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Research Paper

A STATIC APPROACH FOR DETERMINATION OF BENDING FORCE AND SPRING-BACK DURING PUNCHING PROCESS

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Bending is a common metal working process used in sheet-metal forming, such as parts of automobiles, aircraft and ships. In addition, bending is used in many sheet metal forming, such as deep drawing and stamping processes. Bending process is usually followed by some elastic recovery upon unloading, called spring-back. A static approach is described to determine new formulas for the bending force, the shift of neutral-axis and the value of spring-back, in V-die bending process. Due to friction force between the sheet-metal and the die edge, a variable neutral-axis shift is deduced in the elastic zone of the bending length. The amount of shift is, rapidly, decreased with the distance starting from the die edge, along the bending length. Based on the reflexive action in elastic zone, and the residual stresses, in elastic-plastic zone, the amount of spring-back is derived. As a case study, the obtained expressions are applied to compare the values of bending force and the spring-back in, previously, published experimental study on V-die bending conducted on two different die-angles 90° and 120°. The comparison shows that the values of bending force and the spring-back, obtained by the present work, are, in general slightly under-estimating.

Keywords: Statics, Punching, Spring-back, Neutral axis, Yield, Elasto-plastic, Residual stress

INTRODUCTION

Bending is the metal working process by which a straight length is transformed into a curved length. During the bending operation, the outer surface of the sheet metal is in tension and the inside surface is in compression. The strain in the bent material increases with decreasing the radius of curvature. Due to the friction force between the sheet and the die, the neutral axis moves toward the inner surface. Amitabha and Kumar (1995) stated that the position of neutral axis depends on the radius and angle of bend. The neutral axis shift s towards the center of curvature, usually a shift of 5-10% of the

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thickness is assumed for the calculation of strain and stock length. In the present study, an expression for determination of the location of neutral axis is derived considering the coefficient of friction.

In bending process the plastic deformation is followed by some elastic recovery upon unloading called spring-back. Spring-back depends on the modulus of elasticity, strength of material, sheet thickness, die radius and heat treatment of material. Precise prediction of sheet spring-back is very important in die design.

In bending production there are several technical problems, such as prediction of spring-back and punch load (bending force). In dealing with spring-back problem, three approaches have been commonly used: analytical methods, experimental methods and numerical methods. Purely theoretical studies or purely experimental analysis or both of them are encountered. Some researchers prepare information just for the sake of analytical analysis of solid mechanics, and some others use available finite element package programs like ABAQUS and ANSYS.

A survey of the previous research work on spring-back prediction and compensation in die manufacturing industry has been investigated by Alfaid and Xiaoxing (2009). Based on Hill's yielding criterion and plane strain condition, an analytical model for spring-back is proposed by Zhang *et al.* (2007), which takes into account the effects of contact pressure, the length of bending arm between the punch and die, transverse stress, neutral surface shifting and sheet thickness thinning on the sheet spring-back of V-die bending.

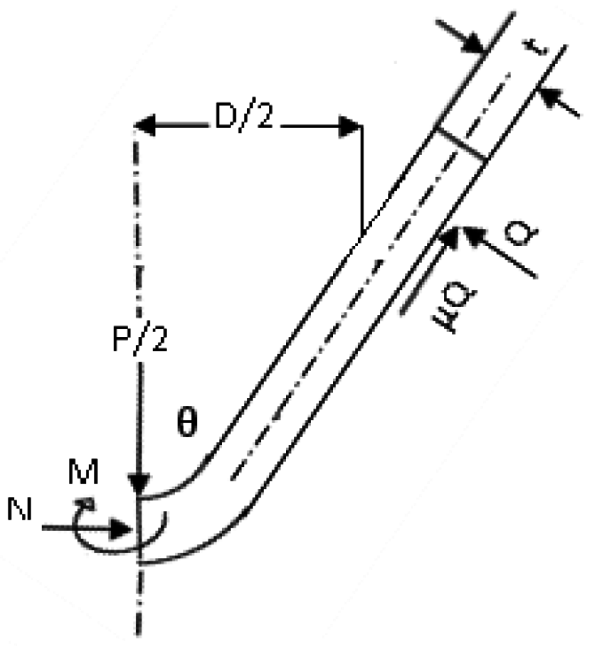
Florica *et al.* (2007) used Finite Element Method (FEM) to evaluate the spring-back, as well as the stress and strain state in the part before and after the spring-back by using ABAQUS Standard. The Stribeck friction model is investigated by Maziar and Zaidi (2009) to predict the spring-back behavior of AA6061-T4 sheets. The amount of spring-back is predicted by Grizelj *et al.* (2010) for a high strength steel plate using FEM for with two different tool geometries for V-die bending.

