# Identification of Kinematic Chains Isomorphism Based on the Distance between Non-binary Vertices

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***Abstract*—This paper proposes a method for identifying isomorphisms between different kinematic chains that is highly efficient, reliable, and simple, with a short CPU running time (KC). In contrast to many methods proposed by researchers in this field, which require significant computing time, particularly in kinematic chains with a large number of bars. Isomorphism identification is critical for designers in order to avoid duplicate solutions and focus all of their energy and creativity on novel, independent kinematic chain solutions. The shortest path between non-binary bars is primarily used in this article to solve the problem of isomorphism identification. The computational complexity and efficiency of the method are evaluated and compared to existing published methods for a variety of cases, including 8-bar, 10-bar, 12-bar, three-complex 13-bar, 15-bar, 28-bar, and 42-bar single-joint kinematic chains. These comparisons demonstrate the superiority of the proposed method.**

***Keywords*—adjacency matrix, isomorphism, distance matrix, invariant identification**

## I. INTRODUCTION

Before beginning work on a new kinematic chain design, we must first check for isomorphism between several potential solutions. This is one of the most difficult and daunting challenges, especially for complex mechanisms based on kinematic chains with many bars. Confusion between isomorphic and non-isomorphic kinematic chains may result in redundant efforts and solutions.

Many methods for overcoming this challenge have been developed by researchers. The following is a summary of various works on the subject published in recent years by researchers. Butcher and Hartman [1] proposed an algorithm that allows for exhaustively enumerating and structurally classifying simple articulated planar kinematic chains using the hierarchical representation by Fang and Freudenstein. In this algorithm, all isomorphic chains are automatically eliminated during the enumeration process, thereby eliminating isomorphism tests on the final set of chains. Pucheta and Cardona [2] used a constrained subgraph-based isomorphism identification approach for structural synthesis. Deng *et al.* [3] proposed a technique for adding subchains for the structural synthesis of planar closed KCs. In studying mixed-loop mechanical systems with simple and multiple joint K-chains, Pozhbelko [4] proposed a unified structure theory. Yan's mechanism synthesis method [5] generates comparable edges and vertices by using permutation groups and combinatorial concepts. Yan's approach is appropriate. Although achieving an efficiency breakthrough is difficult, it provides a foundation for applying and analysing similar edges in mechanism synthesis. In 2002, Wang and Yan [6] used regeneration rules to divide KC similarity into symmetry, transfer, row, and irregular similarities. The detection of link similarity was then suggested as a method for selecting functional components. Yang *et al.* [7] proposed the incident matrices technique for determining topological network isomorphism. When there are several comparable values in rows and arrays, discrimination becomes difficult. Dargar *et al.* [8] developed a method based on the concept of weighted structural matrices to demonstrate that a link and a chain are isomorphic at the same time. However, their utility has yet to be proven. Lohumi *et al.* [9] proposed a computerised loop-based method for determining isomorphism in planar kinematic chains but provided no mathematical justification. Ambekar and Agrawal [10] proposed a coding-based method for identifying isomorphisms in 1986. Fang and Freudenstein [11] proposed the stratified coding method in 1990. The Fang and Freudenstein approach was based on a stratified or hierarchical representation of KCs, beginning with the most abstract level (the simplified graph), moving to an intermediate representation (the contracted graph), and finally to the most detailed (the monochrome graph, or, for KCs with different types of joints, the colored graph). A stratified adjacency matrix represents the kinematic chain, which can then be condensed into a special code known as the stratified code. Tang and Liu [12] used the degree code of an adjacency matrix approach to identify isomorphic KCs, while Shin and Krishnamurthy [13] established the standard code method in 1994. Rai and Punjabi [14] proposed an

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identification isomorphism method based on link connection numbers and entropy in 2018. Rai [15] published a binary coding technique in 2019 to assess isomorphism in various KCs. Uniqueness and decodability are, theoretically, advantages of coding-based isomorphic identification techniques. Most proposed methods, however, can only reliably locate KCs with 10 bars or less [16]. In 1970, Buchsbaum and Freudenstein [17] used graph theory for the first time to synthesise PGT mechanisms. Tsai [18] used the characteristic polynomial of epicyclic gear trains to determine rotational and translational isomorphism in such mechanisms in 1987. A random number technique was used to compute the characteristic polynomial. Although less reliable, this method is more efficient. Kim and Kwak [19] developed an edge permutation-based technique for detecting isomorphism in PGTs in 1990. Hsu [20] proposed a structural code for detecting PGT mechanisms in 1994. Pathapati and Rao [21] examined previous isomorphism detection methods in 2002. They stated that one of the major reasons for the contradictory synthesis results of PGTs is that the isomorphism detection methods fail in some cases. In 2003, Rao [22] proposed a genetic approach to discovering PGT isomorphism. The use of the high-order adjacent link value to find isomorphisms among kinematic chains was proposed by Leiying He *et al.* [23]. Moha Shadab Alam *et al.* [24] identified isomorphisms by using the square of the degree link in a short path between every pair of vertices and the weight of the vertices. Zongyu Chang *et al.* [25] develop a simple method based on the eigenvector and eigenvalue to identify kinematic chain isomorphism.

Despite extensive research on mechanism synthesis methods, the limitation of these methods remains due to the existence of isomorphic solutions.

The proposed method is useful to solve the similarity between different kinematic chain configurations, which is the main problem facing innovative regenerative design and mechanism synthesis fields.

As a result, this paper attempts to develop an isomorphism identification method that uses the shortest path between non-binary vertices and their links with binary vertices. This new method drastically reduces execution time while detecting isomorphism in complex configurations with many vertices.

## II. Adjacency Matrix

A graph could be used to model a kinematic chain. In this case, bars are represented as vertices, while joins are the graph's edges.

