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Research Paper

MATHEMATICAL MODELLING OF ELECTROOSMOTIC FLOW THROUGH A SLIT MICROCHANNEL UNDER THE INFLUENCE OF TIME VARYING ELECTRICFIELD

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Electrokinetics involves the study of liquid or particle motion under the action of an electric field. The applications of electrokinetics in the development of microfluidic devices have been widely attractive in the past decade. Electrokinetic devices generally require no external mechanical moving parts and can be made portable by replacing the power supply by small battery. Therefore, electrokinetic based microfluidic systems can serve as a viable tool in creating a Lab-On-a-Chip (LOC) for use in biological and chemical assays. Here analytical procedure is carried out to find out the solution of electroosmotic flow. The governing equations including the linearized Poisson-Boltzmann equation, the Cauchy momentum equation are solved to seek velocity distribution and flowrate. Then effect of electricfield frequency on velocity profile is studied. The effect of frequency,electric field and electric double layer thickness on flow rate is also studied. The problem is first treated by using analytical methods, but the quantitative estimates are obtained numerically with the help of the software MATHEMATICA. The study reveals that the flow is oscillatory and maximum positiveflow is obtained for electricfield frequency of 20 Hz and as electricfield strength increases, flow rate also increases.

Keywords: Electrical double layer, Electroosmotic flow, Zeta potential

INTRODUCTION

A lab on a chip device is a micro scale laboratory with an array of micro channels, electrodes, sensors and electronic circuits. Electrodes are placed at different locations so that by applying electric field we could control the liquid flow and other operations on the chip. Almost every application which can do on room sized laboratories can be duplicated by LOC devices. So a reduction in samples and reagents, very short reaction time, portability, etc., is possible with LOC devices. We can

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also perform several applications like pumping, mixing, thermal cycling, separating etc. For example in a typical heart attack situation, the patient would require sudden diagnostic such as electrocardiogram and blood test. If blood test is conducting on a common laboratory, it will require a greater time and it may affect their life. But a LOC device could provide rapid diagnosis of the patient's cardiac condition. The recent development in electro kinetics and LOC device brings a strong demand of understanding electro kinetic phenomena and micro fluidics.

When electric fields are applied across capillarities or micro channels, bulk motion is obtained. The velocity of this motion is linearly proportional to the applied electric field, and is depends on the material used to construct the micro channel and also the solution in contact with the channel wall. This motion is referred as electro osmosis and stems from electrical forces on ions in the electrical double layer, a thin layer of ions that is located near a wall exposed to an aqueous solution. If the fluid velocity is interrogated at micrometer resolution, the fluid flow in a channel of uniform cross-section appears to be uniform. If the fluid velocity is interrogated at nanometre resolution, the fluid velocity would be uniform far away from the wall, but it would decay to zero at the wall over a length scale ª ranging from approximately 0.5-200 nm.

This fluid flow can be immensely useful in micro fluidic systems, because it is often much more straight forward experimentally to address voltage signals sent to electrodes rather than implement and control a miniaturized mechanical pressure pump. This flow however comes with its own complications, as it s velocity distributions is different from pressure driven flow , it is sensitive chemical features at the interface , and because the act of applying electric fields can also move particles relative to fluid or cause joule heating throughout the fluid. This phenomenon provides an effective way for mixing applications. By applying an external oscillatory electric field, a time dependant secondary flow can be excited. It is found that the mixing is improved due to an enlargement and folding of the virtual interface between the liquids. Thus diffusion can act more efficiently and virtual interface between the liquids can be maximized in area by cooling an optimum excitation frequency (Meisel and Ehrhard, 2006). The analytical solutions for channel and pipe flows of visco elastic fluids under the mixed influence of electro kinetic and pressure forces are found out (Afonso *et al.*, 2009). The analyses are restricted to cases with small EDL. The viscoelastic fluids used are described by Phan Thien and Tanner (PTT) model with linear Kernel for the stress coefficient function and zero second normal stress difference and FENE – P model. In the absence of an imposed pressure gradient, the electro osmotic flow exhibits a plug like velocity profile. When visco elastic flow is induced by a combination of both electrical and pressure potential, in addition to a single contributions from these two mechanisms, there exist an extra term in velocity profile which is absent for the Newtonian case where superposition principle applies. This non linear term can be contribute significantly to the total flow rate ie, under conditions of favourable pressure gradient it act as a drag reducer and for adverse pressure gradients, it will act as a drag increaser. The exact solution of unsteady flow due to an infinite plate oscillating in its own plane is found out.