The spring-back and spring-go phenomena in a V-die bending process has been investigated by Thipprakmas (2010). He investigated, also, the effects of process parameters, including radius and height of the punch. Spring-back behavior was investigated at various temperatures ranging from RT to 300 °C and various rolling directions by performing experimental tests and ANSYS FEA software (Barouzeh and Mondali, 2011a and 2011b). The results indicate that the amount of spring-back decreases and formability increases with increasing the temperature. An algorithm for inverse spring-back modeling using bending theory and FE modeling is presented by Behrouzi *et al.* (2008). Some researchers have suggested the bending force based on bending moment. Many parameters such as material properties and die profile affect the bending moment. An analytical approach cannot correctly describe the amount of moment in the bending process. Thus, die designers use a simple equation to determine the bending force (Farsi and Behrooz, 2011). Thus, a simple and reasonable formula for bending force is suggested in the present work.

DETERMINATION OF BENDING FORCE

Considering the free body diagram of half the sheet metal, under punching, Figure 1, three equilibrium equations can be written:

Figure 1: Free Body Diagram of the Half Sheet Metal



$$N - Q\cos\theta + \mu Q\sin\theta = 0 \qquad ...(1)$$

$$Q\sin\theta + \mu Q\cos\theta - \frac{P}{2} = 0 \qquad ...(2)$$

$$M - Q\frac{D}{2}\csc\theta - \mu Q\Delta = 0 \qquad ...(3)$$

where $\Delta = (R + t) - (R - t)\csc\theta$

The equilibrium Equations (1)-(3), can be solved for N, Q and M in terms of bending force P, punch and die dimensions. The reactive force Q and the bending moment M are obtained as:

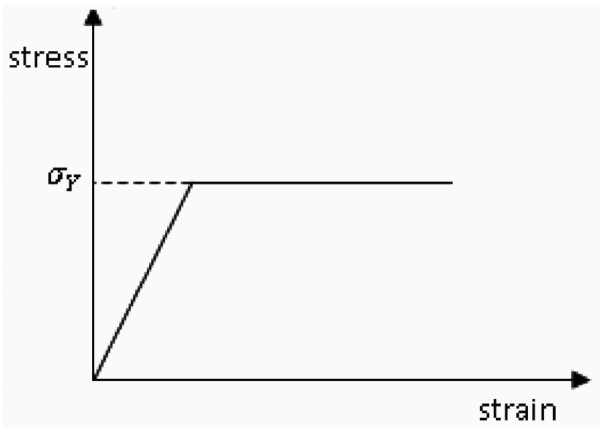
$$Q = \frac{P}{2\left(\sin\theta + \mu\right)\cos\theta}$$

$$M = \frac{PD}{2}\left[\frac{1}{1 + \mu\sin\varphi - \cos\varphi}\right] \qquad ...(4)$$

where $\varphi = 2\theta$ is the die angle.