The adjacency matrix is a square matrix with n-by-n dimensions (n is the number of vertices) that describes the relationship between different vertices in a graph by assigning a value of 1 to the elements $a_{ij}$ if two vertices are connected (or adjacent) and 0 otherwise.

$$A = [a_{ij}]_{n\times n} \quad \text{n is the number of vertices}$$

$$a_{ij} = \begin{cases} 1 & \text{if vertex i is adjacent to vertex j} \\ 0 & \text{otherwise} \end{cases}$$

## III. Distance between Non-binary Vertices

The distance between two vertices is defined as the number of joints on the shortest path between two vertices, i and j. This article calculates only the distance between non-binary vertices using a simple adaptation of the Floyd-Warshall method, which was originally developed to find the shortest path between all pairs of vertices in a weighted graph.

The Floyd-Warshall method is used to calculate the matrix distance between non-binary vertices. It begins by extracting the non-binary vertices connections from the adjacency matrix and calculating the distance between the vertices.

### A. Application Example

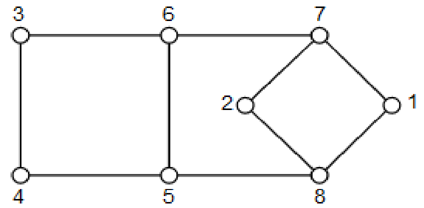


Figure 1. Eight bar KC.

The non-binary vertices connection matrix is obtained by removing the rows and columns from the adjacency matrix that correspond to the binary vertices.

The adjacency matrix and the extracted non-binary vertices connection matrix are:

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

$$And \quad Nbc = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \tag{2}$$

The distance matrix of non-binary vertices connection is:

$$DNbc = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 3 \\ 1 & 2 & 3 & 0 \end{bmatrix} \tag{3}$$

## IV. Process Method

The main idea behind this method is to extract the non-binary vertices connection (Nbc) and the columns of non-binary vertices connection (CNbc) from the adjacency matrix, calculate the distance between the non-binary vertices as shown above (DNbc), sort the DNbc, and compare it to the sorted DNbc of other kinematic chains. If two sorted DNbc are not identical, the corresponding kinematics are also not isomorphic. If not, include the DNbc in the adjacency matrix instead of the submatrix corresponding to the non-binary vertices to create a new matrix that includes the distance between the non-binary vertices. AmDNbc is the name given to this new matrix. This is the first step in the procedure.

The second step is to post multiply AmDNbc by CNbc and then pre-multiply the result by the transpose of CNbc. Finally, the rows and columns of the resulting matrix are sorted in ascending order. The obtained matrix is denoted MI, which stands for Matrix Identity. Two isomorphic kinematic chains have identical MI.

Two non-isomorphic KCs may lead to the same matrix identity MI in some cases. For example, when the only difference between two KCs is a path with a different number of binary bars in series. To make the proposed method more general, first change the binary bar series to two binary bars directly connected and remove the rows and columns from the adjacency matrix that correspond to the intermediate bars in this series, then add k to the Aac element of the adjacency matrix. The number of joints (edges) between vertices a and b is represented by Aac, which is as follows:

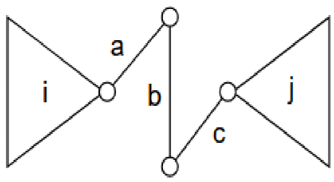


Figure 2. Series of binary vertices between two non-binary vertices.

In this example, Aac=2.

The summary of the method is as follows:

(1) Let us refer to the product of AmDNbc and CNbc as matrix C, which is n by nb (n total number of bars and nb the number of non-binary bars)
- The elements of C that correspond to a binary bar i and non-binary j are obtained by counting the number of common neighbors i and j share
- The elements of C corresponding to a non-binary bar i and non-binary j are the sum of the number of binary bars adjacent to i and j and the sum of distances between i and all non-binary bars neighbors with j.

Note: C(i,i)=degree(i).

(2) The matrix formed by multiplying the transpose of CNbc by the matrix formed in point 1 (named C) is a nb by nb matrix and is obtained as follows: elements corresponding to non-binary bars i and j are the sum of elements C(i,k), where k are all j's neighbors.

The two kinematic chains are isomorphic if the sum of the distances between the non-binary vertices adjacent to each other non-binary vertices and the number of bars in paths of length 2 between the binary vertices and each other non-binary vertices are equal for each of the two corresponding vertices in the two kinematic chains. These two points demonstrate how well this method works in determining whether two kinematic chains are equivalent or not.

Application on KC of Fig. 1:

The AmDNbc and the CNbc are:

$$AmDNbc = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 & 2 \\ 1 & 1 & 0 & 0 & 2 & 1 & 0 & 3 \\ 1 & 1 & 0 & 0 & 1 & 2 & 3 & 0 \end{bmatrix} \quad (4)$$

$$CNbc = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (5)$$

The matrix identity is:

$$MI = sort(CNbc' \times AmDNbc \ \times CNbc)$$

So, MI corresponding to the KC of Fig. 1 is:

$$MI = \begin{bmatrix} 0 & 1 & 4 & 5 \\ 0 & 1 & 4 & 5 \\ 4 & 4 & 5 & 9 \\ 4 & 4 & 5 & 9 \end{bmatrix} \quad (6)$$

## A. *Application Examples*

In this subsection, as all previous methods in the literature identify the isomorphism of eight-bar configurations, we apply the proposed method to eight-bar configurations (see Fig. 3); we then apply the proposed method to a fifteen-bar kinematic chain (Figs. 4, 5, and 6), such that most isomorphism identification methods in the literature have failed to distinguish; and finally, we show the ability of the proposed method to identify the isomorphism of kinematic chains with a large number of vertices (Figs. 7, 8, and 9); as an example, the twenty-bar kinematic chains.