Oldroyd fluid A and B are non Newtonian fluid but they show constant viscosity. The main features of the general solution are the fluid initially at rest, oscillates harmonically in the x – direction. The envelope of these oscillations grows with increasing time‘t’ and decreases within the distance from the surface (Hayat *et al.*, 2001). The solution obtained represents a transverse wave: its velocity is perpendicular to the direction of propagation. Also transverse waves can occur in an Oldroyd – B fluid, but they are rapidly damped as moved away from the solid surface. The exact and approximate solutions are presented for fully developed pipe and planar flows of multimode differential viscoelastic equations (Cruz *et al.*, 2005). The exact solutions are semi analytic and pertain to models based on PTT, Giesekus and FENE-P equations, whereas the approximate solutions are fully analytical and concern the PTT and FENE-P models. In all cases due account is taken of the presence of Newtonian solvent (Cruz and Pinho, 2007). The analytical approximate solutions was obtained with a perturbation technique, but calculations with these formulae are limited to low values of flow Deborah number for the PTT model and also for FENE-P model. In spite of its length the analytical solution is advantageous at low Deborah numbers because it eliminates the need for an iterative numerical procedure. The electro osmotic flow of a third grade fluid between micro parallel plates is considered (Akgül and Pakdemirli, 2008). Equations of motion are derived and made dimensionless. Temperature and velocity profiles are calculated using approximate solutions and by perturbation technique. Effect of non Newtonian parameter, joule heating, and viscosity index and electro kinetic parameters on the velocity and temperature profiles is discussed. Approximate solutions are contrasted with numerical solution. Two different cases ie constant viscosity and variable viscosity cases are treated separately. Analytical solutions are contrasted with the numerical solutions of the original equations. Within the validity range, solution matches with each other.

Electro viscous effects in a power law non-Newtonian fluid taken to be a symmetric 1:1 electrolyte solution are investigated for fully developed pressure driven flow through a cylindrical micro fluid pipe (Bharti *et al.*, 2009). The Poisson Boltzmann equation for the electrical potential, and the Navier stokes equation with electrical forcing and power law fluid rheology, are solved numerically using a finite difference method. The induced axial electrical field shows only a weak dependence on the power law index with the dependence being greatest for shear thinning liquid. The electro viscous effect is stronger in shear thinning and weaker in shear thickening liquids, than it is when the liquid is Newtonian. The frequency dependent flow of electrolyte between pairs of parallel plate microelectrodes is analyzed (Talapatra and Chakraborty, 2008). It is found that the impact of double layer overlap on ac electro osmosis flow turns out to be more predominant at frequencies of the order relaxation frequency of electrode electrolyte system. Closed form expressions are derived to depict the influence of EDL overlap on ac electro osmosis flow patterns. If the applied potential is below the ionization potential, electrolysis doesn't occur at electrode surfaces. The EDL formed behaves in a capacitive increases resulting in strong potential gradients across the electrode surface. A satisfactory explanation of ac electro osmotic fluid flow characteristic between parallel plate micro channels under overlapped EDL conditions could be obtained.

Non Newtonian fluids such as blood, colloids and cell suspensions are often manipulated in micro fluidic devices and exhibit extraordinary flow behaviour, not existing in Newtonian fluids (Xiao-Xia Li and Ze Yin, 2012). An analytical solution of transient velocity for electro osmotic flow generalized Maxwell fluids through both a micro parallel channel and a micro tube using method of Laplace transform is found out. The solution involves analytically solving the linearized PB equations together with Cauchy momentum equation and the general Maxwell constitutive equation. Using the method of Laplace transform the transient EOF velocity profiles are obtained. The computational results show that velocity profile depend on the normalized relaxation time, the EOF velocity approaches steady and flow needs longer time to attain steady status with the increase of normalized relaxation time. The velocity and temperature distributions of the thermally fully developed electro osmotic flow through a rectangular micro channel are obtained (Jie Su and Yongjun Jian, 2012). Based on linear PB equation, Navier stokes equation and thermally fully developed energy equation, analytical solution of normalized velocity, temperature and nusselt number are derived using the method of variable separation. They greatly depend on the electro kinetic width, width to height ratio and joule heating to heat flux ratio. By numerical computations, it is clear that for prescribed electro kinetic width (k), increased heating to heat flux ratio (s) yields greater variation of the temperature over the rectangular micro channel cross section. The dependence of temperature on's' is more significant for small k, while at a larger k, the temperature profiles are almost identical.

