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Research Paper

ANALYTICAL MODEL FOR PREDICTION OF FORCE DURING 3-ROLLER MULTIPASS CONICAL BENDING AND ITS EXPERIMENTAL VERIFICATION

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Conical sections and shells are produce using 3-roller conical bending machines. In this conical bending process, the blank is given static bending by placing the blank over the bottom rollers and lowering the top roller. The rollers are rotated than to get roll bending action. Static bending of the plate requires larger force and it is done in multiple stages to lower down the value of required force [Gajjar et. al., 2008]. The total deflection of the top roller required is divided in steps to get the multipass bending. The machine is designed on the basis of maximum reaction forces during bending. In this paper an analytical model is proposed for the prediction of bending force during the multiple pass 3-roller conical bending. Multipass bending experiments are carried out to validate the developed model. Experimental results do not match exactly with the analytical results. So a correction factor for each pass has been found out and applied to the analytical results. The corrected analytical results match with experimental results quite satisfactorily. The model derived can be effectively used to study the effect of various parameters on the bending force and can be helpful to the researchers working in this area.

Keywords: 3-roller conical bending, Force prediction, Experimental validation, Internal bending moment, External bending moment

INTRODUCTION

For construction of various structures as well as integral part of machines various conical sections are widely used. Such conical sections are manufactured by various methods and 3-roller conical bending process is one such process. It consists of two bottom rollers and a top roller. Metal plates with specified contours are rolled without decrease in thickness to get the desired cone angle. The plate undergoes plastic deformation and it is cold forming process and hence it has higher

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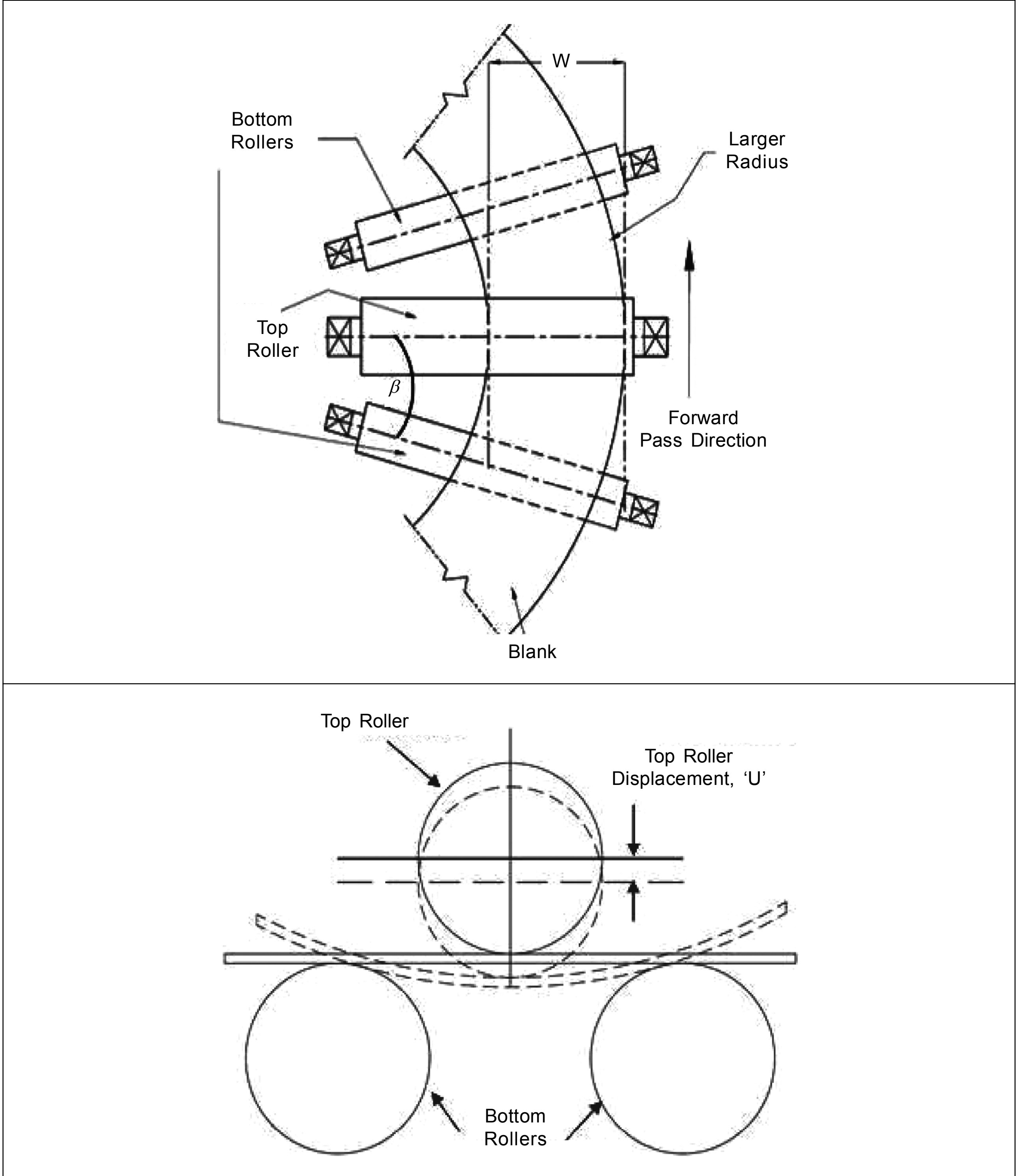
dimensional accuracy. 3-roller conical bending process has four stages, (a) static bending, (b) forward rolling, (c) backward rolling, and (d) unloading. In the first stage the plate is kept

