International Journal of Mechanical Engineering and Robotics Research

ISSN 2278 – 0149 www.ijmerr.com
Vol. 2, No. 1, January 2013
© 2013 IJMERR. All Rights Reserved

Research Paper

ANALYSIS OF UNSTEADY HEAT CONDUCTION THROUGH SHORT FIN WITH APPLICABILITY OF QUASI THEORY

Tejpratap Singh[1]*, Sanjeev Shrivastava[1] and Harbans Singh Ber[2]

*Corresponding Author: **Tejpratap Singh,** ✉ tejpratap50@yahoo.com

The paper is based on the analysis of unsteady heat conduction through short fin with applicability of quasi theory. “The area exposed to the surrounding is frequently increased by the attachment of protrusions to the surfaces, and the arrangement provides a means by which heat transfer rate can be substantially improved. The protrusions are called fins”. The fins are commonly used on small power developing machine as engine used for motor cycle as well as small capacity compressor. Earlier, Work under steady state conduction had been carried out extensively. Unsteady heat conduction analysis for the fins is being done for calculation of heat transfer. Unsteady Closed form solutions had been derived earlier by various researchers. Exact solutions are given for the unsteady temperature in flux-base fins with the method of Green’s Functions (GF) in the form of infinite series for three different tip conditions. The time of convergence is improved by replacing the series part by closed form solution. The present study supplies a new approach to calculate the thermal performance of the short fin. For the short fin case, exact fin solution and a quasi-steady solution is presented. Numerical values are presented and the conditions under which the quasi-steady solution is accurate are determined. Dimensionless temperature distribution is presented for both quasi steady theory and exact fin theory.

Keywords: Unsteady heat, Green function, Protrusion, Heat transfer

INTRODUCTION

The laws which are governing heat transmission are very important to the engineers in the design, construction, testing and operation of heat exchange apparatus. Heat transfer is the study of the rate at which energy is transferred across a surface of interest due to temperature gradients at the

[1] Department of Mechanical Engineering (Thermal Engineering), Shri Shankracharya College of Engineering and Technology (SSCET, Bhilai), Chhattisgarh, India.

[2] Department of Mechanical Engineering (Thermal Engineering), National Institute of Technology (NIT, Raipur), Chhattisgarh, India.

surface, and temperature difference between the different surfaces. This variation in temperature is governed by the principle of energy conservation which when applied to a control volume or a control mass, states that the sum of the flow of energy and heat across the system, the work done on the system and the energy stored and converted within the system is zero. The mechanical engineer deal with problems of heat transfer in the field of internal combustion engines, steam generation, refrigeration and heating and ventilation. To estimate the cost, the feasibility and size of the equipment necessary to transfer a specified amount of heat in a given time, a detailed heat transfer analysis must be made. The dimensions of boilers, heaters, refrigerators and heat exchangers depend not only on the amount of heat to be transmitted but rather on the rate at which heat is to be transferred under given conditions. Thermal system contains matter or substance and this substance may change by transformation or by exchange of mass with the surroundings. To perform a thermal analysis of a system, we need to use thermodynamics, which allows for quantitative description of the substance. This is done by defining the boundaries of the system, applying the conservation principles and examining how the system participates in thermal energy exchange and conversion.

The unsteady response of fins is important in a wide range of engineering devices including heat exchangers, clutches, motors and so on. Heat conduction is increasingly important in various areas, namely in the earth sciences, and in many other evolving areas of thermal analysis. A common example of heat conduction is heating an object in an oven or furnace. The material remains stationary throughout, neglecting thermal expansion as the heat diffuses inward to increase its temperature. The importance of such conditions leads to analyze the temperature field by employing sophisticated mathematical and advanced numerical tools. The section considers the various solution methodologies used to obtain the temperature field. The objective of conduction analysis is to determine the temperature field in a body and how the temperature varies within the portion of the body. The temperature field usually depends on boundary conditions, initial condition, material properties and geometry of the body. Why one need to know temperature field. To compute the heat flux at any location, compute thermal stress, expansion deflection, design insulation thickness, heat treatment method, these all analysis leads to know the temperature field. The solution of conduction problem involves the functional dependence of temperature on space and time coordinate. Obtaining a solution means determining a temperature distribution which is consistent with the conditions on the boundaries and also consistent with any specified constraints internal to the region.

Many researchers have contributed in unsteady heat conduction through fins.

Donaldson and Shouman (1972) studied the transient temperature distribution in a convecting straight fin of constant area for two distinct cases, namely, a step change in base temperature, and a step change in base heat flow rate. The tip of the fin is insulated. The authors developed the equations for the transient temperature distribution and the heat flow rate for the two aforementioned cases, and present their results graphically. Also included is a summary of their experimental work to verify their results for the case of a step function in heat flow rate.

Chapman (1959) who studied the transient behaviour of an annular fin of uniform thickness subjected to a sudden step change in the base temperature. His interest in circular annular fins temmed from the numerous applications of these type of fins, especially on cylinders of air cooled internal combustion engines. Chapman (1959) developed equations that give the temperature distribution within the fin, the heat removed from the source and the heat dissipated to the surroundings, all as functions of time. These equations in graphical form are very useful for the design engineers.

Suryanarayana (1975 and 1976) also studied the transient response of straight fins of constant cross-sectional area. However, rather than using the separation of variables technique followed by Donaldson and Shouman, he utilized the Laplace transforms in order to develop the solutions for small and large values of time, when the base of the fin is subjected to a step change in temperature or heat flux. The tip of the fin is insulated in addition, the use of the Laplace transforms made it easier for Suryanarayana to develop solutions for the case of a fin subjected to a sinusoidal temperature or heat flux at its base.

Suryanarayana (1976) has provided an analysis of the heat transfer that takes place from one fluid to another separated by a solid boundary with fins on one side.

Aziz and Na (1980) considered the transient response of a semi-infinite fin of uniform thickness, initially at the ambient temperature, subjected to a step change in temperature at its base, with fin cooling governed by a power-law type dependence on temperature difference. The choice of a semi-infinite geometry enabled the transformation of the governing nonlinear partial differential equations into a sequence of similarity type linear perturbation equations. Aziz and Na also discussed the applicability of the results to finite fins.