The analytical solution of the time periodic EOF of the general Maxwell fluids between micro parallel plates under the Debye-Huckel approximation is presented (Quan-Sheng Liu and Yong-Jun Jian, 2011). The solution involves analytically solving the linearized Poisson-Boltzmann equation, together with Cauchy momentum equation and the general Maxwell constitutive equation considering electro osmotic forces as the body forces. With the method of separation of variables, the velocity profile and volumetric flow rates for the time periodic 1-D EOF are obtained analytically.

GOVERNING EQUATIONS

Theoretical Model and Analysis of EDL

The relation between the electrical potential $\cdot$ and local net charge density per unit volume $^\circ_e$ at any point in the solution is described by Poisson equation

$$\nabla^2 \cdot = \frac{-^\circ}{\in £_0} \qquad ...(1)$$

From Boltzman distribution, the number concentration of type *i* ion in symmetric electrolyte solution is:

$$n_i = n_{i\infty} e^{\frac{Zie\cdot}{KbT}} \qquad ..(2)$$

where $n_{i\infty}$ = bulk ion concentration

Z_i = valency

e = charge of a proton

$°_e$ is propotional to the concentration difference between symmetric cations and anions.

$$°_e = ze\left(n^+ - n^-\right) \quad ...(3)$$

$$= -2zen\infty \sinh\left[\frac{Ze\cdot}{KbT}\right] \quad ...(4)$$

The net volume charge density,

$$°_e = \Sigma zien_i \quad ...(5)$$

$$= e\Sigma z_i n_{i\infty} e^{\frac{Zie\cdot}{KbT}} \quad ...(6)$$

So equation reduces to Poisson Boltzman equation.

$$\nabla^2\cdot = \frac{2zen0}{££_0}\sinh\left[\frac{Ze\cdot}{KbT}\right] \quad ...(7)$$

$$\nabla^2\cdot = \frac{-e}{££_0}\sum Z_i n_{i\infty} e^{\frac{Ze\cdot}{KbT}} \quad ...(8)$$

By defining Debye Huckel parameter,

$$K^2 = \frac{2ze2n\infty}{££_0 KbT} \quad ...(9)$$

And non dimensional electrical potential

$$\cdot' = \frac{ze\cdot}{KbT} \quad ...(10)$$

The final equation becomes

$$\nabla^2\cdot' = k^2 \sinh\cdot' \quad ...(11)$$

AC electroosmotic flows in a rectangular microchannel

Electroosmotic flow induced by unsteady applied electric fields or electroosmotic flow under an alternating electrical (AC) field has its unique features and applications

This section reviews a theoretical approach that was taken to investigate the time periodic electro osmotic flow in a rectangular micro channel . An analytical solution to the velocity field is presented for a linearized Poisson-Boltzmann double layer model and an applied sinusoidal electric field. As explained previously, when a liquid comes into contact with a solid, the formation of an interfacial charge causes a rearrangement of the local free ions in the liquid and produces a thin region of non-zero net charge density near the interface, referred to as the Electrical Double Layer (EDL). The application of an external electric field then results in a net body force on the free ions within the EDL inducing a bulk fluid motion called electro osmotic flow. For electro osmotic flows of incompressible liquids, the Navier-Stokes equations may take the following form,

$$°\cdot\frac{DV}{Dt} = -\nabla P + \nabla\cdot{}^2 + F \quad ...(12)$$

where ° is the density, V denotes the velocity vector, p is the pressure, ² represents the stress tensor, and F is the body force.

Assuming that the body force acts in the x direction, this term is replaced by $°_e E_x$, where $°_e$ represents the local net charge density, and E_x is the external electric field intensity applied along the axis direction. Considering steady-state and hydrodynamically fully developed flow, the shear stress for power-law fluid becomes

$$^2 = «\frac{dV}{dy} \quad ...(13)$$

Applying the above values, the momentum equation becomes,

$$° \frac{\partial v}{\partial t} = -\frac{\partial P}{\partial x} - « \frac{\partial 2V}{\partial y2} + °_e E_0 e^{i\sim t} \quad ...(14)$$

$$° \frac{\partial v}{\partial t} = -\frac{\partial P}{\partial x} - « \frac{\partial 2V}{\partial y2} + E_0 £ \frac{d2\cdot}{dy2} e^{i\sim t} \quad ...(15)$$