Figure 1: 3-Roller Conical Bending Setup: Schematic Representation



between top roller and bottom rollers as shown in Figure 1 and the top roller is given vertical displacement to get the required bend. In next stage the bottom rollers are driven using motors in forward direction to get the roll bending of the plate. Similarly the rollers are driven in reverse direction to get better dimensional accuracy of the final product. The bent plate is than unloaded by raising the top roller.

Mechanics involved in the conical bending is very complex. It can be observed that there will be 3-dimensional force pattern at the roller plate interface during the conical bending. In this paper an attempt is made to get the analytical model for bending force prediction for such complex mechanics.

Various researchers have worked for the development of mathematical models for cylindrical bending process (Wang *et al.*, 1993). Hua *et al.* have proposed a method of determining the plate internal bending resistance at the top roll contact for multi-pass four-roll thin-plate bending operations (Hua *et al.*, 1995). For continuous single-pass four-roll thin plate bending a model was proposed by Hua and Baines, considering the equilibrium of the internal and external bending moment at and about the plate-top roll contact (Baines *et al.*, 1997). Lin *et al.* had considered varying radius of curvature for the plate between the rollers and proposed a mathematical model to simulate the mechanics in a steady continuous bending mode for four-roll thin plate bending process (Hua and Lin, 1999). Hua and Lin also investigated Influence of material strain hardening on the mechanics of steady continuous roll and edge-bending mode in the four-roll plate bending process (Hua and Lin, 2000).

For continuous multi-pass bending of cone frustum on 3-roller bending machines with non-compatible (cylindrical) rollers, Gandhi *et al.* had reported the formulation of springback and machine setting parameters (Gandhi *et al.*, 2008 and 2009). They incorporated the effect of change of flexural modulus during the deformation in the formulation to study the effect on springback prediction. For plane strain flow of sheet metal subjected to strain rate effects during cyclic bending under tension Sanchez presented an elastic–plastic mathematical model (Sanchez, 2010). He also included Bauschinger factors in the model for stress reversal. Chudasama and Raval have reported analytical model of force prediction for single pass in static bending stage during 3-roller conical bending process (Chudasama and Raval, 2011).

The roll bending process is used for years, it can be observed from the literature reviewed that conical bending process is untouched area as far as force prediction is concerned. Even in the industries the normal practice of plate roller bending still heavily depends upon the experience and the skill of the operator. Working to templates, or by trial and error, yet remains a common practice (Gandhi, 2009). For conical bending investigation related to conical bending as far as machine setting is concerned is done by Gandhi *et al.* (2008 and 2009), but few references are available to addressed the problem of bending force prediction for roll-bending so far as the knowledge of the authors is concerned. Considering this, in this paper an attempt is made to develop force prediction model for multipass 3-roller conical bending process which will be useful to the researchers to understand the complex mechanics of the process.

ANALYTICAL MODEL

Hua et. al. have concluded that it is very difficult to have a single mathematical model that takes into account all the complexities of the bending process (Hua *et al.*, 1995). As 3-roller conical bending is also a bending process it is very difficult to get a mathematical model for of 3-roller conical bending process taking all the factors into consideration. A realistic simplification is thus necessary.

So following simplifying assumptions have been made for formulation of the relation for force prediction:

Assumptions

- Plate is always having line contact with the roller which is parallel to roller axis during the process.
- The forces acting during the bending are larger than the self weight of the plate. So the self weight of the plate is neglected.
- The shift of the neutral plane is zero, i.e., it is considered to be at the center line of the plate thickness.
- Frictional force at the bottom roller and the plate interface is always tangent to the roller surface.
- Rollers are assumed to be rigid. Roller material and plate material is assumed to have stable microstructure throughout the deformation process.
- Deformation occurs under isothermal conditions and '*E*', i.e., Modulus of Elasticity remains constant during the process.
- As cone angle considered being small.
- As cone angle considered being small blanks for the cone frustum bending are selected such that $((w/t) > 8)$ and hence plane strain conditions maintained (Marciniak, 1992; and Wang *et al.*, 1993).
- Plane section remains plane, before and after the bending. Blank thickness (t) remains constant during and after the bending.
- Baushinger effect is neglected. Blank is having uniform/constant radius of curvature for the supported length of the blank between two bottom rollers.
- Further simplifying assumptions are discussed as and when required during the formulation.

