



Research Paper

RELIABILITY EVALUATION OF REPAIRABLE COMPLEX SYSTEMS AN ANALYZING FAILURE DATA

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Reliability is one of the important parameters, which contributes to customer satisfaction. So every aspect of design and manufacturing, quality engineering and control directly influences the reliability of a product. As the importance of reliability is growing in the market scenario, and as the cost of maintaining the equipment is increasing, it would necessitate every organization to focus its interest to increase the reliability of their equipment as well as their products. Before going for the reliability improving acts, it is necessary and good to measure the current performance. As there are many more difficulties in analyzing complex systems, which have repairable components, focus of the project work is concentrated on it. In the present work attempts is made to collecting data, identifying the data type and evaluate the reliability of some systems. We have used for power-law for evaluate the reliability of some systems. The effort is put evaluate complex system reliability from the component reliabilities that have been evaluated by analyzing field failure data. The analysis of reliability measurement is mainly focused on repairable units, for which different maintenance polices are available. By using 'Bottom-Up' approach it has been tried to evaluate the system reliability of complex repairable systems with help of component reliabilities. For all these problems and difficulties, a program in C-language is written.

Keywords: Reliability evaluaton, Bottom-up approach, Failure data

INTRODUCTION

In today's technological world nearly everyone depends upon the continued functioning of a wide array of complex machinery and equipment for their everyday health, safety,

mobility and economic welfare. We expect our cars, computers, electrical appliances, lights, televisions, etc., to function whenever we need them-day after day, year after year. When they fail the results can be catastrophic: injury, loss

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of life and/or costly lawsuits can occur. More often, repeated failure leads to annoyance, inconvenience and a lasting customer dissatisfaction that can play havoc with the responsible company's marketplace position.

It takes a long time for a company to build up a reputation for reliability, and only a short time to be branded as "unreliable" after shipping a flawed product. Continual assessment of new product reliability and ongoing control of the reliability of everything shipped are critical necessities in today's competitive business arena.

BASIC TERMS AND MODELS USED FOR RELIABILITY EVALUATION

Reliability theory developed apart from the mainstream of probability and statistics, and was used primarily as a tool to help nineteenth century maritime and life insurance companies compute profitable rates to charge their customers. Even today, the terms "failure rate" and "hazard rate" are often used interchangeably.

The following sections will define some of the concepts, terms, and models we need to describe, estimate and predict reliability

Reliability or Survival Function

The Reliability Function $R(t)$, also known as the Survival Function $S(t)$, is defined by:

$R(t) = S(t)$ = The probability a unit survives beyond time t .

Since a unit either fails, or survives, and one of these two mutually exclusive alternatives must occur, we have

$$R(t) = 1 - F(t), F(t) = 1 - R(t)$$

Calculations using $R(t)$ often occur when building up from single components to subsystems with many components. For example, if one microprocessor comes from a population with reliability function $R_m(t)$ and two of them are used for the CPU in a system, then the system CPU has a reliability function given by

$$R_{cpu}(t) = R_m^2(t)$$

Survival is the Complementary Event to Failure

A different approach is used for modeling the rate of occurrence of failure incidences for a repairable system. In this chapter, these rates are called repair rates (not to be confused with the length of time for a repair, which is not discussed in this chapter). Time is measured by system power-on-hours from initial turn-on at time zero, to the end of system life. Failures occur at given system ages and the system is repaired to a state that may be the same as new, or better, or worse. The frequency of repairs may be increasing, decreasing, or staying at a roughly constant rate.

Let $N(t)$ be a counting function that keeps track of the cumulative number of failures a given system has had from time zero to time t . $N(t)$ is a step function that jumps up one every time a failure occurs and stays at the new level until the next failure.

Every system will have its own observed $N(t)$ function over time. If we observed the $N(t)$ curves for a large number of similar systems and "averaged" these curves, we would have an estimate of $M(t)$ = the expected number (average number) of cumulative failures by time t for these systems.

