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Research Paper

GENETIC ALGORITHM FOR PROJECT SCHEDULING AND RESOURCE ALLOCATION UNDER UNCERTAINTY

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This paper provide a solution approach for planning, scheduling and managing project efforts where there is significant uncertainty in the duration, resource requirements and outcomes of individual tasks. Our approach yields a nonlinear (GA) optimization model for allocation of resources and available time to tasks. This formulation represents a significantly different view of project planning from the one implied by traditional project scheduling, and focuses attention on important resource allocation decisions faced by project managers. The model can be used to maximize any of several possible performance measures for the project as a whole. We include a small computational example that focuses on maximizing the probability of successful completion of a project whose tasks have uncertain outcomes. The resource allocation problem formulated here has importance and direct application to the management of a wide variety of project-structured efforts where there is significant uncertainty.

Keywords: Project scheduling, Resource constraints projects, Genetics algorithms, Uncertainty

INTRODUCTION

In Engineering and business, projects are subjected to a multitude of uncontrollable factors that affect their successful completion and are difficult to be predicted precisely or with certainty in the planning phase. Weather variations, workers' productivity variation, resources availability, failures of equipment, customer's acceptance or refusal at different phases of a project, extent of maturity of adopted technology and budget uncertainty are few examples of these uncontrollable factors. Uncertainty manifests itself in the estimation of activities durations, in estimating cost, in determining resource requirements, in the precedence order of different activities and even in the outcome of some activities: whether these activities will be successfully

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accomplished or they will be failures entailing consideration of different courses of action and/or rework. Problems of project planning and management in case of lack of information and uncertainty have attracted the attention of researchers since long time ago. Project Evaluation and Review Technique, known as PERT was the first model (Goldberg, 1989) that considers the randomness of activity durations. The Three-valued estimation of activity duration (optimistic, most likely and pessimistic) is one of the powerful features of PERT, since it is easy to collect them from experienced staffs. PERT evaluates the mean and variance of each activity duration in terms of the three values by assuming the Beta distribution as the probability distribution for all different activities. PERT uses the same technique of the Critical Path Method CPM (Forward and Backward Passes) in evaluating the project completion time, defining the critical path(s) and the floats (total and free) for each activity, all based on the mean activity duration.

PERT applies the Central Limit theorem and considers the project completion time as a random variable distributed according to normal distribution. The probability of successfully completing a project at that time defined by PERT or less is nearly 50%, which means that there is a risk of failing to complete the project with nearly 50% probability. Having a determinately defined critical path(s) contradicts the assumption of the randomness of activities durations. This and other facts, stated later, call for further investigations for building models more realistic than PERT (Van Slyke, 1963; Hillier and Lieberman, 1995; and Feng *et al.*, 1997). It is worthwhile to state here the limitations of PERT that motivated the development of other more realistic models:

- Assuming Beta distribution to model the duration of all project activities without any regard to the different natures of different activities.
- Determination of the project completion time using averages of durations of activities with no account for their variances.
- Getting determinately defined critical path(s) is in contradiction with the assumption of randomness of durations of activities. Under Uncertainty any path could be critical but of course with different probability.
- The assumption of Normality of the project completion time is approximately valid only in case of having large number of activities. The normal distribution with its unlimited ends is not the proper model.
- PERT is not capable of modeling uncertainties in the order of precedence and in outcomes of activities.
- Time-Cost tradeoffs under uncertainty in an attempt to enhance the probability of project successful completion cannot be performed by using PERT.
- The activities are assumed independent and hence the project variance is taken as the sum of variances of activities on the critical path. Correlations between activities are ignored in PERT.
- The precedence relationships between activities are limited only by one discipline-the start to finish discipline. Other precedence relationships such as: finish to

start, start to start and finish to finish could not be included in PERT model.

Building mathematical models that are capable of overcoming all limitations of PERT is an extremely difficult, if not impossible task. The advent and fast progress of building computational models enables researchers to introduce Simulation Modeling as more reliable models to tackle such problems. Monte Carlo Simulation was first introduced since sixties of the last century (Van Slyke, 1963). Monte Carlo Simulation enabled planners to use different probability distributions for durations of different activities, to introduce what is called criticality index for different paths and different activities and to fit probability distributions to project completion time based on statistics obtained from several simulation runs. Monte Carlo Simulation has the common drawback of all simulation techniques, i.e., the statistical nature of the results and moreover it is not capable to perform constrained resource-based scheduling and time-cost optimization analysis. Discrete-event Simulation (Hong *et al.*, 2004) is more powerful technique and may allow the use of other models of uncertainties such as fuzzy models of activity duration.