Based on the above assumptions the bending force equation can be derived by equating the external bending moment required to bend the plate and internal bending moment induced in the plate.

External Bending Moment

In 3-roller conical bending process first step is bending of the plate by lowering down the top roller. In this step it is required to exert force on the top roller to lower it, which will bend the plate. This vertical force exerted on the top roller will give the external bending moment required to bend the plate.

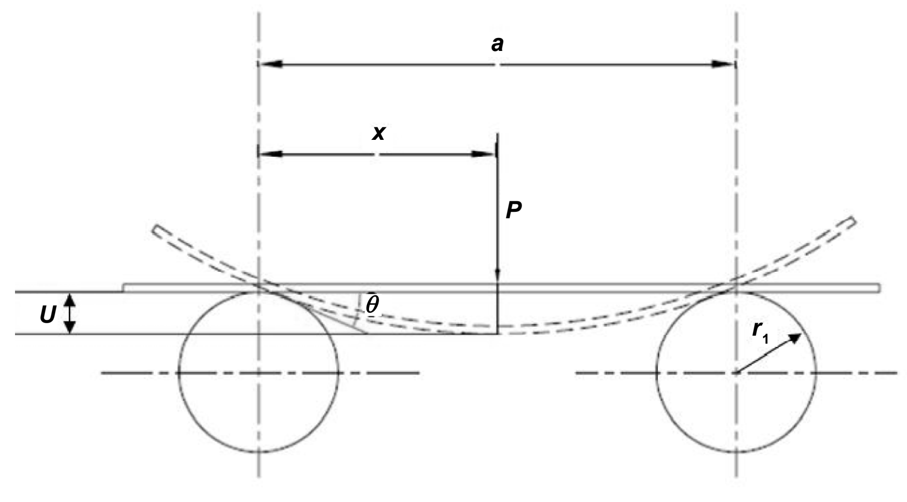
External bending moment over the plate exerted by the rollers can be derived by considering the geometry of the setup during bending. The force pattern during conical bending is complicate and needs to be simplified. During the bending process there will be reaction forces on the bottom roller because of the top roller load.

In the derivation of external bending moment over the plate, forces in vertical plane passing through the three rollers and perpendicular to the top roller axis is considered. If axial forces on the bottom rollers are resolved, they will not have any component along the vertical plane, as bottom rollers are inclined only in horizontal plane as explained earlier. So axial force on the bottom rollers will not affect the derivation of the external bending moment derived earlier by Chudasama and Raval (2011). Reaction on the top roller will have three components in mutually perpendicular directions. Axial force on the top roller will also not affect the external bending moment as it is not inclined in the vertical plane. In the present analysis vertical component of the force on top roller, 'P', is considered for the calculations.

Figure 2 gives schematic of blank and roller arrangement for static bending. Equation for external bending moment can be derived as reported (Chudasama and Raval, 2011),

$$M_{total} = \frac{P}{2}\left(1 - \frac{x - r_1 \sin\theta}{a - r_1 \sin\theta}\right)\left(\frac{1}{\cos\theta + \mu \sin\theta}\right) \left(x - \tan\theta\left(r_1 - U\right) + \mu\left(x * \tan\theta - r_1\left(\sin\theta * \tan\theta + \cos\theta - 1\right) - U\right)\right) \quad ...(1)$$

Figure 2: Schematic of Blank and Roller Arrangement for Static Bending



It is to be stated here that external bending moment equation for multipass is similar to the single pass equation but the values of the parameters 'U' and 'θ' will be different for multipass.

Internal Bending Moment for Multipass

When the top roller is lowered down and external force is exerted to bend the plate, the material of the plate will resist the deformation of the plate. There will be elastic as well as plastic deformation of the plate and hence there will elastic as well plastic resistance to the deforation. Combined effect of elastic and plastic resistance will give the internal bending moment and can be calculated as below:

The bending moment can be split into an elastic contribution and plastic contribution as discussed earlier and can be calculated by Chudasama and Raval (2011).

$$M_{total} = M_{elastic} + M_{plastic} \quad ...(2)$$

$$= 2\int_{elastic} \sigma_x y\, dy + 2\int_{plastic} \sigma_x y\, dy \quad ...(3)$$

Hill's non-quadratic yield criteria (Hill, 1979) for plane strain deformation in plastic region for isotropic material is,

$$\sigma_x = \frac{2}{\sqrt{3}}\bar{\sigma} \text{ and } \bar{\varepsilon} = \frac{2}{\sqrt{3}}\varepsilon_x \quad ...(4)$$

where, F = Anisotropic constant,

σ_x = Uniaxial stress,

$\bar{\sigma}$ = Effective stress,

$\bar{\varepsilon}$ = Effective strain,

ε_x = Axial strain.

The power law material behaviour considering initial strain å0, is assumed in the plastic region.