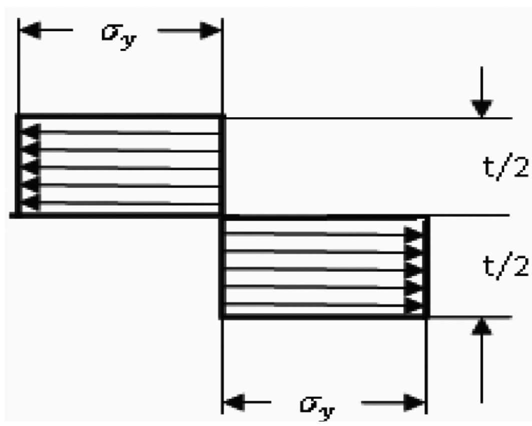
Frequently, the sheet material used for the experiments is of a low carbon type, for which the stress-strain diagram may be considered as elastic/perfect plastic, with yield stress σ_y, Figure 2.

Figure 2: Stress-Strain Diagram for an Elasto-Plastic Material



The minimum value of the bending force P is sufficient to make yielding of the sheet cross-section, under the punch tip. The stress

Figure 3: Stress Distribution at the Yielded Sheet Section



distribution along the sheet thickness of this section is shown in Figure 3 with compressive strain at upper half thickness and tensile stress at lower half.

The bending moment, at this section, is defined as:

$$M_u = \sigma_y wt^2/4 \quad ...(5)$$

where,

σ_y is the yield stress of the sheet material,

w is the sheet width.

t is the sheet thickness.

Introducing the expression of M_u Equation (5), into Equation (4), the minimum bending force is obtained as;

$$P = \frac{\sigma_y wt^2}{D}\left(\frac{2}{1+\mu\sin\varphi - \cos\varphi}\right) \quad ...(6)$$

The term in prances depends on the coefficient of friction and the punch angle φ. In published literatures, this term is considered as die opening factor, which equals 1.3 for rectangular section D.

Considering the bend length L, three different zones of elastic/plastic intermittent can be distinct, starting from position of the die reactive force Q (Figure 4);

Elastic zone, AB with length a, which can be determined from the following equation

$$a = \frac{\sigma_y wt^2}{6Q} \quad ...(7)$$

- Elastic/plastic zone, BC with length b which can be determined as:

$$b = \frac{M_u}{Q} - a \quad ...(8)$$

- Narrow plastic zone, CD which increases as the value of the punch force increases over than the minimum value given by Equation (6)

VARIATION OF NEUTRAL AXIS ALONG THE BENDING LENGTH

Consider a section in the sheet metal at a distance x from the die edge ($0 \le x \le a$) as shown in Figure 5, the normal force N and bending moment M are expressed as:

$$N = \mu Q$$

$$M = Qx + \mu Q\left(\frac{t}{2}\right) \quad ...(9)$$

The normal stress, σ caused by the above actions, N and M can be written as:

$$\sigma = \frac{N}{wt} + \frac{12M}{wt^2}y \quad ...(10)$$

Where $0 \le y \le \frac{t}{2}$

At the neutral axis, the value of the normal stress equals to zero ($\sigma = 0$). Substituting of N and M in Equation (9) into Equation (10) the shift of the neutral axis from the center line of the sheet y is obtained in the form:

$$\frac{y}{t} = \frac{\mu}{6\left[\mu + 2\frac{x}{t}\right]} \quad ...(11)$$

Instead of considering constant value of neutral axis shift ratio (5%-15%) to thickness t, estimated by different authors, Equation (11) indicates that the shift ratio of the neutral axis has its maximum value 16.7% at $x = 0$ and decreases as x increases. Variation of the neutral axis along the bending length in elastic zone for different coefficient of friction is shown in Figure 6.