A non-repairable population is one for which individual items that fail are removed permanently from the population. While the system may be repaired by replacing failed units from either a similar or a different population, the members of the original population dwindle over time until all have eventually failed. We begin with models and definitions for non-repairable populations. Repair rates for repairable populations will be defined in a later section.

The theoretical population models used to describe unit lifetimes are known as Lifetime Distribution Models. The population is generally considered to be all of the possible unit lifetimes for all of the units that could be manufactured based on a particular design and choice of materials and manufacturing process. A random sample of size n from this population is the collection of failure times observed for a randomly selected group of n units.

A lifetime distribution model can be any probability density function (or PDF) $f(t)$ defined over the range of time from $t = 0$ to $t = \infty$. The corresponding cumulative

distribution function (or CDF) $F(t)$ is a very useful function, as it gives the probability that a randomly selected unit will fail by time t . The Figure 1 shows the relationship between $f(t)$ and $F(t)$ and gives three descriptions of $F(t)$.

1. $F(t)$ = The area under the PDF $f(t)$ to the left of t .
2. $F(t)$ = The probability that a single randomly chosen new unit will fail by time t .
3. $F(t)$ = The proportion of the entire population that fails by time t .

The Figure 1 also shows a shaded area under $f(t)$ between the two times t_1 and t_2 . This area is $[F(t_2) - F(t_1)]$ and represents the proportion of the population that fails between times t_1 and t_2 (or the probability that a brand new randomly chosen unit will survive to time t_1 but fail before time t_2).

Note that the PDF $f(t)$ has only non-negative values and eventually either becomes 0 as t increases, or decreases towards 0. The CDF $F(t)$ is monotonically increasing and goes from 0 to 1 as t approaches infinity. In other words, the total area under the curve is always 1.

The 2-parameter Weibull distribution is an example of a popular $F(t)$. It has the CDF and PDF equations given by:

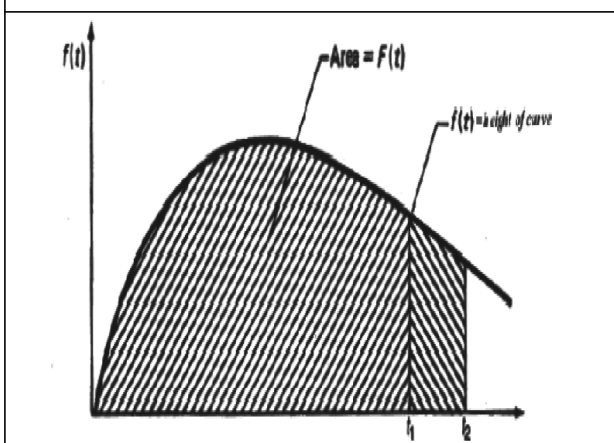
$$F(t) = 1 - e^{-\left(\frac{t}{\alpha}\right)^\gamma}, f(t) = \frac{\gamma}{t} \left(\frac{t}{\alpha}\right)^{\gamma-1} e^{-\left(\frac{t}{\alpha}\right)^\gamma}$$

where γ is the 'shape' parameter and α is a scale parameter called the characteristic life.

Censoring

When not all units on test fail we have censored data:

Figure 1: The Relation Between Function ($f(t)$) and Time



Consider a situation in which we are reliability testing n (non repairable) units taken randomly from a population. We are investigating the population to determine if its failure rate is acceptable. In the typical test scenario, we have a fixed time T to run the units to see if they survive or fail. The data obtained are called Censored Type 1 data.

Censored Type 1 Data

During the T hours of test we observe r failures (where r can be any number from 0 to n). The (exact) failure times are $t_1, t_2, \dots, t_r$ and there are $(n - r)$ units that survived the entire T -hour test without failing. Note that T is fixed in advance and r is random, since we don't know how many failures will occur until the test is run. Note also that we assume the exact times of failure are recorded when there are failures.

This type of censoring is also called "right censored" data since the times of failure to the right (i.e., larger than T) are missing.

Another (much less common) way to test is to decide in advance that you want to see exactly r failure times and then test until they occur. For example, you might put 100 units on test and decide you want to see at least half of them fail. Then $r = 50$, but T is unknown until the 50th fail occurs. This is called Censored Type 2 data.