So, $\bar{\sigma} = K\left(\varepsilon_0 + \bar{\varepsilon}\right)^n$ ...(5)

From this, $\frac{2}{\sqrt{3}}\sigma_x = K\left(\varepsilon_0 + \frac{2}{\sqrt{3}}\varepsilon_x\right)^n$

(Gandhi *et al.*, 2009) ...(6)

$$\therefore \sigma_x = \frac{\sqrt{3}}{2} K\left(\varepsilon_0 + \frac{2}{\sqrt{3}}\varepsilon_x\right)^n \quad ...(7)$$

For elastic region $\sigma_x = \varepsilon_x \times E$, where E = Elastic constant ...(8)

Considering plane strain bending, E is replaced by E' for plain strain conditions and

$E' = \frac{E}{\left(1-\nu^2\right)}$ (where, ν = Poison's Ratio)

...(9)

For deciding the limits of the integration, the elastic zone thickness is considered upto a distance of y_{ep} from the neutral plane. Inserting the values of σ_x and E' elastic and plastic bending moment in Equation (3), we get,

$$M_{total} = 2\int_0^{y_{ep}} \frac{E}{\left(1-\nu^2\right)}\varepsilon_x y\, dy + 2\int_{y_{ep}}^{t/2} \frac{\sqrt{3}}{2} K\left(\varepsilon_0 + \frac{2}{\sqrt{3}}\varepsilon_x\right)^n y\, dy \quad ...(10)$$

For bending operation,

$$\frac{\sigma}{y} = \frac{E}{R_b} \quad ...(11)$$

where, σ = bending stress, y = distance of outermost fibre from neutral axis, E = elasticity constant, *and* R_b = bend radius.

From this

$$\frac{\sigma_x}{E} = \frac{y}{R_b} = \varepsilon_x \quad ...(12)$$

In Single pass bending, initial radius of the blank will be infinite, i.e., flat plate is considered. But in case of multipass bending, blank will have some initial radius except for first pass. So the radius of curvature for multipass will be $\left(\frac{1}{R} - \frac{1}{R_{f0}}\right)$, where R = final radius of the blank, R_{f0} = initial radius of the blank. Radius of curvature is inverse of the bend radius.

$$\text{So, } B = \left(\frac{1}{R} - \frac{1}{R_{f0}}\right) = \frac{1}{R_b} \quad ...(13)$$

$$\therefore \frac{\sigma_x}{E} = \varepsilon_x = \frac{y}{R_b} = yB \quad ...(14)$$

Now considering only elastic moment,

$$M_{elastic} = 2\int_0^{y_{ep}} E'(yB)y\,dy \quad ...(15)$$

$$M_{elastic} = 2\,[E'B]\left[y^3/3\right]_0^{y_{ep}} \quad ...(16)$$

$$= \frac{2}{3}\,[E'B]\,y_{ep}^3 \quad ...(17)$$

Now considering plastic moment,

$$M_{plastic} = 2\int_{y_{ep}}^{t/2} \frac{\sqrt{3}}{2}\,K\left(\varepsilon_0 + \frac{2}{\sqrt{3}}\,\varepsilon_x\right)^n y\,dy \quad ...(18)$$

$$= 2\int_{y_{ep}}^{t/2} \frac{\sqrt{3}}{2}\,K\left\{\varepsilon_0 + \frac{2}{\sqrt{3}}y\left(\frac{1}{R} - \frac{1}{R_{fo}}\right)\right\}^n y\,dy \quad ...(19)$$

$$= \sqrt{3}\int_{y_{ep}}^{t/2} K\left\{\varepsilon_0 + \frac{2}{\sqrt{3}}By\right\}^n y\,dy \quad ...(20)$$

From Equations (17) and (19)

$$M_{Total} = \frac{2}{3}\,[E'B]\,y_{ep}^3 + \sqrt{3}\int_{y_{ep}}^{t/2} K\left\{\varepsilon_0 + \frac{2}{\sqrt{3}}By\right\}^n y\,dy \quad ...(21)$$

where $y_{ep} = \frac{\sigma R}{E}$ (Chudasama and Raval, 2011)

Expression for Bending Load *P*

Equating Equations (1) and (21), and rearranging,

$$M_{Total} = \frac{2}{3}\,[E'B]\,y_{ep}^3 + \sqrt{3}\int_{y_{ep}}^{t/2} K\left\{\varepsilon_0 + \frac{2}{\sqrt{3}}By\right\}^n y\,dy$$

$$= P\left(1 - \frac{x - r_1 \sin\theta}{a - r_1 \sin\theta}\right)\left(\frac{1}{\cos\theta + \mu\sin\theta}\right)$$

$$\left(x - \tan\theta(r_1 - U) + \mu(x * \tan\theta - r_1(\sin\theta * \tan\theta + \cos\theta - 1) - U)\right) \quad ...(22)$$

$$P = \frac{\frac{2}{3}\,[E'B]\,y_{ep}^3 + \sqrt{3}\int_{y_{ep}}^{t/2} K\left\{\varepsilon_0 + \frac{2}{\sqrt{3}}By\right\}^n y\,dy}{\left(1 - \frac{x - r_1\sin\theta}{a - r_1\sin\theta}\right)\left(\frac{1}{\cos\theta + \mu\sin\theta}\right)X} \quad ...(23)$$

where, $X = (x - \tan\theta(r_1 - U) + \mu(x * \tan\theta - r_1(\sin\theta * \tan\theta + \cos\theta - 1) - U))$