### 1) *Identification of isomorphism between three eight-bar configurations*

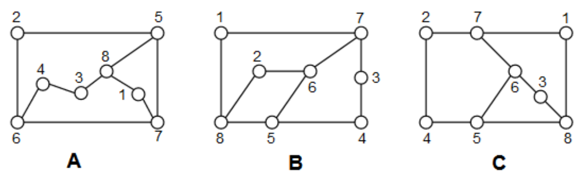


Figure 3. Three eight bars kinematic configurations, such as B and C isomorph and A non-isomorph with B and C.

It is obvious that the vertices of graphs B and C correspond. To obtain graph B, simply permute vertices 2 and 3 in configuration C. This is not the case when A and B or A and C are concerned. The sorted distance matrices between non-binary vertices for the three kinematic chains are identical:

$$SDNbc = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 1 & 2 & 3 \end{bmatrix} \quad (7)$$

However, A, B, and C are not necessarily isomorphic. The identity matrices for the three KCs, on the other hand, are used to determine which KCs are isomorphic, as follows:

$$MI_A = \begin{bmatrix} 0 & 2 & 4 & 6 \\ 0 & 2 & 4 & 6 \\ 2 & 4 & 6 & 10 \\ 2 & 4 & 6 & 10 \end{bmatrix} \tag{8}$$

$$MI_B = \begin{bmatrix} 0 & 2 & 3 & 5 \\ 0 & 2 & 4 & 6 \\ 3 & 4 & 5 & 9 \\ 4 & 4 & 6 & 9 \end{bmatrix} \tag{9}$$

$$MI_C = \begin{bmatrix} 0 & 2 & 3 & 5 \\ 0 & 2 & 4 & 6 \\ 3 & 4 & 5 & 9 \\ 4 & 4 & 6 & 9 \end{bmatrix} \tag{10}$$

Therefore, only A and B are isomorphic.

*2) Isomorphism identification of fifteen bar configurations*

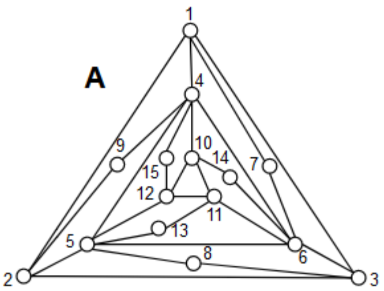


Figure 4. fifteen bars kinematic chains configurations.

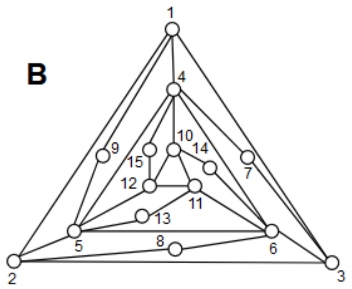


Figure 5. fifteen bars kinematic chains configurations non-isomorphic with Fig. 4.

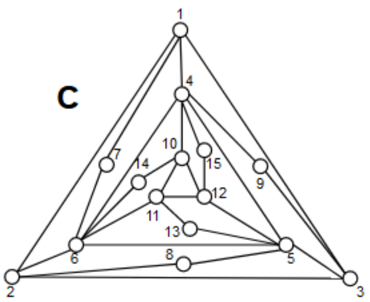


Figure 6. fifteen bars kinematic chains configurations isomorphic with Fig. 4.

Permuting vertex 2 and 3 in graph A in Fig. 4 results in graph C in Fig. 6, but there is no similarity between vertices in graph B in Fig. 5 and the other two graphs. These three kinematic chains have identical SDNbc, but the identity matrices determined the isomorphism and non-isomorphism between the three graphs.

$$SDNbc_{A\&B\&C} = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 & 1 & 2 & 2 & 2 & 2 \\ 0 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 \\ 0 & 1 & 1 & 1 & 2 & 2 & 2 & 3 & 3 \end{bmatrix} \tag{11}$$

$$MI_{A\&C} = \begin{bmatrix} 10 & 13 & 13 & 18 & 18 & 19 & 19 & 20 & 23 \\ 10 & 13 & 13 & 18 & 18 & 19 & 19 & 20 & 23 \\ 10 & 13 & 13 & 18 & 18 & 19 & 19 & 20 & 23 \\ 10 & 13 & 13 & 18 & 18 & 19 & 19 & 20 & 23 \\ 10 & 13 & 13 & 18 & 18 & 19 & 19 & 20 & 23 \\ 10 & 13 & 13 & 18 & 18 & 19 & 19 & 20 & 23 \\ 18 & 18 & 20 & 20 & 22 & 23 & 23 & 29 & 29 \\ 18 & 18 & 20 & 20 & 22 & 23 & 23 & 29 & 29 \\ 18 & 18 & 20 & 20 & 22 & 23 & 23 & 29 & 29 \end{bmatrix} \tag{12}$$

$$MI_B = \begin{bmatrix} 10 & 13 & 13 & 18 & 18 & 18 & 20 & 20 & 23 \\ 10 & 13 & 13 & 18 & 18 & 18 & 20 & 20 & 23 \\ 10 & 13 & 13 & 18 & 18 & 18 & 20 & 20 & 23 \\ 10 & 13 & 13 & 18 & 18 & 18 & 20 & 20 & 23 \\ 10 & 13 & 13 & 18 & 18 & 18 & 20 & 20 & 23 \\ 10 & 13 & 13 & 18 & 18 & 18 & 20 & 20 & 23 \\ 18 & 18 & 20 & 20 & 22 & 23 & 23 & 29 & 29 \\ 18 & 18 & 20 & 20 & 22 & 23 & 23 & 29 & 29 \\ 18 & 18 & 20 & 20 & 22 & 23 & 23 & 29 & 29 \end{bmatrix} \tag{13}$$

*3) Isomorphism identification of twenty-eight bar configurations*

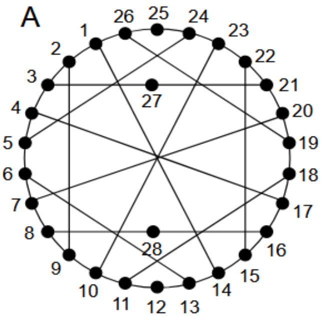


Figure 7. twenty-eight bars kinematic chains.