Mao and Rooke (1994) also used the Laplace transform method to study straight fins with three different transients: a step change in base temperature; a step change in base heat flux and a step change in fluid temperature. Transient fins of constant cross-section have also been studied with the method of Green's functions (Beck *et al.*, 1992, pp. 60-64), a flexible and powerful approach that are applicable to any combination of end conditions on the fin.

Aziz and Kraus (1995) present a variety of analytical results for transient fins, developed by separation of variable and Laplace transform techniques. Results discussed include rectangular fins with three different base conditions, rectangular fins with power law convective heat loss and radial fins, along with several specific examples. Aziz and Kraus also present a comprehensive literature review. The material on transient fins of constant cross-section is also included in a book by Kraus *et al.* (2001, Chap.16).

Kim (1976) developed an approximate solution to the transient heat transfer in straight fins of constant cross-sectional area and constant physical and thermal properties.

The author utilized the Kantorovich method in the variation formulation to provide a simple expression of the exact form of the solution. In some fin applications, Newton's law of cooling is not applicable, and a power-law type dependence of convective heat flux on temperature better describes the cooling process. Such cases include cooling of fins due to film boiling, natural convection, nucleate boiling, and radiation to space at absolute zero.

The work discussed so far has focused on the transient response of fins of simple geometry such as circular annular fins and straight fins. In addition, several simplifying assumptions were utilized such as uniform thickness, constant cross-sectional area, semi-infinite length, insulated tip and small fin thickness-to-length ratio to ensure one dimensional heat conduction. Recently, work has included fins of various shapes and cross-sections, two and three dimensional heat transfer, and practical applications of finned heat exchangers.

Campo and Salazar (1996) explored the analogy between the transient conduction in a planar slab for short times and the steady state conduction in a straight fin of uniform cross-section. They made use of a hybrid computational method, known as the Transversal Method Of Lines (TMOL), to arrive at approximate analytical solutions of the unsteady-state heat conduction equation for short times in a plane having a uniform initial temperature and subjected to a Drichlet boundary condition. The resulting solutions are suitable for obtaining quality short-time temperature distributions within the slab when it is subjected to a Dirichlet boundary condition, or a Robin boundary condition for which the convective heat transfer coefficient is very large and/or the thermal conductivity of the slab material is very small.

In an application type study, Saha and Acharya (2003) conducted a detailed parametric analysis of the unsteady three-dimensional flow and heat transfer in a pin-fin heat exchanger. The work was motivated by the desire to enhance the perform-of compact heat exchangers, which are designed to provide high heat transfer surface area per unit volume and to alter the fluid dynamics to enhance mixing. There have been several numerical studies of transient fins combined with complicating factors, such as natural convection (Hsu and Chen, 1991; and Benmadda and Lacroix, 1996), spatial arrays of fins (Tafti *et al.*, 1999; and Saha and Acharya, 2004) and phase change materials (Tutar and Akkoca, 2004).

There are few publications on transient experiments for determining heat transfer coefficients in fins. Mutlu and Al-Shemmeri (1993) studied a longitudinal array of straight fins suddenly heated at the base. The instantaneous heat transfer coefficient was found at one point on the fin as a ratio of the measured temperature to the measure heat flux. There are several papers on inverse technique for determination of heat transfer coefficients from temperatures measured in compact bodies suddenly placed in a Convection environment (Stolz, 1960; and Osman and Beck, 1990). In these studies, the heat transfer coefficient is found from a systematic comparison between the transient data and a mathematical model of the heat conduction in the body of interest.

EXACT UNSTEADY SOLUTIONS OF FINS FOR THREE DIFFERENT TIP CONDITIONS

Consider a straight fin initially in equilibrium with the surrounding fluid environment at temperature T_e. The fin has a constant cross-sectional area, but may be of any shape (pin, rectangular, etc.). For time $t > 0$ a steady heat flux is applied to the base of the fin. The temperature in the fin satisfies the following equations.

$$\frac{\partial^2 T}{\partial x^2} - m^2(T - T_e) = \frac{1}{\alpha}\frac{\partial T}{\partial x}\ ;\ 0<x<L \qquad ...(1)$$

$$\text{At } t = 0,\ T(x, 0) - T_e = 0 \qquad ...(2)$$

$$\text{At } x = 0,\ -k\frac{\partial T}{\partial x} = q_0 \qquad ...(3)$$

$$\text{At } x = L,\ k_2\frac{\partial T}{\partial x} + h_2(T - T_e) = 0 \qquad ...(4)$$

Quantity m is the fin parameter given by $m = \sqrt{\frac{hA_h}{kV}}$. The boundary condition at $x = L$ is a general condition that represents one of three different tip conditions for the fin. For a tip condition of the first kind, setting $k_2 = 0$ and $h_2 = 1$ represents a specified end temperature (at $T = T_e$). For a tip condition of the second kind, setting $k_2 = k$ and $h_2 = 0$ represents an insulated end condition. For a tip condition of the third kind, setting $k_2 = k$ represents convection at $x = L$. Usually the convective coefficient at the end of the fin is taken as same as that along the sides of the fin (i.e., $h_2 = h$ in general). Here, the results will be written out for three tip conditions. For the temperature-end condition (first kind),

$$T(x,t) - T_e = 2\frac{q_0 L}{k}\sum_{n=1}^{\infty}\frac{\cos(\beta_n x/L)}{m^2L^2 + \beta_n^2}$$
$$\times\left[1 - \exp\left[-\left(m^2L^2 + \beta_n^2\right)\alpha t/L^2\right]\right] \qquad ...(5)$$

Where $\beta_n = (n - 1/2)\pi$;

For the insulated end condition (second kind),

$$T(x,t) - T_e = \frac{q_0 L}{k}\frac{\left(1 - e^{-m^2\alpha t}\right)}{m^2L^2} + 2\frac{q_0 L}{k}\sum_{n=1}^{\infty}\frac{\cos(\beta_n x/L)}{m^2L^2 + \beta_n^2}$$
$$\times\left[1 - \exp\left[-\left(m^2L^2 + \beta_n^2\right)\alpha t/L^2\right]\right] \qquad ...(6)$$

Where $\beta_n = n\pi$ and for convective end condition.

$$T(x,t) - T_e = 2\frac{q_0 L}{k}\sum_{n=1}^{\infty}\left(\frac{\beta_n^2 + B_2^2}{\beta_n^2 + B_2^2 + B_2}\right)\frac{\cos(\beta_n x/L)}{m^2L^2 + \beta_n^2}$$
$$\times\left[1 - \exp\left[-\left(m^2L^2 + \beta_n^2\right)\alpha t/L^2\right]\right] \qquad ...(7)$$

Where β satisfies $\beta_n \tan\beta_n = B_2$

And where $B_2 = h_2L/k$.