Equation (23) gives the bending force required to get the required bend radius for multipass conical bending. It is required to integrate the equation for plastic bending moment. In the present case of integration there is no standard formula available for it. Also it is not possible to solve it by using any conventional method like substitution. So it is to be evaluated using Numerical Method.

In Equation (23), there is no term which reflects the effect of bottom roller inclination on the bending force *P*. Bottom roller inclination is set by setting the bending radius at front end and rear end on 3-roller conical bending machine. The relation between rolling radius and bottom roller inclination is given in Equation (24) (Gandhi, 2009). If bottom roller inclination is zero than both the radius will be same, with no top roller inclination, this condition is of cylindrical bending.

$$\beta = \tan^{-1}\left[\left(\frac{A_R - A_F}{2}\right)\left(\frac{1}{\cos\alpha}\right)\left(\frac{\sin^{\varphi/2}}{R_R - R_F}\right)\right] \quad ...(24)$$

where, β = bottom roller inclination,

A_F, A_R = Center distance between bottom rollers at front and rear end respectively

α = Top roller inclination, in the present case it is zero

φ = Cone angle

R_F, R_R = Bending radius at the front end and rear end respectively

To get the value of *P*, i.e., top roller load, from the above Equation (23) along with the material properties and geometrical parameters of the machine setting, the value of coefficient of friction μ at roller plate interface is required. Value of angle θ can be calculated from the geometrical configurations of the machine setting.

EXPERIMENTATION

For verification of the developed analytical model for prediction of force during multipass cone frustum bending, experimentation is carried out. Details of experimentation are discussed in subsequent sections.

Experimental Setup

To validate the analytical models of force prediction developed in earlier section it is required to do the experiments. 3-roller conical bending machine is used to carry out

Figure 3: 3-Roller Conical Bending Machine Setup; and Arrangement of Load Cell to Measure the Bending Force

(a)

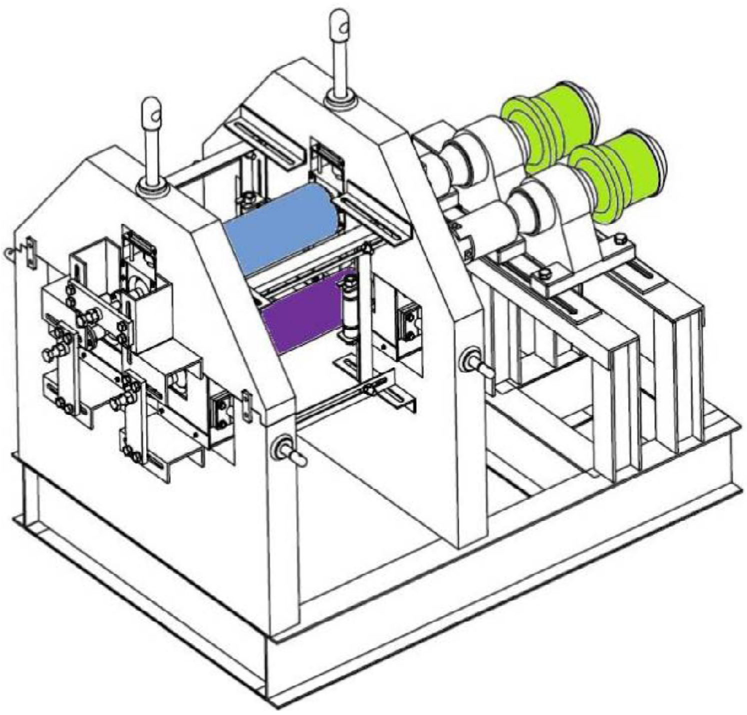


Figure 3 (Cont.)

(b)

the experiments. It is shown in Figure 3. The bottom rollers of the machine are driven by two independently driven electric motors. The rollers are attached with the motors using universal joints as shown in Figure 3a. Bottom rollers can be set at angle as the spherical roller bearings are provided at the roller supports. The top roller can be lowered down using power screws as the bearing blocks for top rollers can slide vertically Figure 3b. The setup is facilitated with the load cells at the top roller as well bottom roller bearings to measure the reaction in 3 mutually perpendicular planes. Figure 3b shows the location of load cell to measure the bending force over the top roller. These load cells are attached to the online data acquisition system which gives the force variation during the continuous roll bending process.