Censored Type 2 Data

We observe $t_1, t_2, \dots, t_r$, where r is specified in advance. The test ends at time $T = t_r$, and $(n - r)$ units have survived. Again we assume it is possible to observe the exact time of failure for failed units.

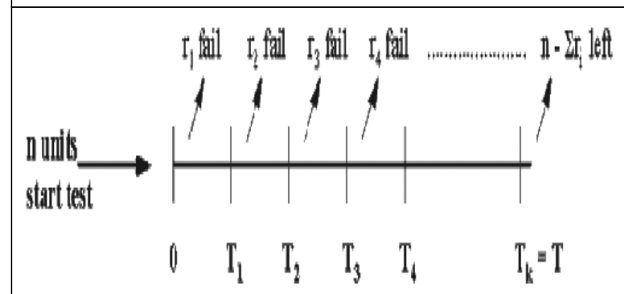
Type 2 censoring has the significant advantage that you know in advance how many

failure times your test will yield—this helps enormously when planning adequate tests. However, an open-ended random test time is generally impractical from a management point of view and this type of testing is rarely seen.

Readout or Interval Data

Sometimes exact times of failure are not known; only an interval of time in which the failure occurred is recorded. This kind of data is called Readout or Interval data and the situation was shown in the Figure 2.

Figure 2: Basics for Analysing Failure Data



PLOTTING RELIABILITY DATA

Graphical plots of reliability data are quick, useful visual tests of whether a particular model is consistent with the observed data. The basic idea behind virtually all graphical plotting techniques is the following: Points calculated from the data are placed on specially constructed graph paper and, as long as they line up approximately on a straight line, the analyst can conclude that the data are consistent with the particular model the paper is designed to test.

If the reliability data consist of (possibly multicensored) failure data from a non repairable population (or a repairable population for which only time to the first failure is considered) then the models are life

distribution models such as the exponential, Weibull or lognormal. If the data consist of repair times for a repairable system, then the model might be the NHPP Power Law and the plot would be a Duane Plot. The kinds of plots we will consider for failure data from non-repairable populations are:

- Probability (CDF) plots
- Hazard and Cum Hazard plots

Probability Plotting

Probability plots are simple visual ways of summarizing reliability data by plotting CDF estimates vs. time on specially constructed probability paper.

Commercial papers are available for all the typical life distribution models. One axis (some papers use the y-axis and others the x-axis, so you have to check carefully) is labeled "Time" and the other axis is labeled "Cum Percent" or "Percentile". There are rules, independent of the model or type of paper, for calculating plotting positions from the reliability data. These only depend on the type of censoring in the data and whether exact times of failure are recorded or only readout times.

When the points are plotted, the analyst fits a straight line through them (either by eye, or with the aid of a least squares fitting program). Every straight line on, say, Weibull paper uniquely corresponds to a particular Weibull life distribution model and the same is true for lognormal or exponential paper. If the points follow the line reasonably well, then the model is consistent with the data. If it was your previously chosen model, there is no reason to question the choice. Depending on the type of paper, there will be a simple way to find the

parameter estimates that correspond to the fitted straight line.

Plotting Positions: Censored Data (Type 1 or Type 2).

At the time t_i of the i^{th} failure, we need an estimate of the CDF (or the Cum. Population Percent Failure). The simplest and most obvious estimate is just $100 \times i/n$ (with a total of n units on test). This, however, is generally an overestimate (i.e., biased). Various texts recommend corrections such as $100 \times (i - 0.5)/n$ or $100 \times i/(n + 1)$. Here, we recommend what are known as (approximate) median rank estimates: Corresponding to the time t_i of the i^{th} failure, use a CDF or Percentile estimate of $100 \times (i - 0.3)/(n + 0.4)$.

Plotting Positions: Readout Data Let the readout times be $T_1, T_2, \dots, T_k$ and let the corresponding new failures recorded at each readout be $r_1, r_2, \dots, r_k$. Again, there are n units on test. Corresponding to the readout time T_j , use

a CDF or Percentile estimate of $\frac{100 \times \sum_{i=1}^j r_i}{n}$.