Figure 4: Elastic/Plastic Intermittent

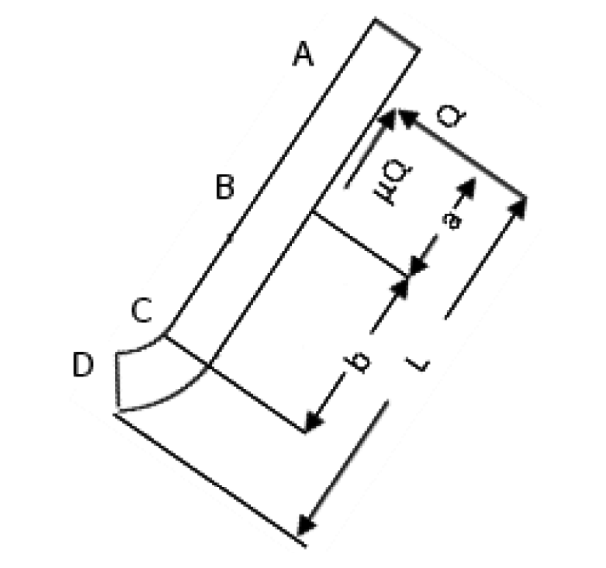


Figure 5: Location of Neutral Axis

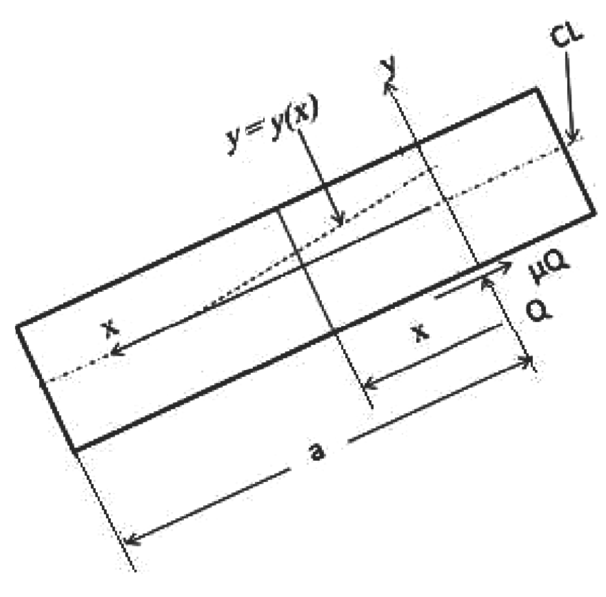
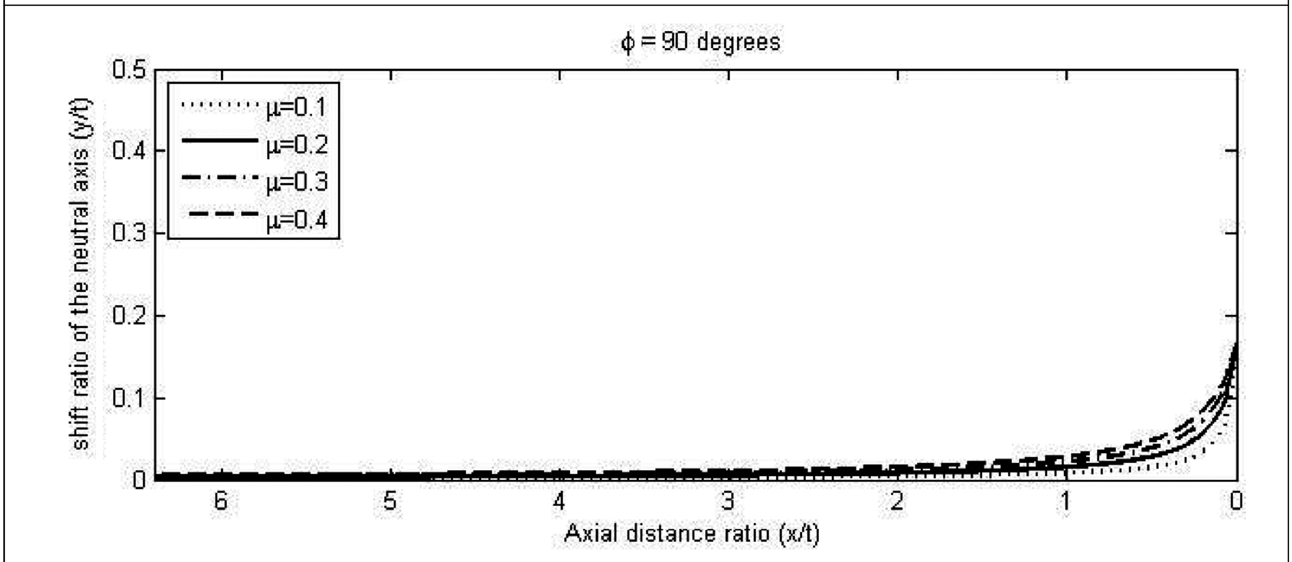