Material Selection

Structural steel FE 410 WA is widely used for the various structural applications and is easily available in Indian commercial market. Hence, structural steel of material grade FE 410 WA as per IS 2062 (2006) is selected for cone frustum bending experiments. Mechanical properties and chemical composition of the structural steel FE 410 WA is given in Tables 1 and 2 respectively.

Table 1: Mechanical Properties of Structural Steel Material Grade FE 410 WA (IS 2062, 2006)

Tensile Strength Minimum (MPa)	Yield Stress, Minimum (MPa)			% Elongation at Gauge Length 5.65, Minimum
	<20	20-40	>40	
410	250	240	230	23

Table 2: Chemical Composition, Structural Steel of Material Grade FE 410 WA (IS 2062, 2006)

Carbon	Manganese	Sulphur	Phosphorous	Silicon	Carbon Equivalent
0.23 ± 0.02	1.5 ± 0.05	0.05 ± 0.005	0.05 ± 0.005	0.40 ± 0.03	0.42

Note: In percentage maximum values.

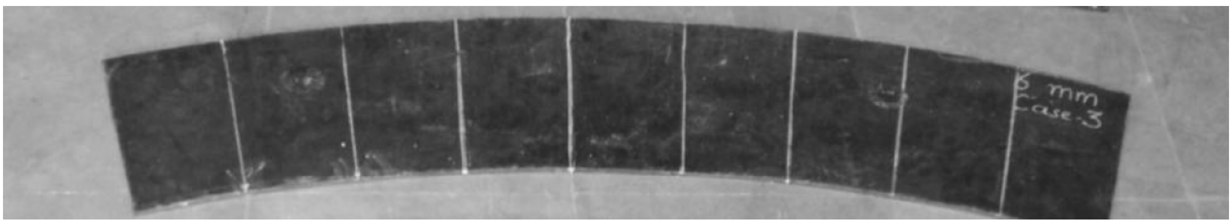
Blank Geometry

Plate thickness for the plate to be bend are selected as per the availability in the commercial market. The plates were taken and the thicknesses were measured over the length at 6 points on each side using vernier calliper. The average of the reading were taken as the thickness of the plates. The thickness 't' of the plates are: 5.84 mm, 7.87 mm, 8.85 mm, 11.84 mm and 13.97 mm. Blank dimensions are taken from the earlier work done by Gandhi regarding parametric investigation of 3-roller conical bending process (Gandhi, 2009). Generators are marked on the blank as per the requirements of the experiments. Figure 4 shows the photograph of the one of developed blanks (plate thickness 't' = 5.84 mm, β = 1.86°) used for the cone frustum bending experiments.

Experimental Procedure

As per the experimental planning presented in this section, cone frustum bending experiments were performed for the verification of the developed analytical models. First the bottom rollers are inclined at required inclination by sliding the bearing blocks at required distance for the inclination of β = 1.86°. The blank is than loaded between top and bottom rollers. To perform the bending the top roller is than lowered down at required distance for the first pass using the power screws. The bending forces at the top roller bearing are measured with the help of load cells and the data acquisition system. Then bottom rollers are rotated in one direction for forming the radius over the entire length of the blank in the forward pass followed by the reverse rotation of rollers during the reverse pass. Reverse pass is necessary for forming of the initial blank length to the desired radius which is not formed during forward pass and for the uniformity of the radius. After reverse pass the plate is unloaded by raising the top roller upward. After completion of this first pass. Again the plate is kept between the rollers and the top roller is lowered further for second pass. The distance travelled by the top roller in second pass will be more than the distance travelled in the first pass. Again forward and reverse

Figure 4: Blank Before Bending (Thickness 't' = 5.84 mm, β = 1.86°)



rolling is done and the cycle repeats. In present case the final dimensions of bending is achieved in 5 passes and the bending force required during each pass is measured to get the experimental bending force (Figure 5).

Experiments have been performed as per the plan by different thickness plates. Data of force required for the roll bending have been acquired through the Data Acquisition System (DAS) as discussed earlier. Data has been processed and the sample results for plate thickness 0f 7.87 mm thick plate are shown in Table 3.

Figure 5: Plate After Bending

Table 3: Sample Result Table for t = 7.87 mm for β = 1.86°

Pass Number	Top Roller Position U (mm)	Top Roller Force (N)
1.	20	899
2.	25	1007
3.	30	681
4.	35	931
5.	40	1176

Data obtained through the experimentation is to be used for the validation of the developed analytical model of force prediction, in the subsequent section.

VALIDATION OF THE DEVELOPED ANALYTICAL MODEL

Analytical model for prediction of force during 3-roller conical bending process for multipass static bending process have been developed in analytical model section. Analytical calculations were done using Simpson's 3/8th Rule for numerical integration. Experiments of 3-roller conical bending have been performed to verify the analytical models and have been reported in expermentation section. In this section developed bending force models are validated using the data obtained from experiments.