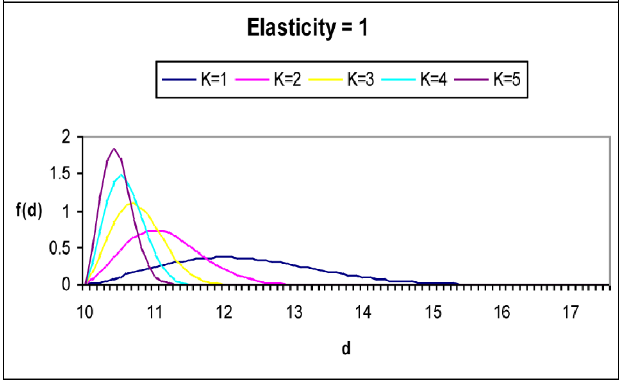
Because of non-linearity, commonly noticed, in Time-Cost Optimization (TCO) and resource allocation problems, linear programming cannot be used. Recently, Genetic Algorithms (GA) as search engines are widely used in solving TCO problems, constrained resource allocation and resource leveling problems Goldberg (1989), Feng *et al.* (1997) and Tarek (1999).

The present work is mainly concerned with the evaluation of the probability of success of projects under uncertainty. This problem was considered before by Turnquist and Linda (2002). They proposed Weibull distribution to model activity duration because of the universality of this distribution. Also in Turnquist and Linda (2002), a modification was introduced in Weibull distribution in order to account for the effect of increasing resources levels on the activity duration. The Modified Weibull distribution takes the following form Turnquist and Linda (2002):

$$f(d) = \frac{\beta}{\left(\frac{\eta}{K^E}\right)} \left(\frac{d - d_{\min}}{\frac{\eta}{K^E}}\right)^{\beta - 1} \exp - \left(\frac{d - d_{\min}}{\frac{\eta}{K^E}}\right)^{\beta} \quad ...(1)$$

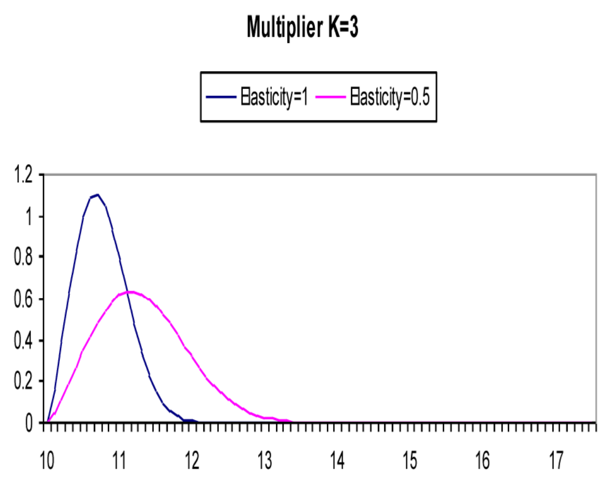
K and E are the two newly introduced parameters that defining resource multiplier and the elasticity of the activity duration to be compressed as more resources are applied to it respectively. The effect of allocating more resources (K times the normal resource level) to activities is shown in Figure 1.

Figure 1: Weibull Density Function f(d) with Different Resource Multiplier K



The concept of elasticity ($E <= 1$) as a measure of the capability to compress the activity duration by allocating more resources of a certain type is clearly illustrated in Figure 2.

Figure 2: Weibull Density Function f(d) with Two Different Values of Elasticity Under the Same Resource Multiplier



Activities with zero elasticity to a specified resource would not respond to any increase in level of that resource. This could be explained by unavailability of space needed by more resource to operate such as for example welding in confined spaces in shipbuilding. It should be emphasized that an activity could have different elasticity's with different types of resources. Later, this point will be pursued analytically.

PROBLEM FORMULATION

Notations and Symbols

a_i: Optimistic evaluation of duration of i^{th} activity

b_i: Pessimistic estimation of duration of i^{th} activity

d_i: Duration of i^{th} activity

d_{oi}: Minimum duration of i^{th} activity

E_{qi}: Elasticity of ith activity to the application of resource q

K_{qi}: Resource multiplier applied to i^{th} activity

K_i: Resultant resource multiplier applied to i^{th} activity

L_{qi}: Upper limit of resource type q multiplier to be applied to activity i

m_i: Most likely estimation of duration of i^{th} activity

N: Number of activities

$pred_j$: Set of predecessors to j^{th} activity

Q: Number of resources types

R_{qu}: Available amount of q^{th} resource in the u^{th} period

S_i: Start of i^{th} activity

T: Project completion time

U_q: Number of contiguous periods of resource q availability

η_i, β_i: Weibull parameters of duration of i^{th} activity

τ_{uq}: Start of u^{th} period of q^{th} resource

Given a project with N activities (i = 1, 2, …, N). The precedence order of the activities is given. Activity duration di is random and distributed according to Modified Weibull distribution given in (1). There are Q types of resources (manpower, cash, equipment, …, etc.) available to finish the project in a predefined completion time T. Resources availability along the project time is limited by the allowable resource levels at different periods. The length of these periods and associated with them allowable levels of resources differ from resource to another. Therefore, the project time is subdivided into a number of equal or non-equal contiguous periods (u_q = 1, 2, …, U_q) as regards to each resource type q. It is required to determine the time window d_i granted to complete each activity aiming at maximizing the probability that the project could be completed in time T or to minimize the probability of failure to complete the project in time T and to determine start times of each activity S_i and resource multipliers K_{qi} so that the consumption of resources of different types will not exceed the allocated amounts of these resources.

