# Modal Analysis of Simply Supported Tapered Pipe Transporting Fluid with an Edge Crack Using Finite Element Method

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***Abstract*—The crack is one of the most common types of failure in pipelines that convey fluid, and early detection of the crack may assist to avoid the piping system from experiencing catastrophic damage, which would otherwise be fatal. The influence of flow velocity and the presence of a crack on the performance of a tapered simply supported pipe containing moving fluid is explored using the finite element approach in this study. ANSYS software is used to simulate the pipe as Bernoulli's beam theory. In this paper, the fluctuation of natural frequencies and matching mode shapes for various scenarios owing to changes in fluid speed and the presence of damage is discussed in detail. The findings demonstrated that the presence of a fracture reduces the stiffness of the systems, resulting in a decrease in the basic natural frequencies. This loss is more pronounced when the fracture is further away from the nodal locations for each mode. Finally, it is demonstrated that increasing flow velocity reduces natural frequencies.**

***Keywords*—damage detection, finite element, tapered pipe, vibration characteristics**

## I. INTRODUCTION

Pipes are used in a broad variety of applications, including fuel gas storage, ventilation conduits, and heat exchanger power plants, nuclear reactor and oil refineries. Pipes may experience high vibration, which may cause damage and, in some cases, jeopardize the ability to operate safely and reliably. There have been various studies in which vibration frequency-based damage diagnostics has been used to ensure the safety of structures and pipelines when they are placed into operation [1–10], and the natural frequencies are the most appealing since they are the most easily measured of the frequencies available. This necessitated an examination of the situations in which a crack is formed and allowed to grow until it causes the flow of liquid from a pipe and results in considerable losses. It is possible to detect damage expansion and assess the pipe's safety status using the natural frequency change when fractures spread in the pipes.

Free and forced vibrations in pipes above ground were studied by Housner [1]. He found that vibrations in the pipeline were not affected by the fluid flow, according to the proposed solution based on elementary beam theory. For free and steady-state vibrations, substantial amplitudes may occur even if the amount of damping is too low. The basic mode of a pipeline carrying a transporting liquid was studied by Long [2] using analytical and experimental approaches for the studying of free transverse vibrations. The governing differential equation of motion was solved utilizing infinite power series. The results show that when flow rate increases, there is a little decrease in frequency. Kirubakaran [3] employed image analysis to identify pipeline fracture patterns. The results were utilized to create a mathematical morphological operator for fracture edge identification. However, Gaith [4] investigated cracks being detected in simply supported FRP beams using finite element modeling.

The influence of liquid density in a pipe on vibration characteristics such as the fundamental circular carrying laminar flow and demonstrating cross-sectional change was examined by Al-Hashimy *et al*. [5]. The governing equation of an elastically hinged conduit of infinite length transporting a fluid under pressure was numerically solved by Stein *et al*. [6]. The impact of the ensuing internal pressure is also highlighted. An undamped system's stability is studied for the effects of flow velocity, foundation modulus, and internal pressure; critical flow speeds are also established. A strain rate dependent damage model was used by Oikonomidis [7] to construct a model for predicting fracture development in natural gas pipelines. Experimentation on a notched pipe concurred their model correct.

A novel non-linear model of a pipeline transporting a fluid and clamped at both ends was presented by Lee and Chung [8]. The equations of motion for non–linear Lagrange strain theory and the Euler–Bernoulli beam theory were provided using the extended Hamilton principle for both axial and lateral vibrations. The Galerkin technique is implemented in solving the equations. Using