Applying $V = V_0$ Sinμt and $E = E_0 e^{i\sim t}$

$$« \frac{\partial 2(V0 \sin \mu t)}{\partial y2} = -° \frac{\partial (V0 \sin \mu t)}{\partial t} - \frac{\partial P}{\partial x} + E_0 £ \frac{d2\cdot}{dy2} e^{i\sim t} \quad ...(16)$$

$$« \sin \mu t \frac{\partial 2(V0)}{\partial y2} - ° \mu \cos \mu t - \frac{\partial P}{\partial x}$$

$$+ E_0 £ K^2 \cdot_w \frac{\cosh(Ky)}{\cosh(KH)} e^{i\sim t} = 0$$

By solving the momentum equation, the velocity V_0 can be obtained.

The above equation can be solved analytically and velocity can be obtained as

$$V_0 = \frac{1}{° \mu (k^2 « + ° \mu \cot[t\mu])} \sin[t\mu]$$

$$\left(k^2 « \in °\pm \mu \cos\left[\frac{h\sqrt{°}\sqrt{\mu}\sqrt{\cot[t\mu]}}{\sqrt{«}} \right] \right) \quad ...(17)$$

Discharge(Q)

$$= \frac{2k^2 « \in \pm \sqrt{«} \sec\left[\frac{h\sqrt{°}\sqrt{\mu}\sqrt{\cot[\mu]}}{\sqrt{«}}\right] \sin[\sim] \sin\left[\frac{\sqrt{°}\sqrt{\mu}\sqrt{\cot[\mu]}}{\sqrt{«}}\right]}{\sqrt{°}\sqrt{\mu}\sqrt{\cot[\mu]}(k^2 « + ° \mu \cot[\mu])} \quad ...(18)$$

RESULTS AND DISCUSSION

AC Electroosmosis

Variation of velocity with time is plotted with constant frequencies.

It is found that upto a time of 0.4 seconds, the magnitude of velocity is negative. It means that upto a time of 0.4 seconds, there will be

Figure 1: Variation of Dimensionless Velocity at Constant Electric Field Frequency

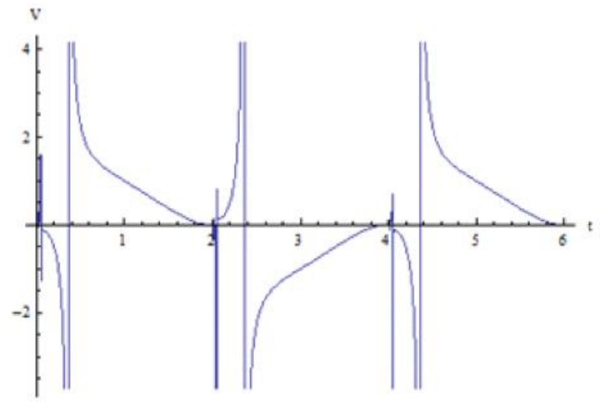


Figure 2: Variation of Normalized EOF Velocity of Newtonian Fluid in the Rectangular Micro Channel (Electric Field Frequency ~ = 10)

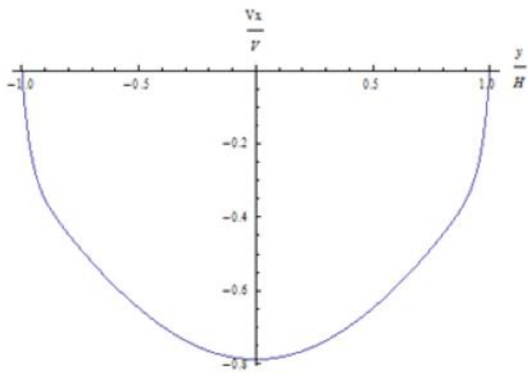


Figure 3: Variation of Normalized EOF Velocity of Newtonian Fluid in the Rectangular Micro Channel (Electric Field Frequency ~ = 15)

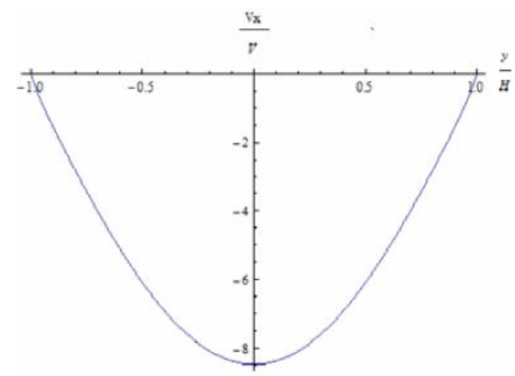


Figure 4: Variation of Normalized EOF Velocity of Newtonian Fluid in the Rectangular Micro Channel (Electric Field Frequency ~ = 20)