Decision Variables

As already discussed, decision variables to be determined in the formulated problem are:

d_i: Allowable time window to complete works on i^{th} activity

S_i: Start time of i^{th} activity

K_{qi}: q^{th} Resource multiplier representing the intensity of resource allocation necessary and sufficient to complete works on i^{th} activity in the predetermined time window d_i with maximum possible probability of success.

The resultant resources multiplier K_i is proposed, in the present work, to be the geometric mean of all multipliers K_{qi} as follows:

$$K_i = \left(\prod_{q=1}^{Q} K_{qi}^{E_{qi}} \right)^{\frac{1}{Q}} \quad \text{...(2)}$$

Objective Function

The Success Probability $PAS_i(d_i,\ K_i)$ of completing the i^{th} activity in time window d_i can be evaluated as follows:

$$PAS_i(d_i,\ K_i) = \int_0^{d_i} f(t,\ K_i)\,dt$$

$$PAS_i(d_i,\ K_i) = 1 - \exp\left(-\left(\frac{d_i - d_{oi}}{\eta_i / K_i} \right)^{\beta_i} \right) \quad \text{...(3)}$$

Since all activities of the project should be completed successfully in order to succeed to finish the project in the target time T, then the probability of the project success $PPS(T)$ is obtained as the product of probabilities of success of all activities:

$$PPS = \prod_{i=1}^{N} PAS(d_i,\ K_i) \quad \text{...(4)}$$

It should be noted here that if there could be optional precedence order as in case of GERT networks, the probability of project success will be determined in a different way taking into account the different options of project completion paths.

Constraints

There are three types of constraints imposed on the decision variables:

Completion Constraint

Let$(N + 1)^{th}$—a dummy activity with zero duration and zero resource requirement to act as an end activity, then

$$S_{N+1} <= T \quad \text{...(5)}$$

Precedence Order Constraints

$$S_j \geq S_i + d_i \qquad \forall i \subset pred_j$$

$$(i = 1, 2, ..., N) \qquad (j = 2, 3, ..., N + 1) \quad \text{...(6)}$$

Resources Availability Constraints

Resources of different types are necessary to complete project works in the predefined project time T. As already stated before, resource of type q is to be made available with predefined quantity R_{qu} in the u^{th} period. The sum of quantities of the q^{th} resource required by all activities in period u should be equal to or less than R_{qu}. In order to calculate the amount of consumed resources in different periods, a resource histogram should be firstly constructed for each resource type. The level of resource q, denoted by r_{qi} is taken constant along the extension of the i^{th} activity duration d_i, i.e., the resource q is assumed to be uniformly consumed by the i^{th} activity. A resource histogram is an arrangement of rectangles with heights r_{qi} and with widths d_i for all activities preserving the precedence order of the activities. A typical resource histogram is depicted in Figure 3 subdivided as regards to the availability of q^{th} resource into four equal contiguous intervals.

The resource level r_{qi} can be evaluated as follows:

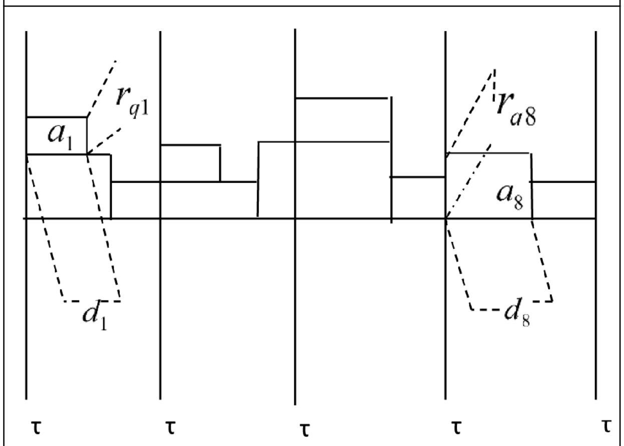
$$r_{qi} = K_{qi} A_{qi} / d_i \qquad S_i \le t \le S_i + d_i$$

$$r_{qi} = 0 \qquad \text{Otherwise} \qquad ...(7)$$

where, A_{qi} is the nominal requirement of i^{th} activity from resource type q. For example how many man days nominally required to complete i^{th} activity?