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the wavelet-based finite element approach, Oke *et al*. [9-10] investigated the dynamic behavior of a composite pipe transporting liquid with interior surface damage. A fluid conveying pipe's theoretical dynamic analysis has been done to investigate its dynamic properties [11]. Additionally, numerical methods have been employed to solve the analytical equations describing these systems, and when compared to the analytical solution, the findings are exact and precise. The investigation of simply supported inclined pipes that are conveying fluid under the influence of thermal stress has led to the development of an analytical model [12]. It should be noted that a number of factors, including aspect ratio and inclination angle, have been taken into account. Additionally, Jabbar et al. [13] analyzed the same circumstances utilizing the Euler beam theory model, where it should be noted that weariness was taken into account. Using the absolute nodal coordinate approach, a dynamic model of free vibration for a rotating pipe delivering fluid has been created [14]. It should be noted that studies have been done on how natural frequencies are affected by fluid and rotational motion. When changing the frequency, amplitude, and flow velocity, an unstable response was seen in a fixed-free L-shaped pipe that transports fluid, therefore a similar method was employed to do nonlinear analysis [15]. Hamilton's approach has been used in dynamic analysis for pipelines with varying internal thickness [16]. In cases where there is good agreement between the analytical results, finite element modeling has been done using ANSYS software. A significant problem in transportation and the design of machining operations is the way that moving media behave on structures and machines. Fluid flow acts as a concentrated tangential follower force at the tip of the pipe, which significantly affects the dynamic characteristics. It is crucial to look into these factors that affect structural dynamics as a result. Therefore, when dealing with vibration, protecting the pipe system is a vital responsibility. There have been various investigations concerning the characteristics and management of vibration in fluid-transport pipework as well as composite mechanical structures [17, 18]. However, the change in dynamic properties of FRP composite laminated cantilever beams has been researched for the purpose of detecting cracks as well as fractures [19–21].

Foundation stiffness and damping were examined by Lottati *et al*. [22] for cantilever and fixed-fixed end pipes. Elastic foundations are shown to raise the critical flow fluid velocity. Stabilizing or destabilizing effects of damping are based on the mass-to-fluid ratio. Tornabene *et al*. [23] implemented the generalized differential to explore the critical flow speeds of pipes conveying fluid and presented the link between the eigenvalue branches and the accompanying unstable flutter modes.

Langre *et al*. [24] examined the stability of a thin flexible cylinder and treated it as a beam in order to better understand its behavior. The stability is investigated by using a finite deference approach to the frequency domain governing equation of motion. The flutter effect is studied using a linear stability analysis of lateral motion as a function of the key factors, which include flow velocity and cylinder length. For the leaking from the pipeline fracture, Zhang [25] developed a prediction code. The model was tested on single and two phases fluid flow at high and low temperatures.

Crack-induced vibration mode shapes were studied by Nguyen [26] for the purpose of crack localization in conjunction with the horizontal and transverse bending vibration [9]. For leak detection in oil and gas pipelines, Lu [27] conducted a thorough review of the latest crack detection technologies. His evaluation contains the merits and disadvantages of each approach, as well as the suitable technical instruments for each of these techniques. Corrosion damage to the pipe produced by cracks and erosion was studied by Mohamed *et al*. [28]. They observed in their analytical study that the equations for the two situations exhibited good agreement with the experimental analysis where the change of vibrating sensitivity of the measuring sensor identifies faults. Using the obtained natural frequencies of pinned-pinned and clamped-clamped pipes, Jweeg *et al*. [29] developed a novel experimental method for determining the critical velocity of pipes carrying fluid. Using the vibration equation for conservative pipes, they deduced the frequency and critical buckling velocity expressions and gave a semi-analytic solution for such pipes. Multiple cracks in lengthy pipelines holding fluid at varying pressures were identified using a method based on measuring the change in natural frequencies [30]. Hamilton's approach was utilized by Kaewunruena *et al*. [31] to construct a finite element model for fluid-conveying maritime risers. They generated fundamental circular frequencies and matching mode shapes for isotropic pipes that transport fluid on elastic layer. Natural frequencies and stability were studied using finite element analysis by Mostafa [32] for pipeline delivering incompressible fluid over a viscoelastic foundations with hinged end. An investigation of the vibration of tubular beams transporting fluid has been carried out by Gaith [33–34]. An investigation on the transverse dynamic response of a pipe carrying fluid with changing cross sectional area was conducted. The pipe is modeled using Euler Bernoulli's beam theory, and the partial differential equations are solved using Galerkin's approach. He analyzed the same pipe lying on a Winkler viscoelastic layer and reported the influence of numerous factors on the stability [35]. The flow of an electrically conducting pair stress fluid that is incompressible and formed by longitudinal and torsional oscillations of a continuously injected its surface was studied in the presence of a radial porous circular cylinder magnetic field [36]. Furthermore, the analytical two-dimensional heat transfer and entropy production properties of axisymmetric, incompressible viscous fluid flow in a horizontal circular conduit are examined [37].