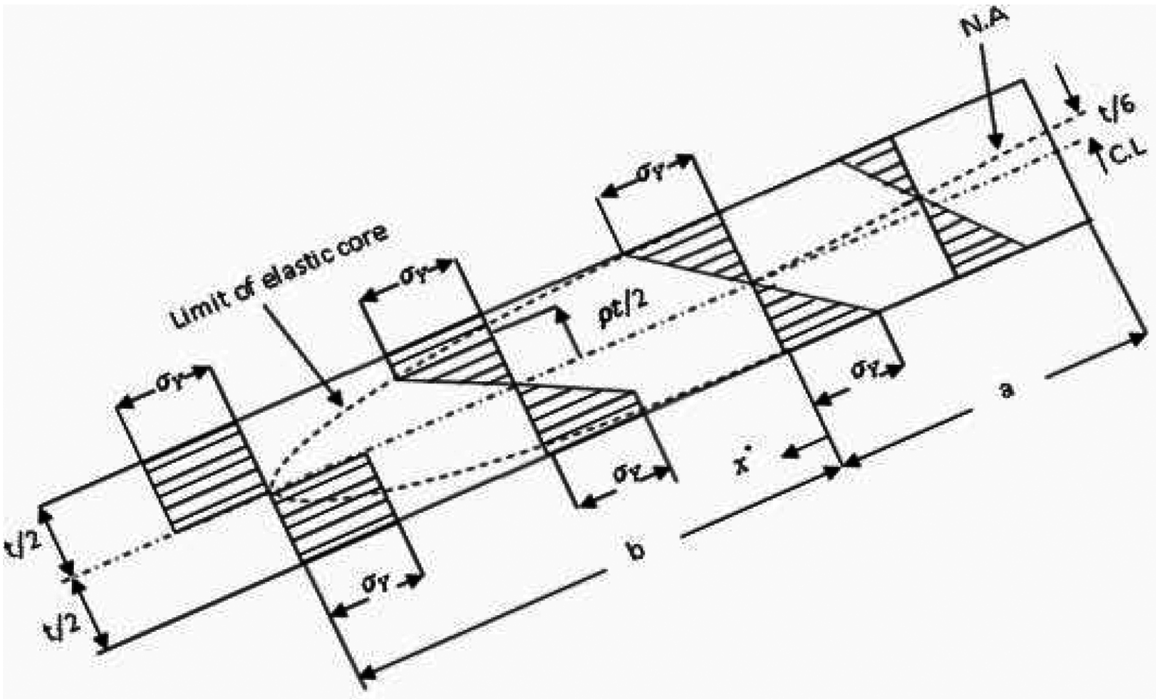
Figure 6: Variation of Neutral Axis, Along the Bending Length in the Elastic Zone ($0 \le x \le a$)



SPRING-BACK ANALYSIS

To determine the amount of spring back in a sheet bending process, the stress-distributions at different zones, shown in Figure 7, are considered, in detail during the punching process and after the punch release. In Figure 7, the stress distributions over different sections along the bending length during punching are detailed. In the elastic part a, all the stresses are elastic, with linear variation along the plate thickness. In the elastic-plastic part, *b*, an outer increasing plastic region is developed, due to the increase of the bending moment, while the core remains elastic. The