It is clear from Figure 3 that within the limits of a period u, project activities may be classified into three categories as regards to their contribution in the resource demand during period u.

Figure 3: Resource *q* Histogram



Category 1: Activities completely embedded inside the period, i.e., their starts is equal to or larger than period start τ^{uq} and their finish is equal to or less than period finish $\tau_{uq} + 1$.

This category contributes with the full value of the required resource $K_{qi}A_{qi}$.

Category 2: Activities partially lie inside the period

$$\tau_{u_q} \le S_i < \tau_{u_q+1} \qquad S_i + d_i > \tau_{u_q+1}$$

$$S_i < \tau_{u_q} \text{ and } \tau_{u_q} < S_i + d_i \le \tau_{u_q+1}$$

This category contributes with a part of the value of the required resource,

$$\alpha_{iu_q} K_{qi} A_{qi}$$

$$0 \le \alpha_{iu_q} \le 1$$

Category 3: Activities lie completely outside the period,

$$S_i + d_i \le \tau_{u_q}$$

Or

$$S_i \ge \tau_{u_q+1}$$

$$\alpha_{iu_q} = 0$$

Based on the above classification, the resources constraints may be expressed in the following form:

$$\sum_{i=1}^{N} \alpha_{iu_q} K_{qi} A_{qi} \le R_{qu}$$

$$0 \le \alpha_{iu_q} \le 1 \qquad ...(8)$$

It should be noted that there is a given upper limit L_{qi} for the resource multiplier K_{qi}.

$$K_{qi} <= L_{qi} \qquad ...(9)$$

α_{iu_q} is a newly factor introduced in our work. It will be called Contribution factor of i^{th} activity in the u^{th} period. Factors α_{iu_q} are not amenable to simple computations because of the step functions expressing resources distributions as seen in Figure 3. In an attempt to circumvent these difficulties in Turnquist and Linda (2002), authors proposed a sophisticated approach to compute factors α_{iu_q}. In this approach, the step functions expressing the distribution of resources are converted into continuous differentiable functions of time written here in a simpler expression than that in the referred work as follows:

$$r_{qi} = \frac{K_{qi}A_{qi}}{2d_i}\left[\tan h(\mu) - \tan h\left(\mu - \frac{1}{w}\right)\right]$$

$$\mu = \frac{t - S_i}{wd_i} \qquad ...(10)$$

As we said; the constant w is introduced in order to counteract the effect of the two singular points at $t = S_i$ and $t = S_i + d_i$ and render the variation of functions r_{qi} gradual rather than abrupt at these points. Figures 4a and 4b illustrate the effect of the constant w.

Figure 4a: *w* = 0.1

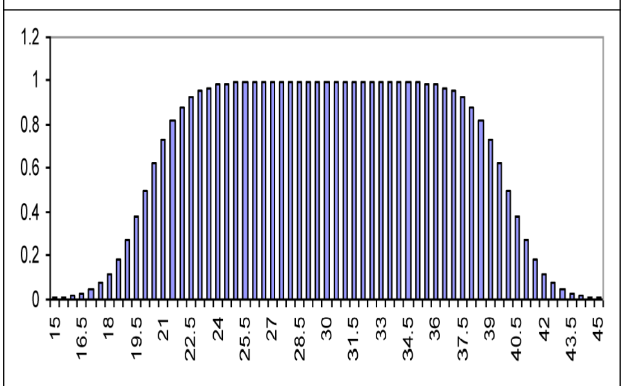
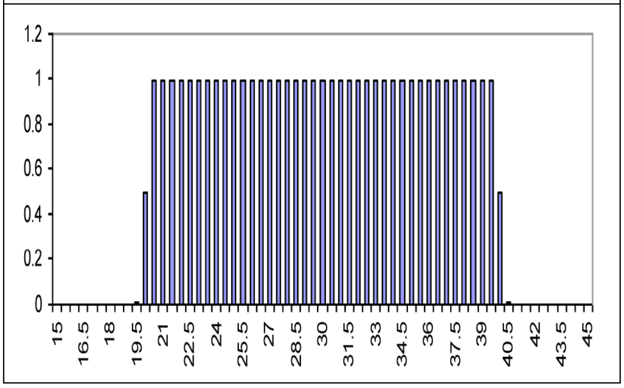


