$F(\omega) = 1 + \mu_1 \operatorname{Re}(j \,\omega W(j \,\omega)) + \mu_2 \operatorname{Re}(W(j \,\omega)) + \omega_2 \operatorname{Re}(W($$

when  $\mu_1 = 0.2 \text{ kg } m^2$  and the following parameters: 
$$\begin{split} T_{\rm e} = 0,001{\rm s}; & k_e = 1\,{\rm V}\cdot{\rm s}/\,rad; & k_m = 1\,{\rm N}\cdot{\rm m}/{\rm A}; \\ R_{\rm aw} = 1\,{\rm Om}; & J_0 = 0,2\,{\rm kg}\ m^2; & J_{\rm min} = 0,1\,{\rm kg}\ m^2; \end{split}$$
 $T_{\rm c} = 0.03$  s. The graphics of functions  $F(\omega)$  were constructed for various values  $\mu_2$  by using software package MathCad, and the fulfillment of the criterion verified by the inequality to zero of these functions.

Fig. 4 shows, that when 
$$\mu_1 = 0.2 \text{ kg } m^2$$
,  
 $\mu_2 \ge 15 \text{ kg } m^2/\text{s}$  stability criterion not satisfied when the  
change of the inertia moment is in the range of 0, 1-0, 3 kg m<sup>2</sup> a  
speed higher than 15 kg  $m^2/\text{s}$ . Fig. 5a shows the transient in  
the researched system at  $\mu_2 = 0.2 \text{ kg } m^2/\text{s}$  K<sub>J</sub> = -0, 1 kg m<sup>2</sup>/s.  
The graphs (Fig. 5b) show that in case of satisfying the  
criterion, the transient is stable and monotonic. In the opposite  
case, when  $\mu_1 = 0.2 \text{ kg } m^2/\text{s}$ ,  $\mu_2 = 20 \text{ kg } m^2/\text{s}$ ,  
 $K_J = -10 \text{ kg m}^2/\text{s}$  at the output of the system there is the  
oscillatory process (Fig. 5. b).

Thus, when forming an associative memory, which is implemented on the basis of its identifier and controller, the level of quantization values of the inertia moment does not exceed 0.2 kg  $m^2$  at the rate of change, less 15 kg  $m^2/s$ .



Figure 4. Graphic interpretation of Criterion 2 for the considered example, when  $\mu_1 = 0.2$  and various values of  $\mu_2$ 



Figure 5. The transient  $\omega(t)$  (1) in the mechatronic module with AMcontroller in case of an input signal  $\omega(t)$  (2) and the changing of inertia moment J(t) (3) for  $\mu 1 = 0.2$ ,  $\mu 2 = 0.2$ , KJ = -0.1 (a) and for  $\mu 1 = 0.2$ ,  $\mu 2 = 20$ , KJ = -10 (b)

### VI. CONCLUSION

Discussed in this paper the criteria of absolute stability of mechatronic modules with AM-controller are precisely formulated and formalized to the convenient graph-analytical techniques. Furthermore, these criteria provide settings the AM-controller, to guarantee stability of the system with a minimum amount of memory required. In addition, the required quality of control is provided if adjust the controller according to the method inverse dynamic problem (*Consequence 1*).

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