Each solution contains a series that should be considered in two parts: a transient part with an exponential factor and a steady part with no exponential factor. Each of the transient series contains an exponential factor with argument $\left(m^2L^2 + \beta_n^2\right)\ \alpha t/L^2$, which defines the rate of decay of the unsteady. The decay rate depends on fin effects (through m^2L^2) and also on the tip condition (through β_n^2).

The series solution for the unsteady temperature in a flux-base fin is developed by the method of Green's functions.

First, a transformation (Ozisik, 1993) is used to remove the fin term from the heat conduction equation. Let

$$T - T_e = We^{-m^2\alpha t} \qquad ...(8)$$

and then transform Equations (1)-(4) to give:

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{\alpha}\frac{\partial w}{\partial t}\ ;\ 0 < x < L \quad ...(9)$$

$$\text{At } t = 0,\ W(x, 0) = 0 \quad ...(10)$$

$$\text{At } x = 0,\ -k\frac{\partial W}{\partial x} = q_0\, e^{m^2 \alpha t} \quad ...(11)$$

$$\text{At } x = L,\ k_2\frac{\partial w}{\partial x} + h_2 W = 0 \quad ...(12)$$

This transformed problem may be solved by the method of Green's functions in the form (Beck *et al.*, 1992, p. 165).

$$W(x, t) = \frac{\alpha}{k}\int_{t'=0}^{t} q_0\, e^{m^2 \alpha t'} G(x, t \mid x' = 0, t')dt' \quad ...(13)$$

The Green's function associated with function W is that for a plane wall, given by Cole (2008).

$$G(x, t \mid x', t') = \frac{x_0(0)x_0(0)}{N_0} + \sum_{n=1}^{\infty}\frac{x_n(x)x_n(x')}{N_n(\beta_n)}\, e^{-\beta_n^2 \alpha(t-t')/L^2} \quad ...(14)$$

The first term (for $n = 0$) is needed only for a type 2 (insulated) boundary at $x = L$. Eigen functions X_n, Eigen values β_n, and norms N_n are determined by the boundary conditions on the fin. For the flux-base fins of interest here, the Eigen functions are:

$$X_n(\beta_n) = \cos\left(\frac{\beta_n x}{L}\right) \quad ...(15)$$

And the eigen values and norms are given in Table 1. The number system in Table 1 for the three cases listed is $X2J$ where J = 1, 2, or 3 to represent tip conditions of the first kind (temperature), second kind (insulated) or third kind (convection), respectively. After the time integral in Equation (13) is evaluated, the transformation in Equation (8) can be reversed to find temperature T in the form:

$$T(x, t) - T_e = \frac{q_0 L}{k}\frac{L}{N_0}\frac{\left(1 - e^{-m^2 \alpha t}\right)}{m^2 L^2} + \frac{q_0 L}{k}\sum_{n=1}^{\infty}\frac{\cos(\beta_n x/L)}{m^2 L^2 + \beta_n^2} \times \left[1 - \exp\left[-\left(m^2 L^2 + \beta_n^2\right)\alpha t / L^2\right]\right] \quad ...(16)$$

Again, the first term is only used when the fin tip is insulated (see Kraus *et al.*, 2001, p. 765) for an independent derivation of the insulated-tip case). The above expression, with the eigen values and norms are given in Table 1, is limited to fins with a specified heat flux at the base (x = 0). However, the same approach could be used for fins with other base conditions with the appropriate plane wall Green's function. The plane-wall Green's functions for the temperature-base fin (type 1 boundary at x = 0) and the fin with the base temperature applied through a contact conductance (type 3 boundary at x = 0) are available elsewhere (see Beck *et al.*, 1992).

Table 1: Eigen Values for Three Different Tip Conditions

Case	$\frac{1}{N_n}$	**β_n or Eigen Condition**
$X21$	2	$\left(n - \frac{1}{2}\right)\pi$
$X22$	2; $n \neq 0$ 1; $n = 0$	$n\pi$
$X23$	$\frac{2\left(\beta_n^2 + B_2^2\right)}{\left(\beta_n^2 + B_2^2 + B_2\right)}$	$\beta_n \tan(\beta_n) = B_2$

Improvement of Series Convergence

It has long been known that classic solutions for the temperature in a body heated on a boundary contain a slowly converging steady-state series (Ozisik, 1993). In this section, the convergence of the transient solution is improved by replacing the steady series by a fully summed form. Although the steady-fin solutions are well known, a unified solution is presented with the method of Green's functions.

The steady temperature satisfies the following equations.

$$\frac{\partial^2 T}{\partial x^2} - \frac{hA_h}{kV}(T - T_e) = 0;\ 0 < x < L \qquad ...(17)$$

At $x = 0$, $-k\frac{\partial T}{\partial x} = q_0$ ...(18)

At $x = L$, $k_2\frac{\partial T}{\partial x} + h_2(T - T_e) = 0$ ...(19)

Again, the boundary condition at $x = L$ represents three kinds of tip conditions. Using the method of Green's functions, the steady-fin temperature has the form Cole (2004).

$$T(x) - T_e = \frac{q_0}{k} G_{X2J}(x,\ x' = 0) \qquad ...(20)$$

The symbol for Green's function G_{X2J} denotes a Cartesian coordinate system symbol X, boundary of the second kind at $x = 0$ (symbol 2), and boundary of type J at $x = L$ (symbol J) for $J = 1$, 2, or 3. This numbering system is used to catalog the many GF available on the Library web site (www.greensfunction.unl.edu). Table below shows the eigen values for three different tip conditions.

Green's function G_{X2J} for the steady-fin is given by Cole (2008).

$$G_{X2J}(x,\ x') = \frac{R\left(e^{-m(2L-|x-x'|)} + e^{-m(2L-x-x')}\right)}{D} + \left(e^{-m|x-x'|} + e^{-m(x+x')}\right)/D \qquad ...(21)$$

Where $D = 2m(1 - R.e^{-2mL})$

Coefficient R is determined by the tip condition:

$$R = \begin{cases} -1 & type\ 1\ at\ x = L \\ 1 & type\ 2\ at\ x = L \\ \dfrac{mL - B_2}{mL - B_2} & type\ 3\ at\ x = L \end{cases}$$

Where $B_2 = h_2L/k$.