Figure 4b: *w* = 0.01



In Figure 4, two plots of the function 0.5 $\left[\tan h(\mu) - \tan h\left(\mu - \frac{1}{w}\right)\right]$ for two values of w(w = 0.1 and w = 0.01) (S_i = 20, d_i = 20). It is noticed that the function $0.5\left[\tan h(\mu) - \tan h\left(\mu - \frac{1}{w}\right)\right]$ has the value of unity as the time t being inside the range $s_i <= t <= s_i + d_i$ while it drops to zero outside this range. The constant w determines the nature of the change of the function at the start and finish of the activity. As w decreases (w < 0.03), the change tends to be rather sharp (4b) while, for bigger values of w the change of the function at start and finish of the activity is rather gradual (4a). This explains the role of the constant w in formula (10). As resource consumption rate r_{qi} is already expressed in the form of a continuous differentiable function, the consumption of resource q during the u^{th} interval can be evaluated by integrating r_{qi} on time over the u^{th} interval and then summing up for all project activities. Proceeding in this way, the resource constraint in (8) will take the form:

$$\sum_{i=1}^{N}\frac{K_i A_{qi}}{2d_i}\int_{\tau_{uq}}^{t(u_q+1)}\left[\tan h\left(\frac{t - S_i}{wd_i}\right) - \tan h\left(\frac{t - S_i - d_i}{wd_i}\right)\right]dt \le R_{qu} \qquad ...(11)$$

$\tau_{(u+1)q}$, τ_{uq} are the times of the start and end of u^{th} interval of resource q

Evaluate the integral I in (11),

$$I = wd_i Ln\left(\frac{\cos hv_2 \cos hv_3}{\cos hv_1 \cos hv_4}\right)$$

where,

$$v_1 = \frac{\tau_{uq} - S_i}{wd_i}$$

$$v_2 = \frac{\tau_{uq+1} - S_i}{wd_i}$$

$$v_3 = \frac{\tau_{uq} - S - d_i}{wd_i}$$

$$v_4 = \frac{\tau_{uq+1} - S_i - d_i}{wd_i}$$

Substituting in (11) we find,

$$\sum_{i=1}^{N} K_{qi} A_{qi}\left(\frac{w}{2} Ln\left(\frac{\cos hv_2 \cos hv_3}{\cos hv_1 \cos hv_4}\right)\right) \le R_{qu} \qquad ...(12)$$

Comparing (8) and (12) we find for factors α_{iu_q} measuring the contribution of the activity i in the consumption of the resources in the interval U:

$$\alpha_{iu_q} = \frac{w}{2} Ln\left(\frac{\cos hv_2 \cos hv_3}{\cos hv_1 \cos hv_4}\right)$$

$$0 \le \alpha_{iu_q} \le 1 \qquad ...(13)$$

It should be noted that, irrespective of the attractiveness of the above approach proposed in Turnquist and Linda (2002), it suffers from computational difficulties of having overflow in calculating hyperbolic and exponential functions. Therefore, another approach is proposed in the present work dealing with the step functions. The approach is clearly presented in the following flow chart in Figure 5. The flow chart may be implemented by means of VBA under Excel.

Formulation Summary

Maximize

$$PPS = \prod_{i=1}^{N} PAS(d_i, K_i)$$

Subject to:

$$S_{N+1} <= T$$

$$S_j \ge S_i + d_i \qquad \forall i \subset Pred_j$$

$$\sum_{i=1}^{N} \alpha_{iu_q} K_{qi} A_{qi} \le R_{qu_q}$$

$$0 \le \alpha_{iu_q} \le 1$$

(i, j = 1, 2, …, N)

(q = 1, 2, …, Q)