Figure 7: Stress-Distributions Over Different Sections of the Bending Length



variation of the elastic core along the sheet thickness is not linear. The bending moment at a section x^* may be expressed in terms of the limit elastic moment, M_y, as:

$$M(x^*) = \left(1 + \frac{x^*}{a}\right) M_y \quad ...(12)$$

where

$$M_y = \frac{\sigma_y w t^2}{6} \quad ...(13)$$

$$0 \leq x^* \leq b \quad ...(14)$$

The reacting bending moment caused by the shown stress distribution, Figure 7, can be deduced to be

$$M(x^*) = \frac{1}{4}\sigma_y w t^2 \left(1 - \frac{\rho^2}{3}\right) \quad ...(15)$$

where ρt is the thickness of the elastic core,

Equating the bending moment expression in Equation (12) with the reactive bending moment, given by Equation (15), the elastic core coefficient, ρ, can be expressed as:

$$\rho^2 = 3 - 2\left(1 + \frac{x^*}{a}\right) \quad ...(16)$$

Considering bending moment expressions (5) and (13), the length of the elastic-plastic zone, b, is related to that of the elastic zone, a, as:

$$\frac{a}{a+b} = \frac{M_y}{M_u} = \frac{2}{3} \quad ...(17)$$

Considering Equation (16), it may be noted that:

at $x^* = 0$, $\rho = 1$ limit of elastic zone.

at $x^* = b = a/2$, $\rho = 0$ end of elastic-plastic zone.

The main amount of spring-back is caused due to the totally reversed stresses in the elastic zone, *a*, and the induced residual stresses in the elastic-plastic zone, *b*.

Spring-Back in Elastic Zone

The differential equation relating the deflection, *y*, with the bending moment *M* as functions of bending length, *x*, is given as, (Farsi and Behrooz, 2011):

$$\frac{d^2y}{dx^2} - \frac{M(x)}{EI(x)} = 0 \qquad ...(18)$$

The integration of Equation (18) leads to determination of the slop angle (*dy*/*dx*) which is the spring-back due to the reversed stresses in elastic zone, *a*. Hence, the elastic spring-back, $\Delta\theta_e$, can be obtained as:

$$\Delta\theta_e = 2\int_0^a \frac{M(x)}{EI(x)}dx \qquad ...(19)$$

where *M*(*x*) is expressed in Equation (9), $I(x) = \frac{wt^3}{12}$ is the second moment of cross-section which may varied due to variation of sheet width, *w*, or thickness, *t*.

Spring-Back in Elastic/Plastic Zone

The residual stresses over a section at distance *x**, in elastic-plastic zone, are obtained from superposition of existed stresses, shown in part *b* of Figure 8, and the elastically reversed stresses, due to release of bending force. The maximum value of the reversed stress is obtained, using Equation (12), as shown in Figure 9.