The above GF may be evaluated at $x' = 0$ and substituted into

The above temperature expression, Equation (20), to give

$$T(x) - T_e = \frac{q_0 L}{k} \frac{\left(R \cdot e^{-m(2L-x)} + e^{-mx}\right)}{mL\left(2 - R \cdot e^{-2mL}\right)} \qquad ...(22)$$

Where coefficient R is given in the previous page.

Alternately, steady-fin solutions may be obtained from computer program TFIN described previously (Cole, 2004) that produce analytical expressions for the steady temperature in fins under a variety of boundary conditions. Program TFIN is also available for download at the Green's Function Library (Cole, 2008).

Next the closed-form steady solutions given above are used in the transient-fin solutions given earlier to replace the slowly converging series by replacing the series steady term with non series steady term as obtained in the Equation (22). The improved-convergence form of the transient temperature in flux-base fins are given by:

For the temperature tip condition (first kind),

$$T(x,\ t) - T_e = \frac{q_0 L}{k} \frac{\left(e^{-mx} - e^{-m(2L-x)}\right)}{mL\left(1 + e^{-2mL}\right)}$$

$$+2\frac{q_0 L}{k}\sum_{n=1}^{\infty}\frac{\cos(\beta_n x/L)}{m^2L^2+\beta_n^2}$$

$$\times\left[\exp\left[-\left(m^2L^2+\beta_n^2\right)\alpha t/L^2\right]\right] \quad ...(23)$$

Where $\beta_n = (n - 1/2)\pi$

For the insulated end condition (second kind),

$$T(x,t)-T_e = \frac{q_0 L}{k}\frac{\left(e^{-m(2L-x)}+e^{-mx}\right)}{mL\left(1-e^{-2mL}\right)} - \frac{q_0 L}{k}\frac{e^{-m^2\alpha t}}{m^2L^2}$$

$$-2\frac{q_0 L}{k}\sum_{n=1}^{\infty}\frac{\cos(\beta_n x/L)}{m^2L^2+\beta_n^2}$$

$$\times\left[\exp\left[-\left(m^2L^2+\beta_n^2\right)\alpha t/L^2\right]\right] \quad ...(24)$$

Where $\beta_n = n\pi$ and for convective tip condition (third kind)

$$T(x,t)-T_e = \frac{q_0 L}{k}\frac{\left(\frac{mL-B_2}{mL+B_2}e^{-m(2L-x)}+e^{-mx}\right)}{mL\left(1-\frac{mL-B_2}{mL+B_2}e^{-2mL}\right)}$$

$$-2\frac{q_0 L}{k}\sum_{n=1}^{\infty}\frac{\cos(\beta_n x/L)}{m^2L^2+\beta_n^2}\left(\frac{\beta_n^2+B_2^2}{\beta_n^2+B_2^2+B_2}\right)$$

$$\times\left[\exp\left[-\left(m^2L^2+\beta_n^2\right)\alpha t/L^2\right]\right] \quad ...(25)$$

Where β satisfies $\beta_n \tan\beta_n = B_2$ and where $B_2 = h_2L/k$

It is instructive to examine these three temperature solutions as a group. Each contains a steady term and each contains an additional term, a non-series transient. However, the insulated-tip solution uniquely contains another term, a non-series transient.

Quasi Steady Solution for Short Fins

The short fin is of interest for our particular application. The exact temperature expression for this case contains two terms: a series steady term and a series transient term. The series contains an exponential factor with argument $m^2L^2 + \beta_n^2$. By comparing these arguments, it is clear that as time increases the series term will decay more rapidly. This suggests that a quasi-steady solution may be constructed of the form

$$T^q(x,t) = T^s(x) + T^L(t)$$

Here T^s is the steady solution term and T^L is the transient solution term. Both the term contain series term. In quasi steady approach, series steady term is being transformed in to a non series term which transforms the expression in to an easily computed algebraic expression. Based on the above discussion of exponential arguments, the quasi-steady solution should be accurate for later time. The numerical results given in the next section are presented with the following dimensionless variables:

$$\theta = \frac{T-T_e}{q_0 L/k} \quad \xi = X = x/L \quad \tau = dt = \alpha t/L^2$$

$$M = \sqrt{Bi}\left(\frac{L}{V/A_h}\right) \quad Bi = \frac{h\left(V/A_h\right)}{k}$$

where θ = Dimensionless temperature

ξ = Dimensionless location

τ = Dimensionless time

M = Fin parameter

Bi = Biot number

With these parameters, the dimensionless quasi-steady temperature for short fin is given by putting the above values in the Equation (25):

$$\theta = \frac{e^{M\xi}\left[(M-B_2)e^{-2M} + e^{-2M\xi}(M+B_2)\right]}{M\left[(M+B_2)-(M-B_2)e^{-2M}\right]}$$

$$-2\sum_{n=1}^{\infty}\left(\frac{\beta_n^2\left(1+\tan^2\beta_n\right)}{\beta_n^2\left(1+\tan^2\beta_n\right)+\beta\tan\beta_n}\right)\frac{\cos(\beta_2\xi)}{\left(M^2+\beta_n^2\right)}$$

$$\exp\left[-\left(M^2+\beta_n^2\right)\xi\right] \qquad \text{...(26)}$$

And the dimensionless exact fin temperature for short fin is given by:

$$\theta = 2\sum_{n=1}^{\infty}\left(\frac{\beta_n^2\left(1+\tan^2\beta_n\right)}{\beta_n^2\left(1+\tan^2\beta_n\right)+\beta\tan\beta_n}\right)$$

$$\times\left[1-\exp\left[-\left(M^2+\beta_n^2\right)\tau\right]\right] \qquad \text{...(27)}$$

Where β_n satisfies $\beta_n \tan\beta_n = B_2$ with dimension less parameter here $\beta_n = n\pi$.

RESULTS AND DISCUSSION

Accuracy of Quasi Steady Solution

The quasi-steady solution is compared with the exact transient solution to determine the conditions under which the quasi-steady solution is accurate. Different dimensionless parameter were considered (M, τ and location) for obtaining the temperature distribution curves. While variation of one parameter was considered the other variables were kept constant as indicated in the graphs. Based on the analysis, quasi-steady solution has been proposed as an accurate solution at large dimensionless times, which is independent of geometry of the problem.