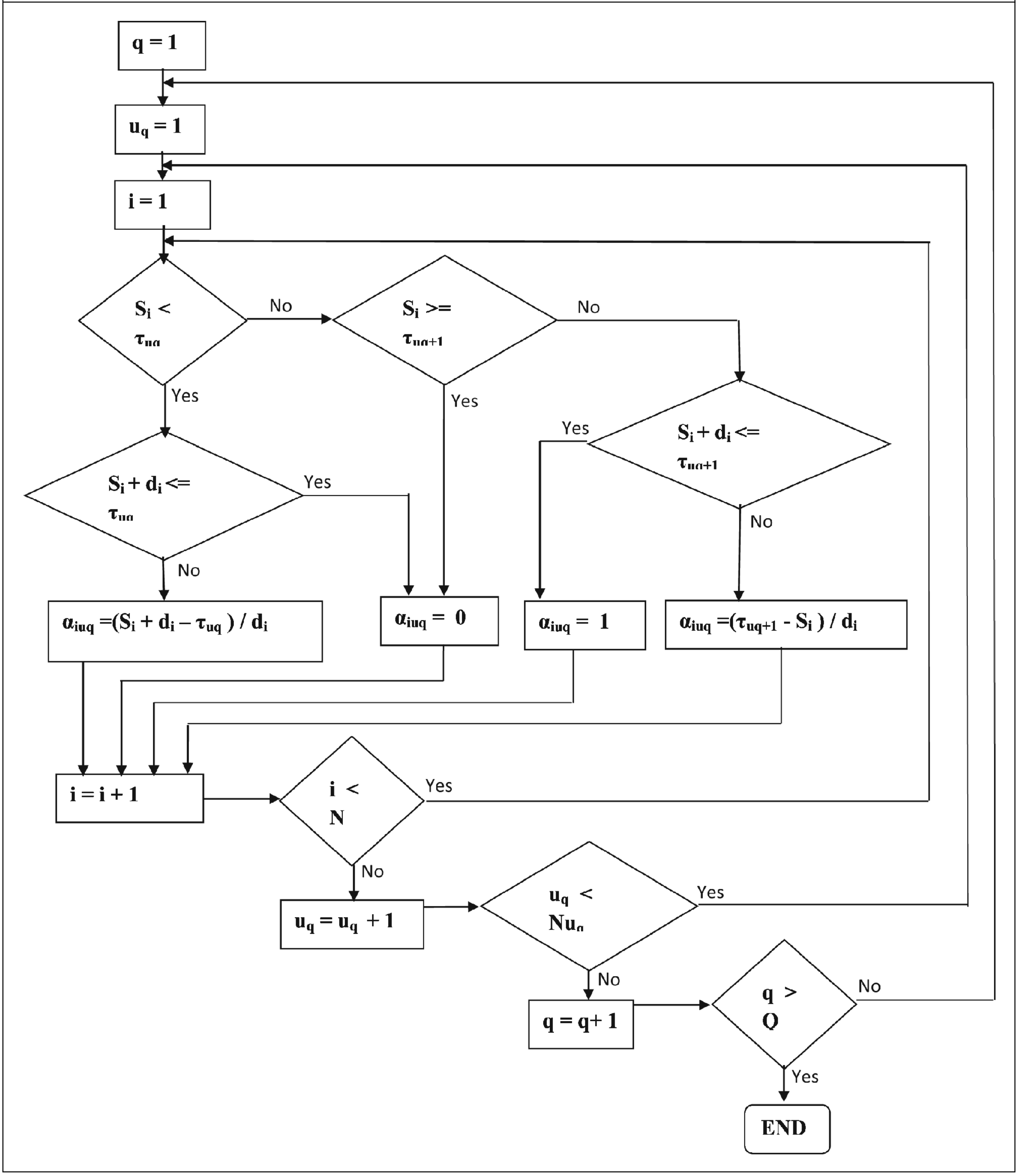
(U_q = 1, 2, …, NU_q) ...(14)

The problem, formulated in (14), is a nonlinear program. The non-linearity is severe and clearly noticed in the objective function and the resource constraints. Solution of such problems with these types of non-linearity is far from being amenable to standard packages of optimization software. Therefore in the present work, Genetic Algorithm approach, as one of the most powerful computational modeling techniques, will be adopted.

A GENETIC ALGORITHM (GA) MODEL

The solution of the problem formulated above is obtained by evaluating the set of decision

Figure 5: The Flow Chart



variables d_i, S_i, K_{qi}(i = 1, 2, …, N), (q = 1, 2, …, Q) that maximize the Objective function (4) and satisfying the constraints (5), (6), (8). GA approach is a search technique searches for an optimum or near optimum solution(s) in a space of solutions. The space of solutions is initially built of a population of chromosomes representing solutions to the problem randomly generated. The space of solutions is evolved by means of the three Genetic Operators namely Crossover Operator, Copying Operator and Mutation Operator. The evolution of the search space and its constituents Chromosomes is governed by a law similar to the Law of Natural Selection in biology, i.e., survival of only the fittest (Goldberg, 1989). A fitness function is applied in order to discard solutions (chromosomes) having lower fitness and keeping only chromosomes with higher fitness. The process of evolution continues until no further improvements could be attained.

Chromosome Structure and Initial Population

Each chromosome consists of $N(Q + 2)$ genes such that the first N genes carry values ξ_i responsible for finding activity durations, the second N genes carry values χ_i responsible for finding starting times of activities and the rest NQ genes carry values δ_{qi} responsible for finding the resource multipliers. The quantities ξ_i, χ_i and δ_{qi} are random numbers ranging from 0 to 1 and uniformly distributed. Next, a method will be developed in order to transfer these random numbers ξ_i, χ_i and δ_{qi} into decision variables d_i, S_i, k_{qi} respectively.