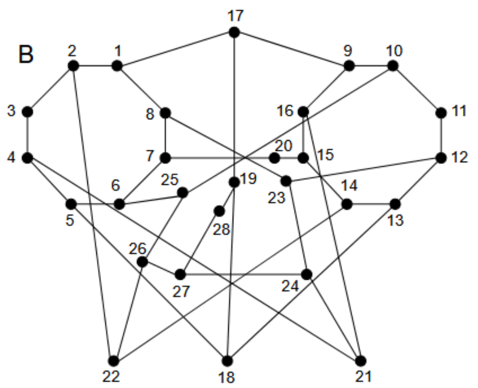


Figure 8. Twenty-eight bars kinematic chains isomorphic with Fig. 7.

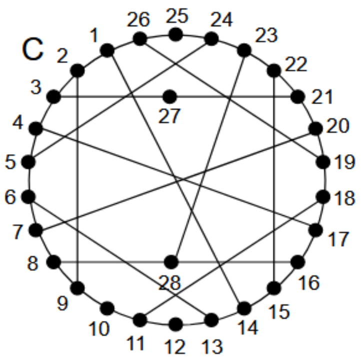


Figure 9. Twenty-eight bars kinematic chains non-isomorphic with Fig. 7.

The identity matrices obtained using the proposed method confirm that the two graphs A and B respectively in Figs. 7 and 8 are isomorphic, which is supported by the literature [26]. However, there is no correspondence between the sorted distance matrices of non-binary vertices of graph C in Fig. 9 and graph A. This confirms that graphs A and C are not isomorphic.

The identity matrices of graphs A and B

$$MI_{A\&B} = \begin{bmatrix} 8 \times 41010101011111212141414141616161616202020202323 \\ 16 \times 1112141416161717202023242424272727272929303033434 \end{bmatrix} \tag{14}$$

The SDNbc of graph C

$$SDNbc_C = \begin{bmatrix} 011122222233333333334444 \\ 3 \times 011122222233333333344444 \\ 011122222233333333444445 \\ 5 \times 011122222233333333444445 \\ 011122222333333334444445 \\ 2 \times 011122222333333344444445 \\ 2 \times 011122222333333344444455 \\ 2 \times 011122223333333444444455 \\ 2 \times 011222233333333444444456 \\ 2 \times 011222233333333444444556 \\ 2 \times 011222233333334444445556 \\ 012233334444444555555566 \end{bmatrix} \tag{15}$$

$$SDNbc_A = \begin{bmatrix} 16 \times 011122222233333333444445 \\ 8 \times 011222233333333444444556 \end{bmatrix} \tag{16}$$

## V. Comparison with Published Methods

### A. *Link Connectivity Number and Entropy Neglecting Tolerance and Clearance Algorithm Method [14]*

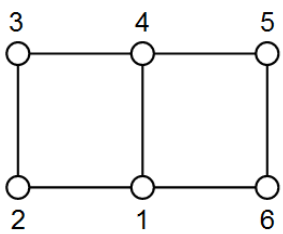


Figure 10. Wall graph kinematic chain.

This isomorphism detection method consists of several steps. Create a link-link connection matrix first, then compute the connectivity number of links for each vertex in the configuration, followed by the sum of all calculated values to provide a new value called the chain's total connectivity. Second, compute the power transmission connectivity. Third, compute the joint connectivity of each joint in the graph configuration, followed by the total joint value of the kinematic chain, which is the sum of all joints' joint connectivity. Finally, the energy transfer rate and power transmission efficiency are evaluated. All the preceding steps are designed to determine two constants of the configuration kinematic chain in question: the connectivity of power transmission and the efficiency of power transmission.

Application of this method on watt chain in Fig. 10

(1) the connectivity number link of each vertex stars by vertex 1 and so on 3.021, 2.011, 2.011, 3.021, 2.011, 2.011
(2) The total connectivity of the chain N=14.086
(3) Power transmission capacity is P=0.769563
(4) The joint connectivity of joint of each joint 3.032, 2.022, 3.032, 3.032, 2.022, 3.032, 4.042
(5) The total joint value of the kinematic chain is J=20.214
(6) Energy transfer rate E= 0.834152
(7) The power transmission efficiency Te= 0.987047

All these steps are required in order to evaluate the two invariant identifications of this simple Watt kinematic chain configuration. In addition, this method doesn't identify the non-isomorphism between configurations in Figs. 4 and 5. This demonstrates the proposed method's simplicity and speed, especially when the number of vertices is large.

The matrix identity and SDNbc of the Watt KC of Fig. 10 are:

$$MI = \begin{bmatrix} 0 & 7 \\ 0 & 7 \end{bmatrix}, and\ SDNbc = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \tag{17}$$

### B. *Genetic Algorithm Method [22]*

Application of this method on the kinematic chain of Fig. 10.

This method necessitates the creation of so-called fitness matrices and the corresponding chain strings. The fitness matrices and chain strings are developed until the problem is solved (first, second, third, fourth generation, etc.), which increases the computer execution time widely (see Table I below). Fortunately, the example in Fig. 10 only required two fitness matrices and two corresponding chain string generations (first and second). Two configurations are isomorphic if they have the same string in each generation.