Model for Multipass Bendig

Analytical model of force prediction for multipass 3-roller conical bending process has been developed in analytical model section. The Equation (23) is rewritten below:

$$P = \frac{\frac{2}{3}\left[E'B\right] y_{ep}^{3} + \sqrt{3}\int_{y_{ep}}^{t/2} K\left\{\varepsilon_0 + \frac{2}{\sqrt{3}} By\right\}^{n} y\,dy}{\left(1 - \frac{x - r_1 \sin\theta}{a - r_1 \sin\theta}\right)\left(\frac{1}{\cos\theta + \mu \sin\theta}\right) X}$$

where, $X = (x - \tan\theta(r_1 - U) + \mu(x * \tan\theta - r_1(\sin\theta * \tan\theta + \cos\theta - 1) - U))$

The values of various parameters involved in the above equation are substituted and the analytical results were obtained. They have been compared. There was large difference in the values of the analytical force and experimental force. It can be due to the simplifying assumptions made during the derivation as well the uncertainties in the experimentation. But it was observed that the trend of the experimental results was matching with the analytical results. So a correction factor was found out. The ratio of experimental value and analytical value was taken for each thickness. Than the average of the ratio for different thicknesses were taken. The average value of the ratio has been taken as the correction factor and applied to the analytical results. Graphs of Experimental force and corrected analytical force have been plotted for each thickness as shown in Figure 6.

It can be observed from the Figure 6a to 6e that as thickness of the plate increases, the value of required bending force also increases. It is reflected in both, corrected analytical bending force as well as experimental bending force. It is quite well represents the actual case. It can also be observed that corrected analytical results are having good agreement with the experimental results except for the first pass and 5.84 mm thickness in Figure 6a. That can be due to some error occurred during the experimentation or some unknown factor. Otherwise the error observed between the experimental and corrected analytical results is in the range of ±20%, which is quite satisfactory. So the analytical model developed here can be utilised for the force prediction with the correction factor for each pass. Though it is not giving the exact results, it can be further modified to give better results.

Figure 6: Variation of Bending Force *P*, with Respect to Number of Passes for Plate Thickness

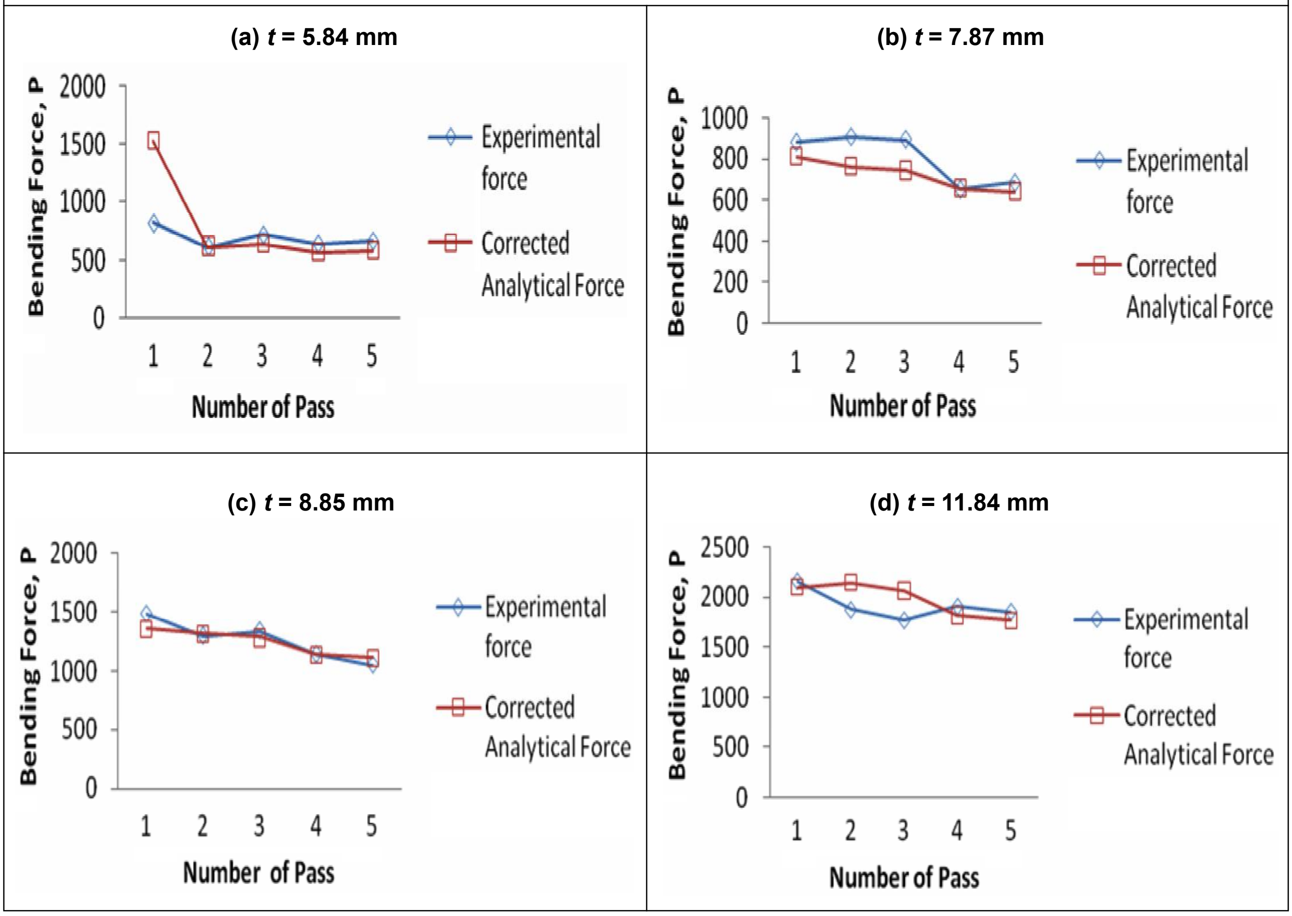
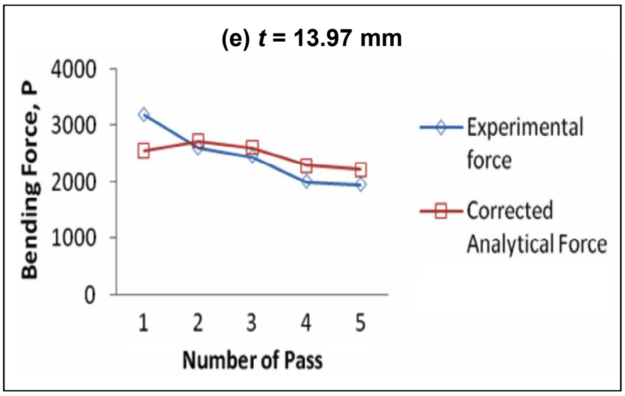