The Inverse Problem of Project Scheduling

Traditionally, the direct problem of project scheduling is to find a project completion time T having durations of all project activities and activities precedence order. On the contrary, in the formulated in this work problem, the project completion time T is given with the precedence order of the activities and required to determine the allowable durations of the activities. This Inverse problem will be solved in the following steps:

- Since the minimum possible duration of activities d_{oi} are given, the minimum possible completion time $T_{\min}$ can be obtained by the direct approach by Critical Path Method (CPM).
- The random quantities ξ_i occupying the first N genes in a chromosome are used as transitional activity durations in a CPM to evaluate a transitional project completion time *Temp*. Note that the precedence order is preserved in evaluating *Temp*.
- The inverse problem is now ready to be solved to find activity durations that result in a completion time T given in the formulated problem. The solution is proposed in the following expression:

$$d_i = d_{oi} + \left(T - T_{\min}\right)\frac{\xi_1}{Temp} \qquad ...(15)$$

Starting Times of Activities

In order to find the values of the second set of decision variables, we proceed as follows:

- Having already determined durations d_i apply CPM to determine the earliest start ES_i and free float FF_i of each activity.
- The random quantities χ_i occupying the second set of N genes in a chromosome are used to determine the starting time of activities by the following expression:

$$S_i = ES_i + \chi_i FF_i \qquad ...(16)$$

Resource Multipliers

The random quantities δ_{qi} occupying the last NQ genes are used to evaluate the resource multipliers as follows:

$$K_{qi} = \delta_{qi} L_{qi} \quad ...(17)$$

The Fitness Function

The optimum or near optimum solution will be found as one of the feasible solution that has the maximum value of, a designed for the purpose, Fitness Function. The feasibility of a solution is realized by satisfying all the constraints in (14). In the framework of GA approach, the infeasible solutions should be penalized by introducing a big negative value depending on the amount of deviation from the right hand side of the unsatisfied constraint. Therefore, the Fitness Function *FIT* for every chromosome will take the form:

$$FIT = \prod_{i=1}^{N} \left(1 - \exp\left(-\frac{d_i - d_{oi}}{\eta_i / K_i} \right)^{\beta_i} \right)$$

$$+ \sum_{q=1}^{Q} \sum_{u_q=1}^{Nu_q} \min\left(\left(R_{qu_q} - \sum_{i=1}^{N} \alpha_{iu_q} K_{qi} A_{qi} \right), 0 \right) \quad ...(18)$$

Determination of Weibull Parameters β_i, η_i

Selection of Weibull distribution—as the law of probability distribution of the activity durations as random variable—is justified by its universality as several well-known distributions could be derived from Weibull distribution as special cases by changing the value of the shape parameter β. Two methods are proposed here to calculate β_i and η_i depending on the input data.

Given Mean μ and Standard Deviation σ of Activity Duration

$$\mu - d_o = \eta \Gamma\left(1 + \frac{1}{\beta}\right) \quad ...(19)$$

$$\sigma^2 = \eta^2 \left[\Gamma\left(1 + \frac{2}{\beta}\right) - \left(\Gamma\left(1 + \frac{1}{\beta}\right)\right)^2 \right] \quad ...(20)$$

d_o is the minimum duration.

Dividing (20) by the square of (19) and performing simple manipulation we get:

$$\frac{\Gamma\left(1 + \frac{2}{\beta}\right)}{\left(\Gamma\left(1 + \frac{1}{\beta}\right)\right)^2} = \frac{(\mu - d_o)^2}{\sigma^2} + 1 \quad ...(21)$$

Equation in (21) is in one unknown β and can be solved easily using one of the tools of Excel – The Goal Seek. Substituting in (19), we find η.

In practice, mean and specially variance of different activities durations are mostly unavailable because of lack of statistics and also uniqueness of projects. The three valued estimations *a*, *m*; *b* of an activity duration which is commonly used in PERT could be collected more easily than mean and variance in special sessions with experienced personnel as optimistic, most likely and pessimistic estimations.

Given a, m, b *Estimations of Activity Duration*

The optimistic estimation *a* will be taken as the minimum duration d_o. The most likely estimation *m* is that duration at which the Weibull pdf attains its maximum value. The pessimistic estimation *b* will be equated to a (1-*R*) percentile (0 < = R < = 1). Therefore, from the differentiation of Equation (1), we get:

$$\left(\frac{m - a}{\eta}\right) = \left(1 - \frac{1}{\beta}\right)^{\frac{1}{\beta}} \quad ...(22)$$

If generally we consider (1-R) percentile, then

$$\exp-\left(\frac{b-a}{\eta}\right)^{\beta}=R$$

$$\left(\frac{b-a}{\eta}\right)=\left(Ln\left(\frac{1}{R}\right)\right)^{\frac{1}{\beta}} \quad ...(23)$$

Dividing (22) by (23), we get

$$\left(\frac{1-\frac{1}{\beta}}{Ln\left(\frac{1}{R}\right)}\right)^{\frac{1}{\beta}}=\frac{m-a}{b-a} \quad ...(24)$$

Equation (24) is in a single unknown β and can be solved by Goal Seek of Excel. Substituting in (22) or (23) we may find η.