Figure 8: Residual Stresses in Elastic/Plastic Zone

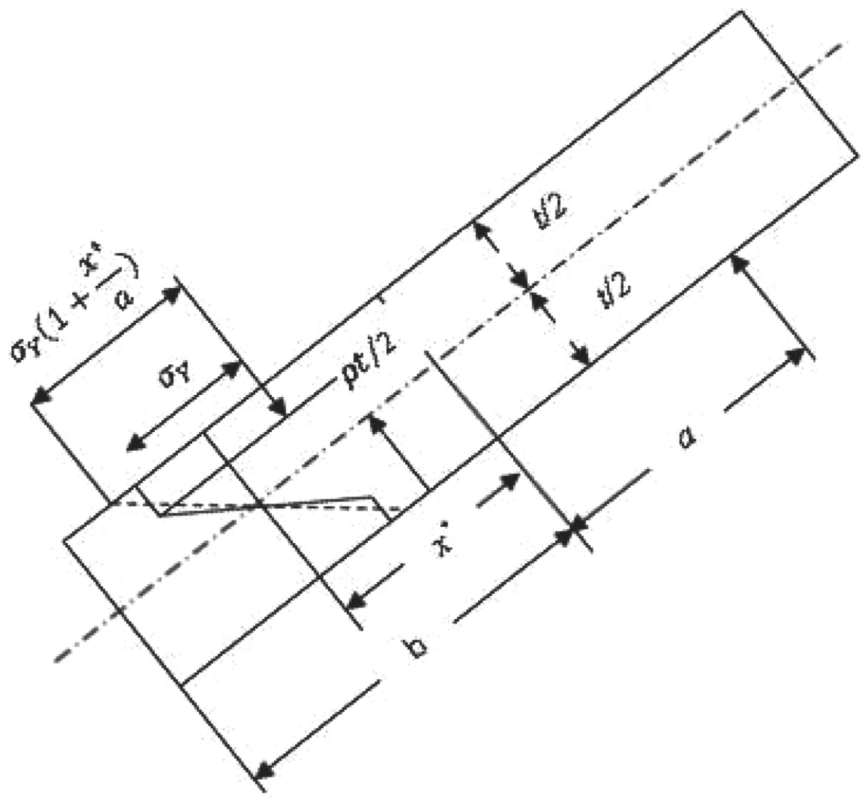
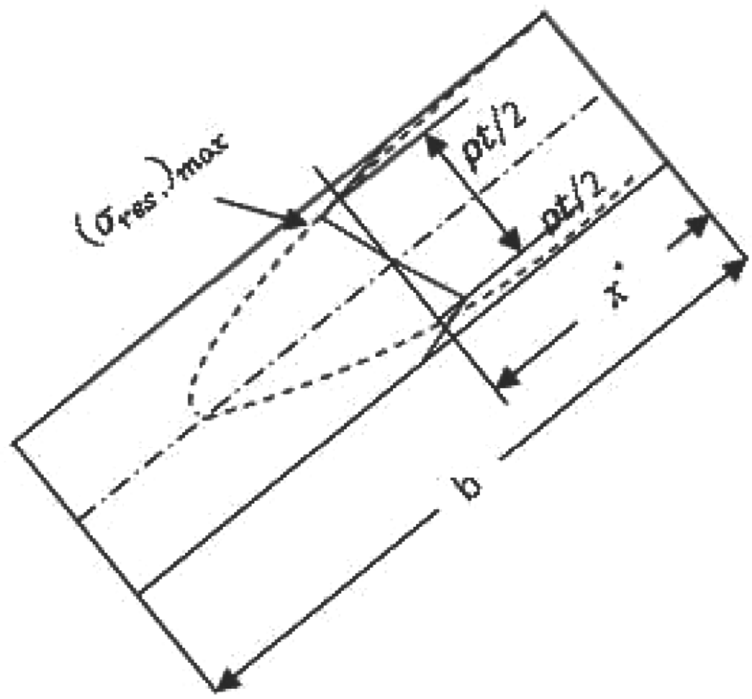


Figure 9: Residual Stresses Over the Elastic Core in Elastic/Plastic Zone



$$\sigma_{\max} = \sigma_y\left(1+\frac{x^*}{a}\right) \quad ...(20)$$

It may be observed that the spring-back is, only, occurred in the elastic core with thickness ρt. Where ρ is given, as a function of x^*, in Equation (16). The maximum value of the residual stress at the elastic core is obtained as:

$$(\sigma_{res})_{\max} = \sigma_y\left[1-\rho\left(1+\frac{x^*}{a}\right)\right] \quad ...(21)$$

which may be expressed in terms of one variable, as:

$$(\sigma_{res})_{\max} = \sigma_y\left[1-\frac{1}{2}\rho\left(3-\rho^2\right)\right] \quad ...(22)$$