Figure 6 (Cont.)



CONCLUSION

An analytical model for multipass 3-roller conical bending process has been derived. External bending moment required to bend the plate has been equated to internal bending moment developed in the plate to get the required formulation for the bending force '*P*'. To validate the developed analytical model, experimentation has been carried out. 3-roller conical bending machine was set for particular value of bottom roller inclination. Different thickness plates were bent using the 3-roller conical bending setup and force required to get the required bend radius have been measured. For the same geometrical as well as material parameters, analytical bending force have been calculated using Equation (23). The value of the analytical bending force and experimental force had large difference. This can be due to the simplifying assumptions made. It can be due to the uncertainties in the experimentations also. But it was observed that the trend of the analytical results was matching with the experimental results. So a correction factor has been obtained for each pass for different thicknesses. The correction factor was applied to the analytical results and corrected analytical force results have been compared with the experimental results. They were in good agreement and the error found to be ±20%.

Though the developed model is not giving the exact results for the bending force, it can be used satisfactorily with the correction factor for each pass. Further the model can be modified by removing the simplifying assumptions like considering the varying radius of curvature of bending instead of constant radius, considering axial forces during bending, etc. The developed analytical model can be useful to the developers of the 3-roller conical bending machines as well as the researchers working in this area.

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APPENDIX

Nomenclatures

w	=	Width of the blank in mm.
t	=	Thickness of the plate in mm.
M	=	Bending moment in N-m.
P	=	Vertical load at the top roller and bending plate interface in *N*.
a	=	Borizontal distance of the bottom roller centers in mm.
x	=	Balf the horizontal distance of the bottom roller centers in mm.
Q	=	Normal force exerted by the plate on the bottom roller at roller plate interface in *N*.
θ	=	Angle between frictional force and horizontal plane at the roller plate interface in radians.
U	=	Vertical distance travelled by the top roller for first stage of static bending in mm.
E	=	Young's modulus in N/mm^2.
K	=	Strength coefficient in N/mm^2.
n	=	Strain hardening exponent.
r_1	=	Radius of bottom roller in mm.
R	=	Radius of curvature of the bent plate in mm.
y	=	Distance of fiber from neutral plane in mm.
μ	=	Coefficient of friction at roller plate interface.
ε	=	Strain.
σ	=	Stress in N/mm^2.
ν	=	Poisson's ratio.
y_{ep}	=	Distance of the fiber upto which elasticity *E* is constant in mm.
χ	=	Curvature of the bend plate between bottom rollers, mm^{-1}.
ε^*	=	Strain at yield point.
E^*	=	The ratio of modulus of elasticity to σ_s.
t_e	=	Thickness of elastic layer in mm.
ε_0	=	Strain of the strip mid-line.
$\bar{\varepsilon}$	=	Effective strain.
$\bar{\sigma}$	=	Effective stress.
β	=	Bottom roller inclination.
A_F, A_R	=	Center distance between bottom rollers at front and rear end respectively.
α	=	Top roller inclination, in the present case it is zero.
φ	=	Cone angle.
R_F, R_R	=	Bending radius at the front end and rear end respectively.