Accuracy of Quasi Steady Solution for *M* = 1 at Dimensionless Locations 0.0, 0.5 and 1.0

Figures 1, 2 and 3 shows the (dimensionless) temperature versus time at three different positions on the fin, all for M = 1.0. For all values of dimensionless time the quasi-steady theory estimates the exact values at x/L = 0 and x/L = 1.0. For all locations the agreement improves as time increases.

Figure 1: Temperature History in a Fin of Constant Cross Section for Both Quasi Steady Theory and Exact Theory for *M* = 1 at Location *x/L* = *X* = 0.0

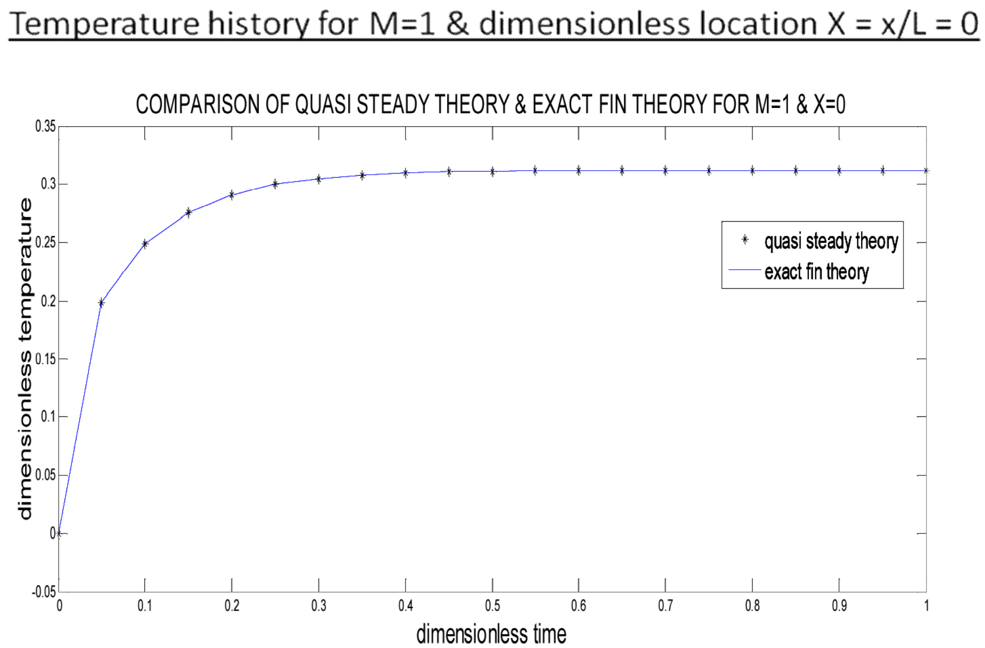


Figure 2: Temperature History in a Fin of Constant Cross Section for Both Quasi Steady Theory and Exact Theory for *M* = 1 at Location *x*/*L* = *X* = 0.5

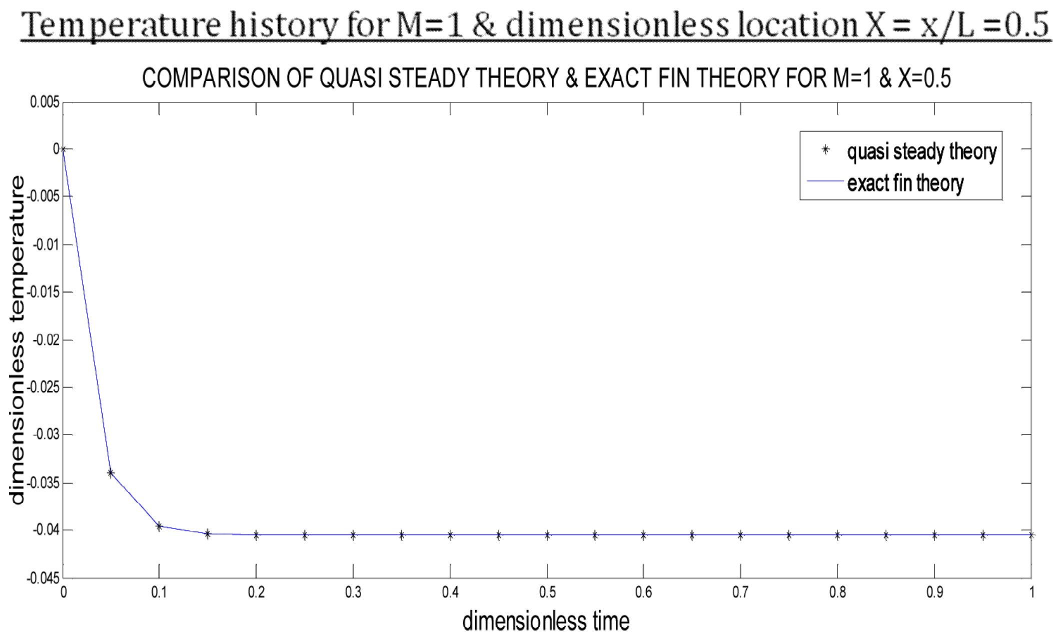
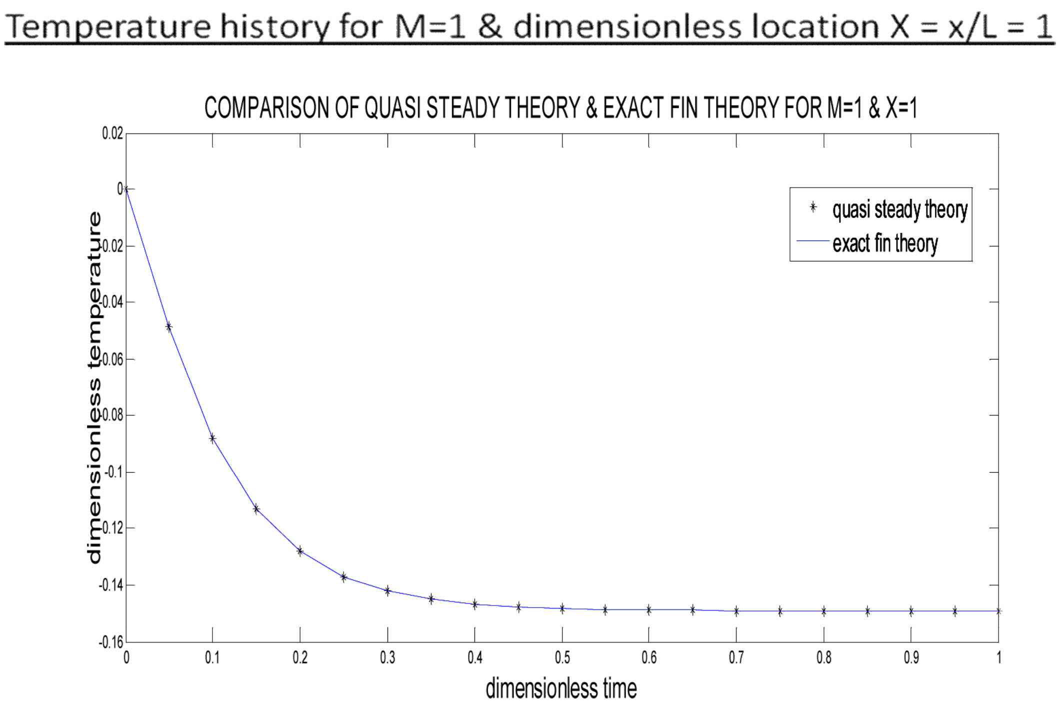