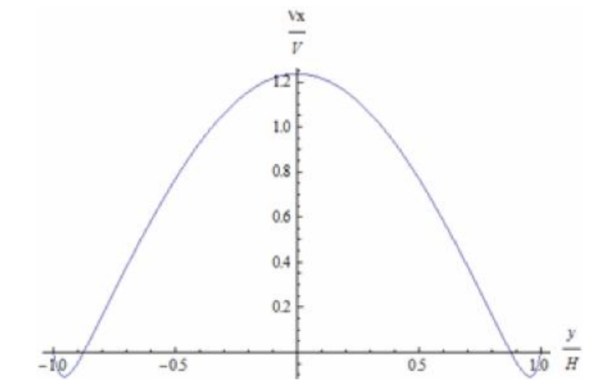


Figure 5: Variation of Normalized EOF Velocity of Newtonian Fluid in the Rectangular Micro Channel (Electric Field Frequency ~ = 30)

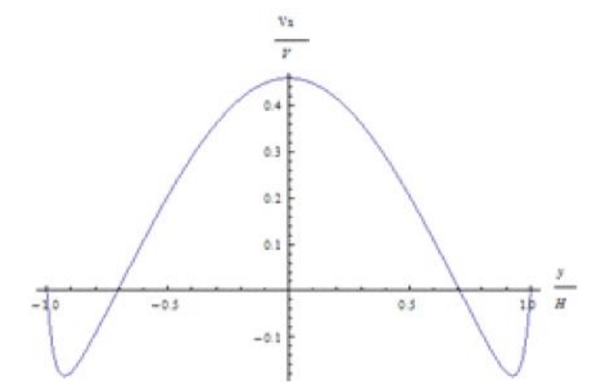


Figure 6: Variation of Normalized EOF Velocity of Newtonian Fluid in the Rectangular Micro Channel (Electric Field Frequency ~ = 50)

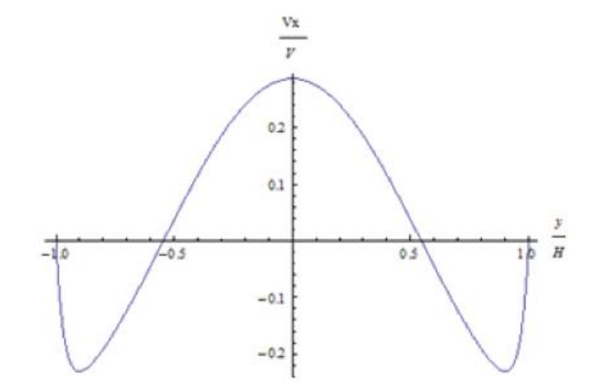
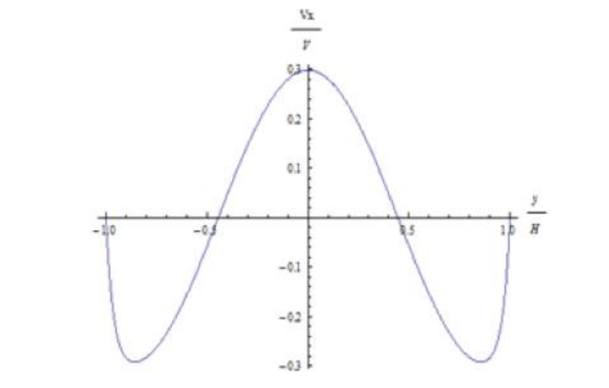


Figure 7: Variation of Normalized EOF Velocity of Newtonian Fluid in the Rectangular Micro Channel (Electric Field Frequency ~ = 75)



reverse flow and after that positive flow occurs. Another peculiarities of the flow is, it is oscillating in nature.

Variation of Velocity with Different Frequency

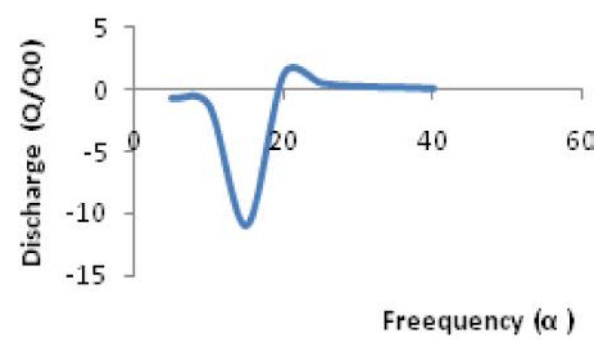
Variation of velocity with different frequency is plotted against channel height.