The first-generation fitness matrix and the second-generation fitness matrix

$$First \begin{bmatrix} 0 & 5 & 1 & 6 & 1 & 5 \\ 5 & 0 & 4 & 1 & 4 & 2 \\ 1 & 4 & 0 & 5 & 2 & 4 \\ 6 & 1 & 5 & 0 & 5 & 1 \\ 1 & 4 & 2 & 5 & 0 & 4 \\ 5 & 2 & 4 & 1 & 4 & 0 \end{bmatrix} \quad Second \begin{bmatrix} 0 & 4 & 2 & 4 & 2 & 4 \\ 4 & 0 & 4 & 2 & 4 & 2 \\ 2 & 4 & 0 & 4 & 2 & 4 \\ 4 & 2 & 4 & 0 & 4 & 2 \\ 2 & 4 & 2 & 4 & 0 & 4 \\ 4 & 2 & 4 & 2 & 4 & 0 \end{bmatrix} \tag{18}$$

First generation chain string = 2[18–6 ,2 (5), 2 (1)] and 4[16–5,2 (4), 2, 1]

Second generation chain string = 6[16–3 (4), 2 (2)]

The matrix identity and SDNbc of the proposed method are:

$$MI = \begin{bmatrix} 0 & 7 \\ 0 & 7 \end{bmatrix}, and\ SDNbc = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix} \quad (19)$$

The proposed method returns the matrix identity of the configuration, demonstrating its unparalleled simplicity and speed. Meanwhile, Raos's method [22] requires more time to compute the first- and second-generation fitness matrices, as well as the corresponding first- and second-generation chain strings. The situation is worst for KCs with large numbers of vertices. Furthermore, if isomorphism cannot be determined after calculating the first- and the second-generation matrices, the iterative process should continue calculating the fitness matrices of subsequent generations.

### C. Isomorphic Identification for Kinematic Chains Using Variable High-order Adjacency Link VALUES [23]

Calculating the high-order adjacency link string of each vertex of the two configurations is the key to detecting the correspondence manner of the pair of corresponding strings to generate the adjacency matrix of one of the two configurations. Two isomorphic KCs have identical adjacency matrices; otherwise, they are not. This process requires a number of steps to determine isomorphism. To begin, compute the high-order adjacency link value $s_i^r$ of each vertex using the following equation:

$$s_i^r = s_i^{r-1} + \frac{1}{10}\sum_{j=1}^{n} s_j^{r-1} d_{i,j} \quad (20)$$

where n is the number of vertices in the configuration, di,j is an element from the ith row and jth column of the adjacency matrix, and r is the maximum length of the shortest path between each vertex indicated $s_i^0 = s_i$, is the initial value of vertex i. Second, if the string $S^r = \{s_1^r, s_2^r, \dots, s_n^r\}$ contains a duplicate value, the KC reassignment is performed [23] until a new string $S_{a,q,l}^r$ with no duplication is obtained. Third, search for pairs of corresponding strings between the two configurations; if none exist, the two configurations are not isomorphic. Determine the correspondence form of two corresponding strings in the fourth step. Finally, using the correspondence form, compute the adjacency matrix of one of the two configurations and compare it to the second matrix. If the two adjacency matrices are identical, the two KCs are isomorphic; otherwise, they are not.

We will Apply this method on fifteen bars configuration of Figs 4, 5

In this case r = 3,

(1) The 0-order adjacency link values of the two KCs A and B in Figs 4, 5 are:

$$S_A^0 = S_B^0 = \{2, 2, 2, 3, 3, 3, 1, 1, 1, 2, 2, 2, 1, 1, 1\} \quad (21)$$

(2) The three-order adjacency link values of the two KCs A and B in Figs. 4, 5 are:

$$S_A^3 = S_B^3 = \{5.526, 5.526, 5.526, 8.306, 8.306, 8.306, 3.183, 3.183$$
$$3.183, 5.526, 5.526, 5.526, 3.183, 3.183, 3.183\} \quad (22)$$

(3) Because there are duplicate values in the three-order adjacency link values of A and B, we must reassign KCA and KCB. KCA becomes as follows after two reassignment procedures.

$$S_{A,2}^3 = \{8.083, 6.297, 6.269, 10.114, 8.781, 8.837, 3.858, 3.282$$
$$3.614, 5.899, 5.599, 5.627, 3.220, 3.252, 3.552\} \quad (23)$$

After two reassignment procedures, we find six forms of the 3-order adjacency link value of KCB and select one as an example.

$$S_{B,2}^3 = \{8.083, 6.269, 6.269, 10.114, 8.837, 8.781, 3.614, 3.282$$
$$3.858, 5.899, 5.597, 5.629, 3.222, 3.250, 3.552\} \quad (24)$$

(4) Because the $S_{A,2}^3$ and the six 3-order adjacency link values of KCB do not correspond, the two configurations are not isomorphic.

Fortunately, there is no correspondence between $S_{A,2}^3$ and the six forms of $S_{B,2}^3$. Otherwise, we must determine how each related string pair ( $S_{A,2}^3$ and each variant of $S_{B,2}^3$) corresponds. In each instance, the adjacency matrix of configuration B is generated and compared to the adjacency matrix of configuration A. In terms of computer execution time, the proposed method in this paper is unquestionably simpler and faster than the exposed method. It immediately returns the aforementioned identity matrices.