Figure 3: Temperature History in a Fin of Constant Cross Section for Both Quasi Steady Theory and Exact Theory for *M* = 1 at Location *x*/*L* = *X* = 1.0



- Figures 1, 2 and 3 shows the (dimensionless) temperature versus time at three different positions on the fin, all for $M = 1$.

- The quasi-steady theory estimates the exact values at x/L = 0.0, 0.5 and 1.0.
- For all locations the agreement improves as time increases.

Accuracy of Quasi Steady Solution For ξ = X = 0 at Fin Parameter M = 0.2, 1.0 and 5.0

Figures 4, 5 and 6 shows temperature versus time at X = 0 for M = 0.2, 1.0 and 5.0. At M = 5.0 the fin transient ends quickly so that this fin reaches steady-state at about τ = 0.1. As M decreases the temperature distribution takes longer and longer to reach steady-state. Fin parameter M may be interpreted as a ratio of thermal resistances. Specifically, M^2 is the thermal resistance along the fin length divided by the convective thermal resistance from the surface of the fin. Thus when M is small, the convective thermal resistance from the surface of the fin is large compared to the thermal resistance along the fin, producing a long, slow transient.

Specific values of the dimensionless temperature in the quasi-steady theory for several values of dimensionless time and several values of fin parameter M, all at different values of x/L (dimensionless location). Dimensionless temperature calculated is then incorporated and analyzed with help of graphs for both the theories. Followings are the results incorporated from the graphs.

- Figures 4, 5 and 6 shows temperature versus time at X = 0 for M = 0.2, 1.0 and 5.0.
- At M = 5 the fin transient ends quickly so that this fin reaches steady-state at about τ = 0.1.
- As M decreases the temperature distribution takes longer and longer to reach steady-state.

Figure 4: Temperature History in a Fin of Constant Cross-Section for Both Quasi Steady Theory and Exact Fin Theory at Location x/L = X = 0 for M = 0.2

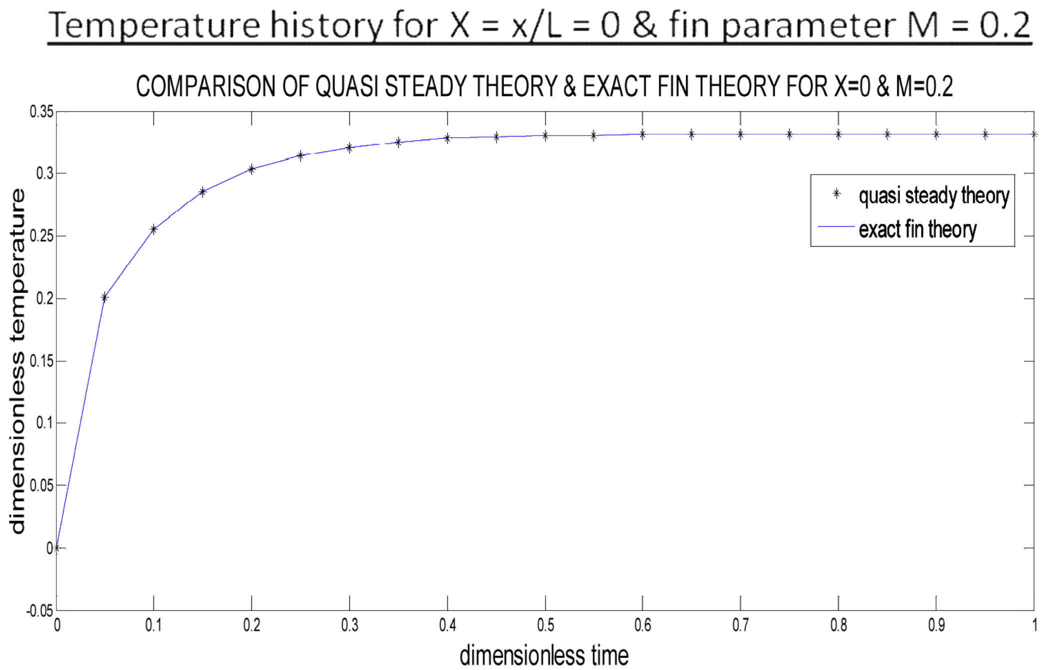


Figure 5: Temperature History in a Fin of Constant Cross-Section for Both Quasi Steady Theory and Exact Fin Theory at Location $x/L = X = 0$ for $M = 1.0$