The spring-back, through the elastic-plastic zone, b, can be obtained as the following expression:

$$\Delta\theta_e/p = 2\int_0^a \frac{M(x^*)}{EI(x^*)}dx^* \quad ...(23)$$

where $M(x^*) = \frac{2(\rho t)^2}{6}(\sigma_{res})_{\max}$

$$I(x^*) = \frac{w(\rho t)^2}{12}$$

Considering the expression (16), relating ρ as function of x^*, the integration on the right hand side of Equation (23) can be performed, which lead to the determination of spring-back in the elastic-plastic zone in the form:

$$\Delta\theta_e/p = \frac{3}{2}\frac{\sigma_y b}{Et} \quad ...(24)$$

The total spring-back is obtained by summation of expressions (19) and (23) as:

$$\text{Spring-back} = \Delta\theta_e + {\Delta\theta_e}/{p} \qquad ...(25)$$

A CASE STUDY

As a case study, the present analysis for bending force and spring-back, is applied to V-bending of sheet metal which experimentally studied by Farsi and Behrooz (2011). The material properties, used in experiments, are given in Table 1, which is that of a low carbon type. The sheet metal sample, the punch and die dimensions are given in Table 2.

The given data in Tables 1 and 2 are introduced into the expressions (6), (19) and (23) to get the values of bending force and spring-back. The obtained results are shown in Table 3, in comparison with that experimentally obtained by Farsi and Arezoo (Farsi and Behrooz, 2011).

Table 1: Material Data

Young's Modulus E(GPa)	Yield Stress (MPa)	Ultimate Stress (MPa)	Poisson's Ratio γ
193	155	298	0.3

Table 2: Sample, Punch and Die Dimensions

Sample	Punch	Die
Bending Length (L) = 80 mm	Angle = 84 degrees	Angle φ = 90, 120 degrees
Width (w) = 50 mm	Tip Radius (R) = 1 mm	Width = 18 mm
Thickness (t) = 0.95mm		

Table 3: Bending Force and Spring-Back Comparison

Die Angle (degrees)	Bending Force (N)		Spring-Back (degrees)	
	Present Study μ = 0.2	Experiment Ref (Farsi and Behrooz, 2011)	Present Study μ = 0.2	Experiment Ref (Farsi and Behrooz, 2011)
90	702	680	1.16	2
120	520	570	0.90	1

CONCLUSION

Depending on a statical approach, an elastic-plastic analysis is presented to determine general expressions for bending force, the amount of shift of the neutral axis and the spring-back, in V-die bending process. Instead of assuming constant amount of neutral axis shift of (5% to 15% of the sheet thickness), a variable expression is deduced, in the elastic zone of bending length. The shift amount decreased, along the bending length, starting with rapid decrease at the die edge. A bending force expression, depending on the die-angle and the coefficient of friction, is deduced. The deep analysis shows that the reflexive action, in elastic zone and the residual stresses, in elastic-plastic zone, have the main reasons for spring-back action. The obtained expressions, for bending force and spring-back, are compared with published experimental results, (Farsi and Behrooz, 2011), on V-die bending with two different die- angles, 90° and 120°. The compared results are in a good agreement, within 9% error.

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APPENDIX

Nomenclature

A	Length of elastic zone
B	Length of elastic-plastic zone
C	Length of plastic zone
D	Die width
E	Young's modulus of sheet metal material
e	Denotes elastic zone
e/p	Denotes elastic-plastic zone
I	Second moment of area of sheet cross-section
L	Bending length
M	Bending moment
M_u	Bending moment over totally plastic section
M_y	Bending moment at starting of surface yielding
N	Normal force
P	Bending force
Q	Reaction force at die edge
R	Radius of punch tip
t	Sheet thickness
w	Sheet width
x	Axial coordinate in the elastic zone
x^*	Axial coordinate in the elastic-plastic zone
y	Shift of the neutral axis
μ	Coefficient of friction
γ	Poisson's ratio
θ	Semi die angle
Φ	Die angle
σ	Normal stress
σ_{res}	Residual stress
σ_y	Yielding stress
ρ	Elastic core coefficient