### D. Isomorphism Identification and Structural Similarity & Dissimilarity Among the Kinematic Chains Based On [WSSP] Matrix [24]

To create the WSSP matrix, add the sum of the squares of the vertices in the short route between vertices i and j and the degree of vertices in the configurations. This matrix's sum is the KC identity. As a result, two kinematic chains with the same identities are isomorphic. The various procedures for developing KC identification will be applied to the example shown in Fig. 11 below:

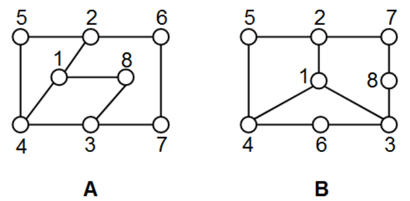


Figure 11. Two non-isomorphic eight bars configuration

(1) Find the degree vector, which is a vector whose elements are the vertices' degrees.

$$d_A = [3\ 3\ 3\ 3\ 2\ 2\ 2\ 2],\ d_B = [3\ 3\ 3\ 3\ 2\ 2\ 2\ 2] \quad (25)$$

(2) Calculate the squared shortest path distance matrix [SSP]

$$[SSP] = \{d_{ij}\}_{n\times n}$$

$$\{d_{ij}\}_{n\times n} = \begin{cases} 1\ if\ i\ and\ j\ directly\ connected \\ 0\ if\ i = j \\ sum\ of\ degrees\ squared\ of\ all\ vertices\ between\ i\ and\ j \end{cases}$$

The $[SSP]_A$ and $[SSP]_B$ are:

$$[SSP]_A = \begin{bmatrix} 0 & 1 & 4 & 1 & 9 & 9 & 13 & 1 \\ 1 & 0 & 8 & 4 & 1 & 1 & 4 & 9 \\ 4 & 8 & 0 & 1 & 9 & 4 & 1 & 1 \\ 1 & 4 & 1 & 0 & 1 & 13 & 9 & 9 \\ 9 & 1 & 9 & 1 & 0 & 9 & 13 & 18 \\ 9 & 1 & 4 & 13 & 9 & 0 & 1 & 13 \\ 13 & 4 & 1 & 9 & 13 & 1 & 0 & 9 \\ 1 & 9 & 1 & 9 & 18 & 13 & 9 & 0 \end{bmatrix} \quad (26)$$

$$[SSP]_B = \begin{bmatrix} 0 & 1 & 1 & 1 & 9 & 9 & 9 & 9 \\ 1 & 0 & 8 & 4 & 1 & 13 & 1 & 4 \\ 1 & 8 & 0 & 4 & 13 & 1 & 4 & 1 \\ 1 & 4 & 4 & 0 & 1 & 1 & 13 & 13 \\ 9 & 1 & 13 & 1 & 0 & 9 & 9 & 13 \\ 9 & 13 & 1 & 1 & 9 & 0 & 13 & 9 \\ 9 & 1 & 4 & 13 & 9 & 13 & 0 & 1 \\ 9 & 4 & 1 & 13 & 13 & 9 & 1 & 0 \end{bmatrix} \quad (27)$$

(3) Determine the relative weight of degree link

The weight of the degree link is the ratio of the degree of the i-th vertex to the degree of the j-th vertex, as shown below:

$$W = \{w_{ij}\}_{n\times n}$$

$$\begin{cases} w_{ij} = \dfrac{d_i}{d_j} \ and \ w_{ji} = \dfrac{d_j}{d_i} \ when \ i \neq j \\ w_{ii} = w_{jj} = 0 \ otherwise \end{cases}$$

$$W_A = W_B = \begin{bmatrix} 0 & 1 & 1 & 1 & 1.5 & 1.5 & 1.5 & 1.5 \\ 1 & 0 & 1 & 1 & 1.5 & 1.5 & 1.5 & 1.5 \\ 1 & 1 & 0 & 1 & 1.5 & 1.5 & 1.5 & 1.5 \\ 1 & 1 & 1 & 0 & 1.5 & 1.5 & 1.5 & 1.5 \\ 0.7 & 0.7 & 0.7 & 0.7 & 0 & 1 & 1 & 1 \\ 0.7 & 0.7 & 0.7 & 0.7 & 1 & 0 & 1 & 1 \\ 0.7 & 0.7 & 0.7 & 0.7 & 1 & 1 & 0 & 1 \\ 0.7 & 0.7 & 0.7 & 0.7 & 1 & 1 & 1 & 0 \end{bmatrix} \quad (28)$$

(4) Calculate the weighted squared shortest path $[WSSP]$, which is the product of the $[SSP]$ matrix and the $[W]$ matrix

$$[WSSP] = [SSP] \times [W]$$

$$[WSSP]_A = \begin{bmatrix} 28.4 & 27.4 & 24.4 & 27.4 & 32 & 32 & 28 & 40 \\ 22.5 & 23.5 & 15.5 & 19.5 & 33.5 & 33.5 & 30.5 & 25.5 \\ 19.5 & 15.5 & 23.5 & 22.5 & 25.5 & 30.5 & 33.5 & 33.5 \\ 27.4 & 24.4 & 27.4 & 28.4 & 40 & 28 & 32 & 32 \\ 39 & 47 & 39 & 47 & 70 & 61 & 57 & 52 \\ 34.1 & 42.1 & 39.1 & 30.1 & 54.5 & 63.5 & 62.5 & 50.5 \\ 30.1 & 39.1 & 42.1 & 34.1 & 50.5 & 62.5 & 63.5 & 54.5 \\ 47 & 39 & 47 & 39 & 52 & 57 & 61 & 70 \end{bmatrix} \quad (29)$$

$$[WSSP]_B = \begin{bmatrix} 28.2 & 27.2 & 27.2 & 27.2 & 31.5 & 31.5 & 31.5 & 31.5 \\ 25.3 & 26.3 & 18.3 & 22.3 & 37.5 & 25.5 & 37.5 & 34.5 \\ 25.3 & 18.3 & 26.3 & 22.3 & 25.5 & 37.5 & 34.5 & 37.5 \\ 27.6 & 24.6 & 24.6 & 28.6 & 40.5 & 40.5 & 28.5 & 28.5 \\ 36.7 & 44.7 & 32.7 & 44.7 & 67 & 58 & 58 & 54 \\ 36.7 & 32.7 & 44.7 & 44.7 & 58 & 67 & 54 & 58 \\ 34.1 & 42.1 & 39.1 & 30.1 & 54.5 & 50.5 & 63.5 & 62.5 \\ 34.1 & 39.1 & 42.1 & 30.1 & 50.5 & 54.5 & 62.5 & 63.5 \end{bmatrix} \quad (30)$$

(5) The KC identity is the structural invariant of a kinematic chain that is the sum of all elements in the [WSSP].