From figures, it is clear that at very low electric field frequency, the flow is reverse in nature. As electric field frequency increases gradually, slightly forward flow occurs. Further increase in frequency result in sinusoidal type flow. Nearer to the channel wall surface reverse flow occurs and at centre line, positive flow occurs. It is observed that the maximum positive flow occurs when electric field frequency is 20 Hz.

Variation of Discharge with Frequency

From Figure 8 it is found that at electric field frequency of 20 Hz, maximum flow velocity occurs and hence discharge. If the frequency is further increased, the net discharge decreases. It is due to sinusoidal nature of velocity profile.

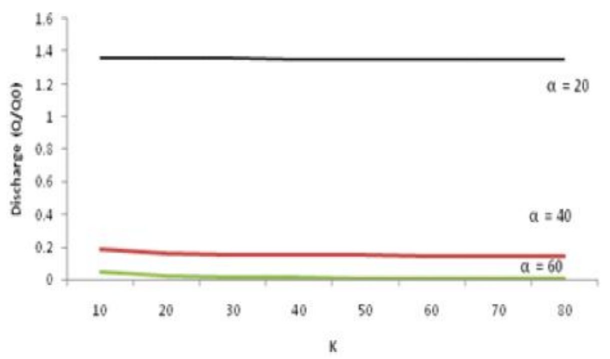
Figure 8: Variation of Normalized Discharge of Newtonian Fluid in the Rectangular Micro Channel with Variation in Freequency



Variation of Discharge with K Value

For AC electro osmosis, it is clear that the variation of k value doesn't affect the discharge significantly, i.e., electrical double layer thickness doesn't have much impact on flow velocity.

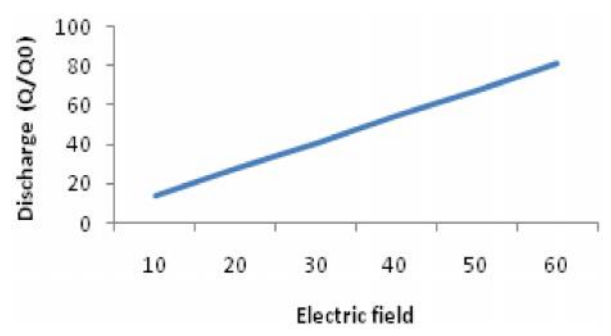
Figure 9: Variation of Normalized Discharge of Newtonian Fluid in the Rectangular Micro Channel with Variation in K Value



Variation of Discharge with Electric Field

From Figure 10, it is clear that discharge increases proportionally to electric field strength. The increase in electric field strength will result in greater body force, which will result in increase in discharge.

Figure 10: Variation of Normalized Discharge of Newtonian Fluid in the Rectangular Micro Channel with Variation in Electricfield



CONCLUSION

The flow behaviour of the electro osmotic flow of the Newtonian fluid in a slit micro channel is studied. An exact solution of the velocity distribution is presented under the Debye-Huckel approximation. This solution is obtained by solving the linear zed Poisson-Boltzmann equation and the modified Cauchy equation. The approximate solutions of the velocity distribution are also obtained by the approximation of the hyperbolic sine functions. For AC electro osmosis, electricfield frequency is the major parameter which determines the nature of the flow. At low frequencies, there is only reverse flow. But at about 20 Hz, maximum positive flows occur and hence discharge. It is most suited for mixing applications.

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APPENDIX

Nomenclature

Eo	External electric field (N/C)
Fx	Driving force due to the interaction of the applied electric Dield (N/m)
H	Half distance between plates (m)
m	Flow consistency index
n	Viscosity index
u	Velocity (m/s)
$n_{i\infty}$	Bulk ion concentration
e	Charge of a proton
Zi	Valency
Kb	Boltzmannconstant
	Greek Symbols
~	Electric field frequency (Hz)
µ	Oscillating velocity frequency (Hz)
«	Dynamic viscosity (Pa s)
°	Density (kg m^{-3})
$°_e$	Net charge density (C/m)
$\cdot_w$	Zeta potential (mV)
$©^{-1}$	Debye length (m)
£	Dielectric constant
Γ	Rate of strain tensor
T	Shear stress (N/m^2)
	Subscripts
ws	Wall