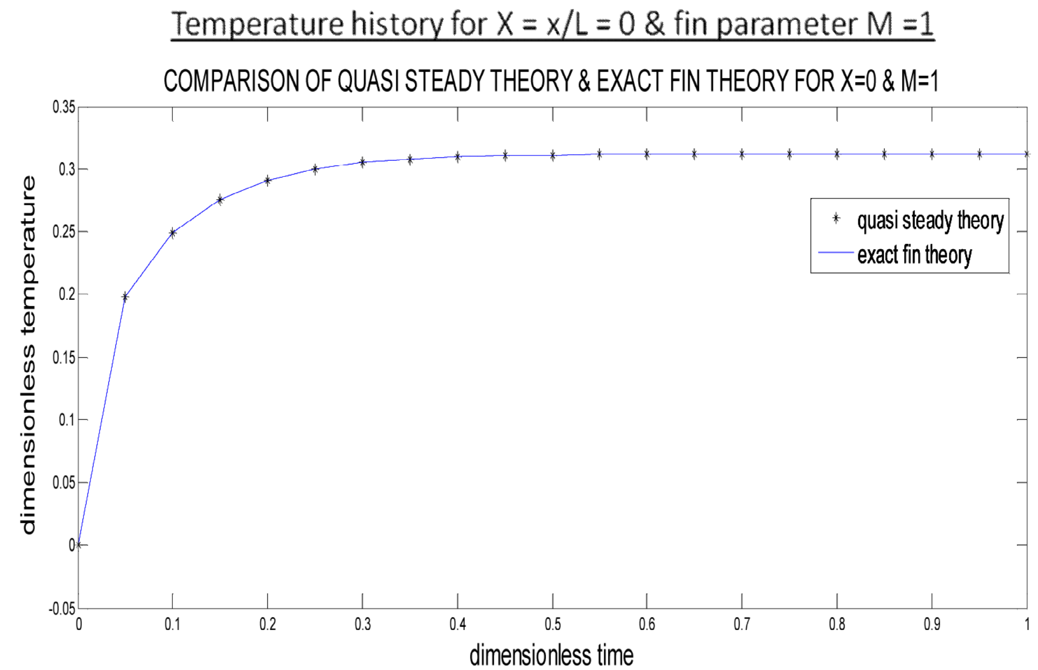
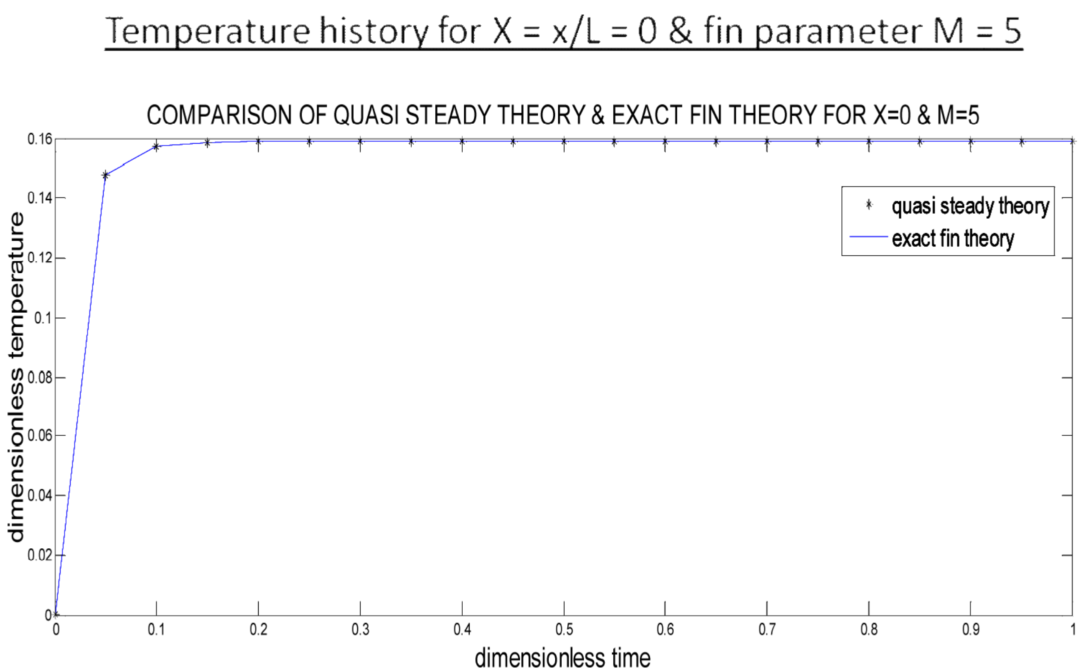


Figure 6: Temperature History in a Fin of Constant Cross-Section for Both Quasi Steady Theory and Exact Fin Theory at Location $x/L = X = 0$ for $M = 5.0$



- Fin parameter M may be interpreted as a ratio of thermal resistances.
- Thus when M is small, the convective thermal resistance from the surface of the

fin is large compared to the thermal resistance along the fin, producing a long, slow unsteady state.

CONCLUSION

A unified theory has been presented for unsteady heat transfer in flux-base fins for three tip conditions. The method may be easily extended to fins with other base conditions. A quasi steady theory has been applied to a case of straight fin with short length tip in the form of a simple, non-series expression for steady term.

- The quasi-steady theory is simple and efficient for computing numerical values compared to the exact series solution.
- A comparison with an exact series solution for the unsteady condition fin shows that the quasi-steady theory is accurate within dimensionless times for all values of the fin parameter *M*.
- The results show that the quasi-steady fin model is a simple way to find heat transfer coefficients for larger dimensionless times.
- Complicated exact unsteady solutions can be simplified for temperature distribution analysis through Green's function method.
- A unified theory is obtained for unsteady heat transfer in flux-base fins for three tip conditions.

Based on the analysis, the following conclusions have been obtained.

- For $M > 1$ the accurate range extends to all dimensionless times except $\tau = 0$.
- The accuracy increases for large dimensionless times, where the sensitivity to heat transfer coefficient is largest.
- For $M = 0.2$, 1.0 and 5.0. For $\tau = 0$ the quasi-steady theory overestimates the exact values at $\xi = X = 0$ and estimates the exact values at $\xi = X = 1.0$. For all locations the agreement improves as time increases.
- At $M = 5$ and $\xi = X = 0$ the fin transient ends quickly so that this fin reaches steady-state at about $\tau = 0.2$. As *M* decreases, the temperature distribution takes longer and longer to reach steady-state.
- When *M* is small, the convective thermal resistance of the fin surface is large compared to the thermal resistance along the fin, producing a long, slow transient.
- For $M \leq 1$ and $M > 1$. There is no error for dimensionless time $\tau > 0$. the region of small error extends to earlier time.
- The quasi-steady and exact temperatures agrees closely except at early time ($\tau = 0$).