The sum of $[WSSP]_A$ is: 2464 and the sum of $[WSSP]_B$ is: 2478

As a result, the two kinematic chains are not isomorphic. This method, however, is more complicated than the proposed method. It takes longer to reach the conclusion because defining the degree of each vertex and the type of vertices on the shortest path between every two pairs of vertices is required, followed by calculating the relative weight of the degree link, and so on. In contrast, the proposed method does not consider the degree of vertices when identifying the isomorphism, demonstrating its simplicity and speed once more.

### *E. Identification Isomorphism by Using Eigenvalues and Eigenvectors [25]*

The eigenvalues and eigenvectors of a graph are fundamental constants used to identify isomorphisms between various kinematic chains. Despite the publication of some counterexamples, such as the two eight-bar kinematic chain graphs in Fig. 12, this method yields different eigenvalues (in absolute) for the two kinematic chains in Fig. 12, confirming that they are not isomorphic.

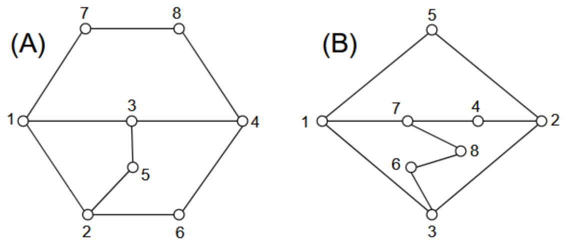


Figure 12. Two isomorphic eight bars kinematic chains

Each column of eigenvalue matrices is sorted in ascending order, as shown below, to facilitate comparison.

$$e_A = \begin{bmatrix} -2.281333395043705 \\ -1.944310733422072 \\ -1.140548094310296 \\ -0.099890015366104 \\ 0.598633976166785 \\ 1.000000000000000 \\ 1.275163171393258 \\ 2.592285090582134 \end{bmatrix} \quad (31)$$

$$e_B = \begin{bmatrix} -2.281333395043705 \\ -1.944310733422072 \\ -1.140548094310296 \\ -0.099890015366104 \\ 0.598633976166784 \\ 0.999999999999998 \\ 1.275163171393258 \\ 2.592285090582136 \end{bmatrix} \quad (32)$$

There is no absolute similarity between the elements of the eigenvalues of the two graphs' kinematic chains. The proposed method, however, recognizes the similarity between graphs A and B in Fig. 12. (The two kinematic chains' identity matrices are identical.)

$$MI_A = \begin{bmatrix} 0 & 2 & 3 & 5 \\ 0 & 2 & 4 & 6 \\ 3 & 4 & 5 & 9 \\ 4 & 4 & 6 & 9 \end{bmatrix} and \ MI_B = \begin{bmatrix} 0 & 2 & 3 & 5 \\ 0 & 2 & 4 & 6 \\ 3 & 4 & 5 & 9 \\ 4 & 4 & 6 & 9 \end{bmatrix} \quad (33)$$

As shown in the preceding example, the proposed method outperforms the method that involves calculating eigenvalues and eigenvectors.

The following table contains a comparison between the execution times of the above methods and the proposed method.

TABALE I. COMPARISON OF EXECUTION TIME WITH SOME LITERATURE METHODS

| Methods | Execution time |
|---|---|
| Genetic algorithm method | $O(n^3 + n^g)$ |
| Isomorphic identification for kinematic chains using variable high-order adjacency link values | $O(n^3 + mn^2)$ |
| Isomorphism identification and Structural Similarity & Dissimilarity Among the Kinematic Chains Based On [WSSP] Matrix | $O(n^3 + n^2)$ |
| Proposed method | $O(n_{Nb}{}^3 + n_b{}^2)$ |

n: vertex number; g: number of generation; m: edges number; $n_{Nb}$: non-binary vertex number; $n_b$: binary vertex number

## VI. CONCLUSION

In this paper, we propose a method to solve the problem of isomorphism identification among kinematic chains. For the last thirty years, isomorphism identification has been a critical issue, particularly in complex configurations with many vertices. Many researchers have made significant contributions to this field. In comparison to recently published articles, the proposed method identifies isomorphisms more efficiently and with significantly less computer execution time.

## NOMENCLATURE

**AmDNbc:** Adjacency matrix with the distance between the non-binary vertices

**CNbc:** Columns of non-binary vertices in the adjacency matrix

**DNbc:** Distance matrix of non-binary vertices connection

**KCs:** Kinematic chains

**MI:** Identity matrix

**Nbc:** Matrix of non-binary vertices connection

**$n_{Nb}$:** Number of non-binary vertices

**$n_b$:** Number of binary vertices

**PGTs:** Planetary gear trains

**SDNbc:** Sorted distance matrix of non-binary vertices connection

**SSP:** Squared shortest path distance matrix

**W:** Relative weight of degree link matrix

**WSSP:** Weighted squared shortest path: [WSSP] matrix

## CONFLICT INTERESTS

The authors declare that they have not competing interests

## AUTHORS' CONTRIBUTION

Mohamed Aly Abdel Kader carried out the research, conceived of the presented idea, conducted the research, analyzed the data and wrote the manuscript. Abdeslam Aannaque supervised the project, provided feedback and guidance, and proofread the manuscript; all authors had approved the final version.