Plotting Positions: Multicensored Data

Hazard and Cum Hazard Plotting

Just commercial probability paper is available for most life distribution models for probability plotting of reliability data, there are also special Cum Hazard Plotting papers available for many life distribution models. These papers plot estimates for the Cum Hazard $H(t_i)$ vs. the time t_i of the i^{th} failure. As with probability plots, the plotting positions are calculated independently of the model or paper used and a reasonable straight-line fit to the points confirms that the chosen model and the data are consistent.

Evaluation of Reliability from the "Bottom-Up" (Component Failure Mode to System Failure Rate)

This section deals with models and methods that apply to non-repairable components and systems. Models for failure rates (and not repair rates) are described.

We use the Series Model to go from components to assemblies and systems. These models assume independence and "first failure mode to reach failure causes both the component and the system to fail".

If some components are "in parallel", so that the system can survive one (or possibly more) component failures, we have the parallel or redundant model. If an assembly has n identical components, at least r of which must be working for the system to work, we have what is known as the r out of n model.

The standby model uses redundancy like the parallel model, except that the redundant unit is in an off-state (not exercised) until called upon to replace a failed unit. Complex systems can be evaluated using the various models as building blocks.

SERIES MODEL

The series model is used to go from individual components to the entire system, assuming the system fails when the first component fails and all components fail or survive independently of one another

The Series Model is used to build up from components to sub-assemblies and systems. It only applies to non-replaceable populations (or first failures of populations of systems). The assumptions and formulas for the Series Model are identical to those for the Competing

Risk Model, with the k failure modes within a component replaced by the n components within a system.

The following 3 assumptions are needed:

1. Each component operates or fails independently of every other one, at least until the first component failure occurs.
2. The system fails when the first component failure occurs.

Each of the n (possibly different) components in the system has a known life distribution model $F_i(t)$.

Add failure rates and multiply reliabilities in the Series Model.

When the Series Model assumptions hold we have: with the subscript S referring to the entire system and the subscript i referring to the i^{th} component.

Note that the above holds for any arbitrary component life distribution models, as long as "independence" and "first component failure causes the system to fail" both hold.

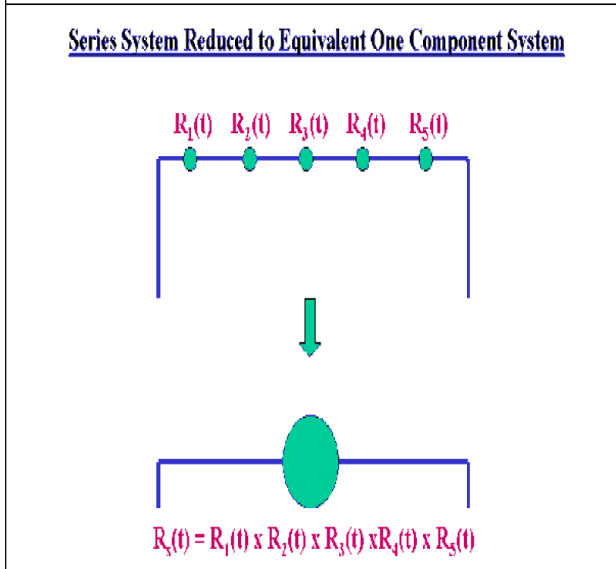
The analogy to a series circuit is useful. The entire system has n components in series. The system fails when current no longer flows and each component operates or fails independently of all the others. The schematic below shows a system with 5 components in series "replaced" by an "equivalent" (as far as reliability is concerned) system with only one component).

$$R_S(t) = \prod_{i=1}^n R_i(t)$$

$$F_S(t) = 1 - \prod_{i=1}^n \{1 - F_i(t)\}$$

$$h_s(t) = \sum_{i=1}^n h_i(t)$$

Figure 3: Series System Reduced to Equivalent One Component System



Parallel or Redundant Model

The parallel model assumes all n components that make up a system operate independently and the system works as long as at least one component still works.