A porous circular cylinder exposed to continuous suction/injection at its surface was investigated using finite difference method by Josula *et al*. [38]. It should be mentioned that the flow is produced by executing longitudinal and torsional oscillations of the porous cylinder. However, Newtonian axisymmetric, viscous heating flow's behavior in a horizontal pipe's two dimensions with regard to thermal transport was analyzed analytically with the flow is exposed to a steady magnetic field [39–40]. The impacts of fracture depth and position, fiber orientation, and fiber volume percentage on the flexibility of cracked fiber-reinforced composite beams are investigated along with corresponding natural frequencies and mode shapes [41].

In this study, analysis of the natural frequencies of the pipes using ANSYS software is utilized to investigate the impact of crack on the frequency of pipe-carrying fluid in the presence of changing fluid velocities and variable crack depths. Cracks in the pipe are evaluated for their size and placement. Finite element analysis is carried out using ANSYS program with the aim to estimate the natural frequencies of a tapered cantilever pipe-carrying fluid in the presence of varying fluid velocities and crack depths. It should be mentioned that at the beginning of the study, the effect of relative crack position is studied for different crack depths at flow velocity of 10 m/s. Moreover, the impact of fluid flow velocity on natural frequencies is investigated for a crack depth 7 mm for different relative crack positions. Finally, at a relative crack position of 0.3 L, the natural frequencies are estimated for different crack depths and fluid flow velocities.

## II. Formulation of the Problem

Fig. 1 shows a hinged pipe with an unequal inlet and output diameter and a clearly tapered length L. The fluid has an input velocity of $\boldsymbol{v}$.

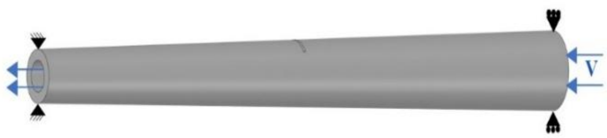


Figure 1. Schematic illustrating fluid conveyance via a hinged pipe with a changing cross section.

Using Hamilton's concept for kinetic and potential energy of the system as a starting point

$$\delta = \int_{t_1}^{t_2} (T - V)\,dt = 0. \quad (1)$$

$$T = T_p + T_f$$

$$= \frac{1}{2}\int_{t_1}^{t_2} \{ m\left(\frac{\partial w}{\partial t}\right)^2 + MV^2 + M\left(\frac{\partial w}{\partial t}\right)^2$$

$$+2MV\frac{\partial w}{\partial t}\frac{\partial w}{\partial x} + MV^2\left(\frac{\partial w}{\partial t}\right)^2 \}dx. \quad (2)$$

Hence, an equation of motion for a simply supported pipe is presented by Gaith and formulated as follows [34]. It should be mentioned that model's kinetic energy T and potential energy V are derived for simply supported boundary conditions as in Benjamin's approach [42].

$$EI\frac{\partial^4 w}{\partial x^4} + \left(2E\frac{dI}{dx}\right)\frac{\partial^3 w}{\partial x^3} + \left(E\frac{d^2 I}{dx^2} + MV^2\right)\frac{\partial^2 w}{\partial x^2} +$$

$$2MV\frac{\partial^2 w}{\partial x \partial t} + (M+m)\frac{\partial^2 w}{\partial t^2} + MV\frac{dV}{dx}\frac{\partial w}{\partial x} = 0. \quad (3)$$

where

$$I(x) = \frac{\pi}{4}\left\{(a + (b-a)\frac{x}{L} + h)^4 - (a + (b-a)\frac{x}{L})^4\right\}. \quad (4)$$

$$V(x) = v_o \frac{b^2}{(a+(b-a)\frac{x}{L})^2}. \quad (5)$$

And applying the boundary conditions

$$w\big|_{x=0} = \frac{\partial^2 w}{\partial^2 x}\bigg|_{x=0} = 0,\ and\ w\big|_{x=L} = \frac{\partial^2 w}{\partial^2 x}\bigg|_{x=L} = 0$$

where $\boldsymbol{E}$ is Young's constant, the pipe and fluid mass per unit are denoted by $M$ and $m$, respectively. $w(x,t)$ is the lateral displacement of the pipe, and the moment of inertia and the fluid velocity function at any point along the pipe are represented by the functions $I(x)$, $V(x)$, respectively. It should be said that $h$ is the thickness of the pipe. $a$ and $b$ are the intake inner diameter and the exit outer and inner diameter, respectively.