## REFERENCE


[1] E. A. Butcher and C. Hartman, "Efficient enumeration and hierarchical classification of planar simple-jointed kinematic chains: Application to 12- and 14-bar single degree-of-freedom chains," *Mechanism and Machine Theory*, vol. 40, pp. 1030–1050, 2005.

[2] M. Pucheta and A. Cardona, "An automated method for type synthesis of planar linkages based on a constrained subgraph isomorphism detection," *Multibody System Dynamics*, vol. 18, pp. 233–258, 2007.

[3] Z. Deng, F. Yang, and J. Tao, "Adding sub-chain method for structural synthesis of planar closed kinematic chains," *Chinese Journal of Mechanical Engineering*, vol. 25, pp. 206–213, 2012.

[4] V. Pozhbelko, "A unified structure theory of multibody open-, closed-, and mixed-loop mechanical systems with simple and multiple joint kinematic chains," *Mechanism and Machine Theory*, vol. 100, pp. 1–16, 2016.

[5] H. S. Yan, "A methodology for creative mechanism design," *Mechanism and Machine Theory*, vol. 27, pp. 235–242, 1992.

[6] Y. X. Wang and H. S. Yan, "Computerized rules-based regeneration method for conceptual design of mechanisms," *Mechanism and Machine Theory*, vol. 37, pp. 833–849, 2002.

[7] F. Yang, Z. Q. Deng, J. G. Tao, "A new method for isomorphism identification in topological graphs using incident matrices," *Mechanism and Machine Theory*, vol. 49, no. 212, pp. 298-307.

[8] A. Dargar, A. Hasan, R. A. Khan, "Some new codes for isomorphism identification among kinematic chains and their inversions," *International Journal of Mechanisms and Robotic Systems*, vol. 1, no. 1, pp. 49-67, 2013.

[9] M. K. Lohumi, A. Mohammad, I. A. Khan, "A computerized loop-based approach for identification of isomorphism and type of mobility in planar kinematic chains," *Sadhana-Academy Proceedings in Engineering Sciences*, vol. 40, no. 2, pp. 335-350, 2015.

[10] A. Ambekar and V. Agrawal, "Canonical numbering of kinematic chains and isomorphism problem: Min code," *Mechanism and Machine Theory*, vol. 22, pp. 453–461, 1987.

[11] W. Fang and F. Freudenstein, "The stratified representation of mechanisms," *Journal of Mechanical Design*, vol. 112, pp. 514–519, 1990.

[12] C. Tang and T. Liu, "The degree code-a new mechanism identifier," *Journal of Mechanical Design*, vol. 115, pp. 627 -627, 1993.

[13] J. K. Shin and S. Krishnamurty, "Development of a standard code for colored graphs and its application to kinematic chains," *Journal of Mechanical Design*, vol. 116, pp. 189–196, 1994.

[14] R. K. Rai and S. Punjabi, "Kinematic chains isomorphism identification using link connectivity number and entropy neglecting tolerance and clearance," *Mechanism and Machine Theory*, vol. 123, pp. 40–65, 2018.

[15] R. K. Rai, "A new algorithm of links labelling for the isomorphism detection of various kinematic chains using binary code," *Mechanism and Machine Theory*, vol. 131, pp. 1–32, 2019.

[16] T. Mruthyunjaya, "Kinematic structure of mechanisms revisited," *Mechanism and Machine Theory*, vol. 38, pp. 279–320, 2003.

[17] F. Buchsbaum and F. Freudenstein, "Synthesis of kinematic structure of geared kinematic chains and other mechanisms," *Journal of Mechanisms*, vol. 5, pp. 357–392, 1970.

[18] L. W. Tsai, "An application of the linkage characteristic polynomial to the topological synthesis of epicyclic gear trains, Journal of Mechanisms," *Transmissions, and Automation in Design*, vol. 109, pp. 329-336, 1987.

[19] J. U. Kim and B. M. Kwak, "Application of edge permutation group to structural synthesis of epicyclic gear trains," *Mechanism and Machine Theory*, vol. 25, pp. 563–574, 1990.

[20] J. K. Shin and S. Krishnamurty, "Development of a standard code for colored graphs and its application to kinematic chains," *Journal of Mechanical Design*, vol. 116, pp. 189–196, 1994.

[21] V. Prasad Raju Pathapati and A. Rao, "A new technique based on loops to investigate displacement isomorphism in planetary gear trains," *Journal of Mechanical Design*, vol. 124, pp. 662–675, 2002.
[22] A. C. Rao, "A genetic algorithm for topological characteristics of kinematic chains," *Journal of Mechanical Design*, vol. 122, pp. 228–231, 2000.
[23] L. Y. He, F. X. Lin, L. Sun, and C.Y. Wu, "Isomorphic identification for kinematic chains using variable high-order adjacency link values," *Journal of Mechanical Science Technology*, vol. 33, no. 10, pp. 1-9, 2019.
[24] M. S. Alam, M. Suhaib, and A. Mohd, "Isomorphism identification and structural similarity & dissimilarity among the kinematic chains based on [WSSP] matrix," *International Research Journal of Engineering and Technology*, pp. 467-474, 2017.
[25] Z. Chang, C. Zhang, Y. Yang, and Y. Wang, "A new method to mechanism kinematic chain isomorphism identification," *Mechanism and Machine Theory*, vol. 37, pp. 411–417, 2002.
[26] H. F. Ding and A. Huang, "Isomorphism identification of graphs: Especially for the graphs of kinematic chains, mechanism and machine theory, vol. 44, no. 1, pp. 122–139, 2009.