The opposite of a series model, for which the first component failure causes the system to fail, is a parallel model for which all the components have to fail before the system fails. If there are n components, any $(n - 1)$ of them may be considered redundant to the remaining one (even if the components are all different). When the system is turned on, all the components operate until they fail. The system reaches failure at the time of the last component failure.

The assumptions for a parallel model are:

- All components operate independently of one another, as far as reliability is concerned.

- The system operates as long as at least one component is still operating. System failure occurs at the time of the last component failure.

The CDF for each component is known

Multiply component CDF's to get the system CDF for a parallel model

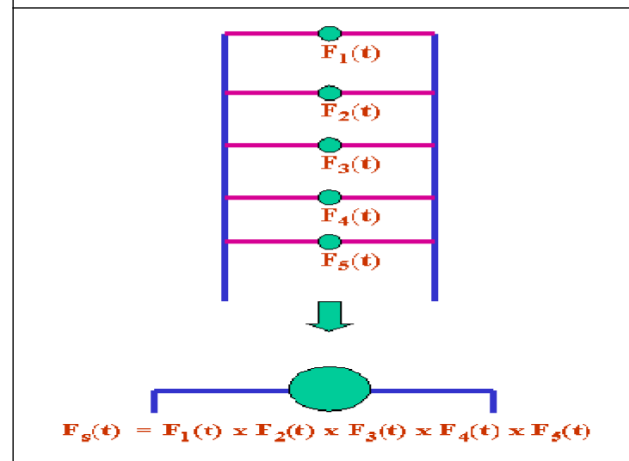
For a parallel model, the CDF $F_s(t)$ for the system is just the product of the CDF's $F_i(t)$ for the components or

$$F_s(t) = \prod_{i=1}^n F_i(t)$$

$R_s(t)$ and $h_s(t)$ can be evaluated using basic definitions, once we have $F_s(t)$.

The schematic below represents a parallel system with 5 components and the (reliability) equivalent 1 component system with a CDF F_s equal to the product of the 5 component CDF's.

Figure 4: Parallel System and Equivalent Single Component



STANDBY MODEL

The Standby Model evaluates improved reliability when backup replacements are switched on when failures occur.

A Standby Model refers to the case in which a key component (or assembly) has an identical backup component in an “off” state until needed. When the original component fails, a switch turns on the “standby” backup component and the system continues to operate.

In the simple case, assume the non-standby part of the system has CDF $F(t)$ and there are $(n - 1)$ identical backup units that will operate in sequence until the last one fails. At that point, the system finally fails.

The total system lifetime is the sum of n identically distributed random lifetimes, each having CDF $F(t)$.

Identical backup Standby model leads to convolution formulas

In other words, $T_n = t_1 + t_2 + \dots + t_n$, where each t_i has CDF $F(t)$ and T_n has a CDF we denote by $F_n(t)$. This can be evaluated using convolution formulas:

$$F_2(t) = \int_0^t F(u) f(t-u) du$$

$$F_n(t) = \int_0^t F_{n-1}(u) f(t-u) du$$

where $f(t)$ is the PDF $F'(t)$

In general, convolutions are solved numerically. However, for the special case when $F(t)$ is the exponential model, the above integrations can be solved in closed form.

Exponential standby systems lead to a gamma Special Case: The Exponential (or Gamma) Standby Model:

If $F(t)$ has the exponential CDF (i.e., $F(t) = 1 - e^{-\lambda t}$), then

$$F_2(t) = 1 - \lambda t e^{-\lambda t} - e^{-\lambda t}$$

$$f_2(t) = \lambda^2 t e^{-\lambda t}, \text{ and}$$

$$f_n(t) = \frac{\lambda^n t^{n-1} e^{-\lambda t}}{(n-1)!}$$

and the PDF $f_n(t)$ is the well-known gamma distribution.