ILLUSTRATIVE EXAMPLES

As a more illustration of the modeling approach outlined before, we apply our approach on a new construction project. For analysis of this example project, we focus on the probability of successfully reaching the end node in 28 months (588 working days), and our formulation of the objective function is $Z(F)=\prod_{n=1}^{n=26}F_n$. The dependence of each $F_i(d_i,\ k_i)$ term on di and ki has been suppressed to simplify the notation. Table 1 summarizes the input data for the 26 tasks. The optimistic duration (a) is the value of d_{oi} for each task.

The most likely (*m*) and the pessimistic time (*b*) to complete each tasks successfully are the basis for specifying the two parameters of the Weibull distribution for each task; given those two values, they solved for η_i and β_i for each task. The elasticity values (E_i) in Table 1 define the percentage reduction in the scale parameter of η_i the distribution of time to successful completion for each task, resulting from a one percent increase in resources applied to the task. The two columns labeled Nominal Person-Days and Nominal Budget (NB) specify the Aqi values for the two resources for each task. We applied our approach on this data, as to find

Table 1: Input Data

Task	a	m	b	NP	NB	E
1	6	9	15	630	60	0.8
2	9	15	22	100	5	1
3	12	18	26	200	7	0.2
4	18	25	34	1800	150	0.5
5	6	10	20	1260	125	0.8
6	8	14	24	420	2	0.2
7	11	15	30	250	15	0.7
8	15	18	30	220	4	0.5
9	18	28	35	220	6	0.4
10	24	30	52	200	4	0.6
11	14	22	30	330	8	0.2
12	20	30	50	300	10	0.2
13	12	17	25	240	9	0.5
14	12	18	25	280	3	0.5
15	17	22	36	800	15	0.8
16	29	35	45	330	8	0.4
17	21	30	40	210	12	0.5
18	6	14	18	180	18	0.2
19	12	15	21	150	12	0.2
20	14	18	26	200	14	0.7
21	17	22	35	440	17	0.8
22	20	25	42	400	12	0.9
23	5	10	14	500	10	0.2
24	17	22	34	560	12	0.3
25	24	30	37	440	18	0.5
26	6	13	18	320	8	0.4

Table 2: Result of Optimization from our Approach

Probability of Success 0.85604

Project Available Time 588

Activity Number	Activity Duration	Activity Start	Activity Total Float	Activity Resource Multiplier	Activity Success Probability
1	57.2408	0.35042	1.16299	0.64843	0.97717
2	58.4038	0	5E-05	2.0111	1
3	42.352	1.92206	16.0519	1.31611	0.99999
4	63.0923	58.4038	4.6E-05	0.6471	0.9747
5	27.9296	121.496	4.6E-05	0.76287	0.99628
6	41.8179	149.426	4.6E-05	1.56137	0.99314
7	58.8207	122.354	48.3069	1.53927	0.9979
8	37.3801	191.244	4.6E-05	2.1238	0.94889
9	46.2816	191.244	50.0871	1.66248	1
10	59.0873	228.624	3.1E-05	2.03212	0.99994
11	47.0753	228.624	12.012	1.51588	0.99142
12	44.5143	287.711	3.1E-05	1.42789	0.99932
13	44.6129	254.143	50.0871	1.2608	0.99027
14	54.7497	296.617	60.6192	1.54293	0.99932
15	49.6194	332.225	27	0.76792	0.99926
16	58.8427	332.225	0	1.82932	0.99991
17	61.1081	391.068	0	1.80904	0.99999
18	43.3313	407.101	27	1.36851	0.98528
19	43.0564	275.699	66.1056	0.97297	0.99904
20	66.7882	374.487	110.81	1.8453	1
21	67.315	350.863	66.1056	1.82511	0.99939
22	44.1771	452.176	0	3.31899	0.99742
23	19.4288	496.353	0	1.36792	1
24	47.6153	496.353	2.19623	1.89728	0.9981
25	63.5773	515.782	0	0.93341	1
26	33.1946	544.947	2.19623	1.9313	1

the probability of completing the project during the available period and resources. We transfer our formulation into code for the Genetic Algorithm. The Genetic Algorithm has been implemented on Visual Basic under Excel. The optimized results for task start times, allowable durations, and resource multipliers, as well as the resulting probabilities for successful completion of each task, are shown in Table 2. This set of values results in a probability of success for the project as a whole of 0.856.

Table 3: The Distribution of Resources Over the Periods

Period	Available Manpower	Consumed Manpower	Available Budget	Consumed Budget
1	2205	957.697	600	595.35
2	2205	1258.79		
3	2205	1787.77		
4	2205	1277.36		
5	2205	1340.94		
6	2205	1844.95		
7	2205	1512.24		
8	2205	1892.04		
9	2205	2048.06		
10	2205	287.272		

Also the distributions of resources over each period are shown in Table 3.

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