SCOPE FOR FUTURE WORK

- It is suggested that the quasi-steady approach could be successfully applied to other fin geometries with different tip conditions or other fins for which exact solutions are difficult to be obtained.
- The results show that the quasi-steady fin approach can be a simple way to find heat transfer coefficient associated with heat loss.
- The heat transfer coefficients obtained by this method are intended for future use as an external boundary condition for more elaborate thermal models.

REFERENCES

1. Aziz A and Na T Y (1980), "Transient Response of Fins by Coordinate

Perturbation Expansion", *Int. J. Heat and Mass Transfer*, Vol. 23, pp. 1695-1698.

2. Aziz A and Kraus A D (1995), "Transient Heat Transfer in Extended Surfaces", *Appl. Mech. Rev.*, Vol. 48, No. 3, pp. 317-349.

3. Beck J V, Cole K D, Haji-Sheikh A and Litkouhi B (1992), "Heat Conduction Using Green's Functions Hemisphere", New York.

4. Benmadda M and Lacroix M (1996), "Transient Natural Convection from a Finned Surface for Thermal Storage in an Enclosure", *Numer. Heat Transf. A*, Vol. 29, No. 1, pp. 103-114.

5. Campo A and Salazar A (1996), "Similarity Between Unsteady-State Conduction in a Planar Slab for Short Times and Steady-State Conduction in a Uniform, Straight Fin", *Heat Mass Transf.*, Vol. 31, No. 5, pp. 365-370.

6. Chapman A J (1959), "Transient Heat Conduction in Annular Fins of Uniform Thickness", *Chem. Eng. Symp.*, Vol. 55, No. 29, pp. 195-201.

7. Cole K D (2004), "Computer Programs for Temperature in Fins and Slab Bodies with the Method of Green's Functions", *Comput. Appl. Eng. Educ.*, Vol. 12, No. 3, pp. 189-197.

8. Cole K D (2008), *Green's Function Library*, available at www.greensfunction.unl.edu (Accessed on May 15, 2008).

9. Cole K D, Tarawneh C and Wilson B (2009), "Analysis of Flux Base Fins for Estimation of Heat Transfer Coefficient", *Int. J. Heat and Mass Transfer*.

10. Donaldson A B and Shouman A R (1972), "Unsteady-State Temperature Distribution in a Convecting Fin of Constant Area", *Appl. Sci. Res.*, Vol. 26, Nos. 1-2, pp. 75-85.

11. Hsu T H and Chen C K (1991), "Transient Analysis of Combined Forced and Free-Convection Conduction Along a Vertical Circular Fin in Micro Polar Fluids", *Numer. Heat Transf. A*, Vol. 19, No. 2, pp. 177-185.

12. Kim R H (1976), "The Kantorovich Method in the Variational Formulation to an Unsteady Heat Conduction Lett.", *Heat Mass Transf.*, Vol. 3, No. 1, pp. 73-80.

13. Kraus A D, Aziz A and Welty J (2001), "Extended Surface Heat Transfer", Wiley, New York.

14. Mao J and Rooke S (1994), "Transient Analysis of Extended Surfaces with Convective Tip", *Int. Commun. Heat Mass Transf.*, Vol. 21, pp. 85-94.

15. Mutlu I and Al-Shemmeri T T (1993), "Steady-State and Transient Performance of a Shrouded Longitu-Dinal Fin Array", *Int. Commun. Heat Mass Transf.*, Vol. 20, pp. 133-143.

16. Osman A M and Beck J V (1990), "Investigation of Transient Heat Coefficients in Quenching Experiment", *J. Heat Transf.*, Vol. 112, pp. 843-848.

17. Ozisik M N (1993), *Heat Conduction*, p. 89, Wiley, New York.

18. Saha A K and Acharya S (2003), "Parametric Study of Unsteady Flow and Heat Transfer in a Pin-Fin Heat Exchanger", *Int. J. Heat Mass Transf.*, Vol. 46, No. 20, pp. 3815-3830.

19. Saha A K and Acharya S (2004), "Unsteady Simulation of Turbulent Flow and Heat Transfer in a Channel with Periodic Array of Cubic Pin-Fins", *Numer. Heat Transf. A*, Vol. 46, No. 8, pp. 731-763.

20. Stolz G Jr. (1960), "Numerical Solution to an Inverse Problem of Heat Conduction for Simple Shapes", *J. Heat Transf.*, Vol. 82, pp. 20-26.

21. Suryanarayana N V (1975), "Transient Response of Straight Fins", *J. Heat Transf.*, Vol. 97, pp. 417-423.

22. Suryanarayana N V (1976), "Transient Response of Straight Fins Part II", *J. Heat Transf.*, Vol. 98, pp. 324-326.

23. Tafti D K, Zhang L W and Wang G (1999), "Time-Dependent Calculation Procedure for Fully Developed and Developing Flow and Heat Transfer in Louvered Fin Geometries", *Numer. Heat Transf. A*, Vol. 35, No. 3, pp. 225-249.

24. Tutar M and Akkoca A (2004), "Numerical Analysis of Fluid Flow and Heat Transfer Characteristics in Three-Dimensional Plate Fin-and-Tube Heat Exchangers", *Numer. Heat Transf. A*, Vol. 46, No. 3, pp. 301-321.

APPENDIX

Nomenclatures

		Greek	
A_h	Surface area of fin for convection (m^2)	α	Thermal diffusivity (m^{2}s^{-1})
Bi	Biot number, $h_i\,(V/A_h)/K$	β_n	Eigen value [Equation (14)]
B_2	Biot number, hL/k	θ	Dimensionless temperature
G	Green's function	ξ, X	Dimensionless x-coordinate
h	Heat transfer coefficient (W m^{-2}K^{-1})	τ, dt	Dimensionless time
k	Thermal conductivity (W m^{-1}K^{-1})	**Superscripts**	
L	Length of fin (m)	q	Quasi-steady
N_n	Norm [Equation (14)] (m)	s	Steady state
m	Fin parameter, (m^{-1})		
M	Dimensionless fin parameter = mL		
q_0	Heat flux (W m^{-2})		
Q	Input heat (W)		
T	Temperature (K)		
t	Time (s)		
V	Fin volume (m^3)		
W	Transformed temperature [Equation (13)]		