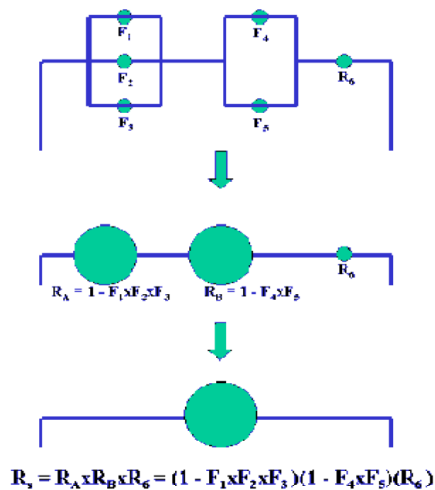
Standby units are an effective way of increasing reliability and reducing failure rates, especially during the early stages of product life. Their improvement effect is similar to, but greater than, that of parallel redundancy. The drawback, from a practical standpoint, is the expense of extra components that are not needed for functionality. Exponential standby systems lead to a gamma lifetime model.

COMPLEX SYSTEMS

Often the reliability of complex systems can be evaluated by successive applications of Series and/or Parallel model formulas.

Many complex systems can be diagrammed as combinations of Series components, parallel components, R out of N components and Standby components. By using the formulas for these models, subsystems or sections of the original system can be replaced by an “equivalent” single component with a known CDF or Reliability function. Proceeding like this, it may be possible to eventually reduce the entire system to one component with a known CDF.

Below is an example of a complex system composed of both components in parallel and components in series is reduced first to a series system and finally to a one-component system.

Figure 5: Complex System Reduced to Equivalent One component SystemComplex System Reduced to Equivalent One Component System**CASE STUDY**

In our project work, the thermal power plant is considered as “Repairable complex system”. As the thermal power plant is electricity, which is a continuously produced product, it would be somewhat difficult to define the failure. But, as the aim of any thermal power plant is to produce the continuous flow of good quality of electricity, any violation to it can be considered as the “Failure” for the system. But it is very difficult to continuously monitor the quality of the product and also it is a cumbersome process, it can’t be defined as failure. The fact that fluctuations in the quality of electricity is inevitable, will also add value to the above statement. So the failure of the

Table 1: Tripping Data of Each Unit

System	Sub System	Component	Sub Component	β	θ	Reliability Values			
						Sub Component	Component	Sub System	System
Power Plant	Unit-1	Boiler	Tube Failures	0.1	166	0.49265	0.09133	0.00854	0.03543
			Boiler Leakages	0.1	11	0.3632			
			Ash Hopper			1			
			Furnace	0.1	456	0.51044			
		Turbine	Turbine			1	1.00		
			Economiser			1			
		Generator	Potential Transformers	0.2	82233.35	0.762684	0.215		
			Bushes			1			
			Brushes	0.2	2199007	0.869			
			Rotor			1			
			Stator			1			
		Transmission	Grid Failure			0.99	0.435		
			Rely	0.1	6	0.4795			
			Current Transformers	0.1	4835632480	0.9167			

Table 1 (Cont.)

System	Sub System	Component	Sub Component	β	θ	Reliability Values			
						Sub Component	Component	Sub System	System
	Unit-2	Boiler	Tube Failures	0.1	271	0.5662	0.16	0.02712	
			Boiler Leakages			1.0			
			Ash Hopper	0.1	257556.15	0.6285			
			Furnace	0.1	46	0.4478			
		Turbine	Turbine			1	1.00		
			Economiser	0.1	6	1			
		Generator	Generator Conditions	0.1	36	0.324	0.1712		
			Potential Transformers			0.762684			
			Bushes	0.3	16910.28	0.7972			
			Brushes	0.2	2199007	0.869			
			Rotor			1			
			Stator			1			
		Transmission	Grid Failure			0.99	0.99		
			Rely			1			
			Current Transformers			1			

system is considered as the obstacle in the production of the electricity. The maintenance taken by maintenance personnel may be obstacle to the production of the electricity. But as it is inevitable to maintain longer life without this obstacle. It is necessary to have measurement technique of reliability so that effective analysis and evaluation of the policies can be made to obtain economical solution. While evaluating the system reliability, the past field failure data and the structure of various subsystems need to be studied, so the failure data here is the tripping data of each unit (Table 1). After identification of basic components, subsystems and

systems, the next system is to get the field data. The contribution of each major and critical sub components, which cause the failure of the system as a whole.

CONCLUSION

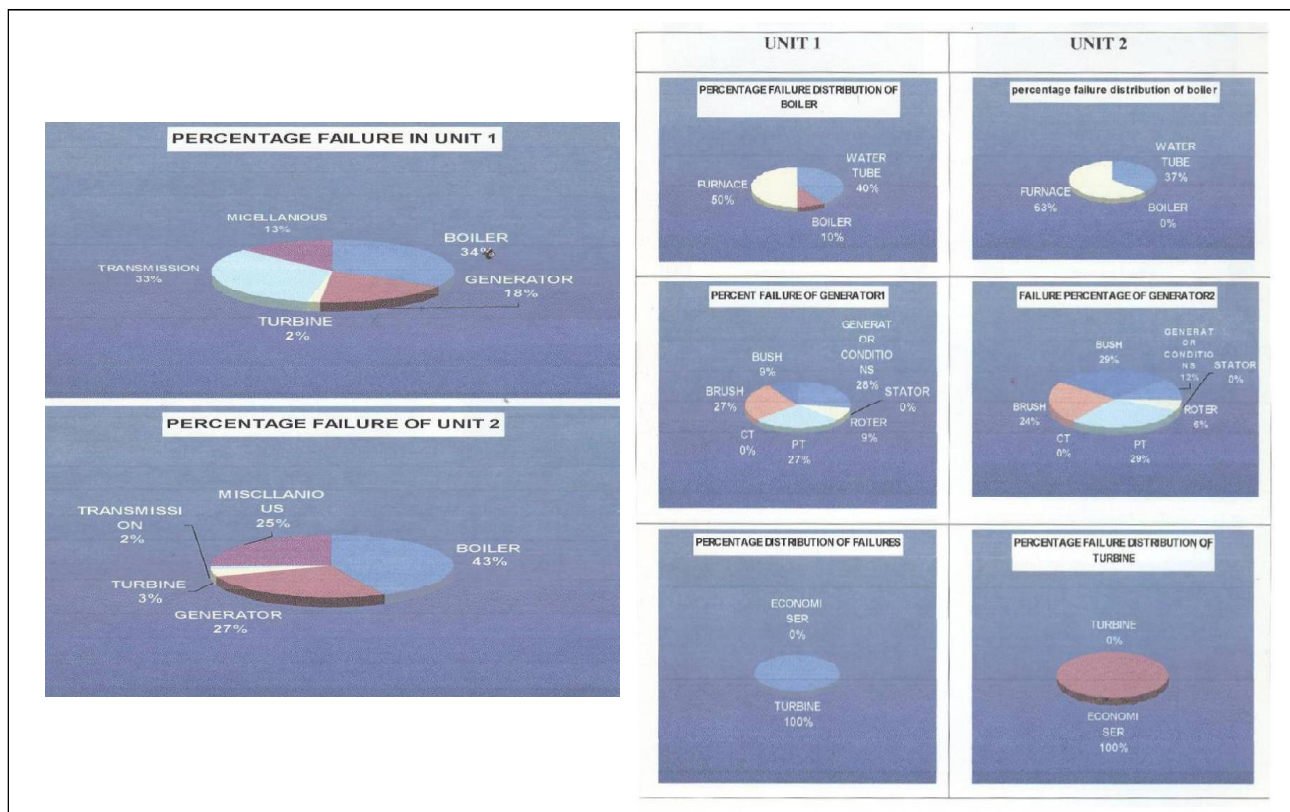
We can achieved after calculating the reliability of each component with respect to the system and then using the pareto analysis, cause and effect diagrams. This may result in quick improvement in the reliability of the system, which may be best result obtainable by any organization. To obtain the effective reliability improvement plan with less cost, by adding few lines to the program, the cost effective best

maintenance policy is achievable. We can predict the reliability of system based on component reliability it can be assessed effectively the component that causes the next failure, so that the inventory can be maintained less, which reduces the cost. The accurate results may be obtained when the field data is complete and easily assessable. 🌀

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APPENDIX 1



APPENDIX 2