## III. Results and Discussion

### A. Finite Element Results

Finite Element Modeling (FEM) is used widely due to its ability of modeling irregular and complex geometrical shapes. Furthermore, FEM can achieve high accuracy without the need of prototype. A finite element model is created using ANSYS software for a simply supported pipe transporting a fluid with defined speed and an outlet diameter equal to 90% of the inlet diameter. A crack of specific depth and location is inserted. The pipe is meshed with 49482 elements and 87114 nodes as shown in Fig. 2.

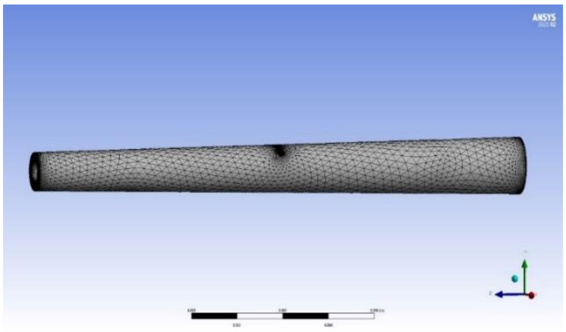


Figure 2. Meshed finite element model.

A specific instance of a pipe with uniform cross section is examined in order to authenticate the finite element model. The results are obtained in terms of first natural frequencies for a uniform simply supported pipe with double cracks at different depths and locations similar to those used in reference [29] and found to be in excellent agreement as shown in Fig. 3.

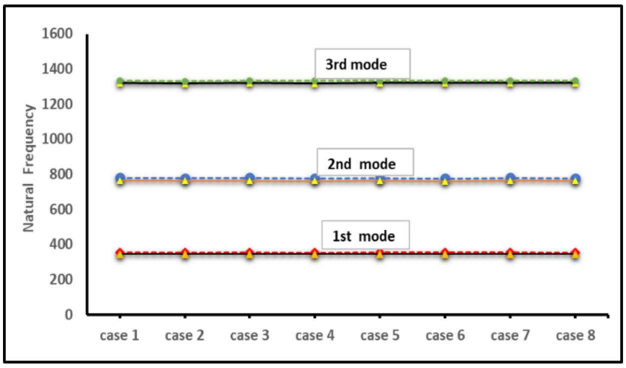


Figure 3. Comparison of first three natural frequencies for the uniform cross-section with double cracks for different cases.

The effect of cross sectional outlet to inlet diameter ratio on the first three natural frequencies for the considered system with fluid velocity ***v***= 5 m/s, is revealed in Fig. 4. The results indicate that by decreasing outlet/inlet diameter ratio, fundamental circular frequencies as well as critical velocities are decreasing. It should be said that this trend was also found by Gaith [18]. However, the geometry and material properties in Table. I are used for the considered pipe system at room temperature where the fluid properties variation is neglected since there is no system temperature change assumption.

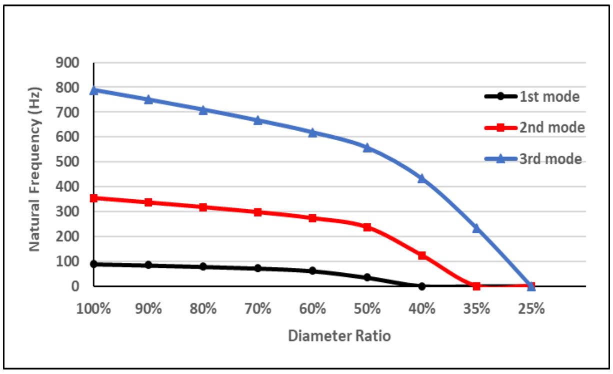


Figure 4. First three natural frequencies versus the outlet/inlet diameter ratios for fluid flow velocity 5 m/s.

TABLE I. THE GEOMETRY AND MATERIAL PROPERTIES FOR THE CONSIDERED PIPE SYSTEM

| | | |
|---|---|---|
| Pipe | Young's Modulus | 68.95 *GPA* |
| | Poisson's Ratio | 0.33 |
| | Length | 800 *mm* |
| | Inlet Do | 27 *mm* |
| | Inlet Di | 17 *mm* |
| | Outlet Do | 21.6 *mm* |
| | Outlet Di | 13.6 *mm* |
| Fluid | Density | 908.2 kg/m$^3$ |

The first mode shapes for a simply supported pipe with edge crack using ANSYS are presented in Fig. 5 for flow velocity, crack depth and crack position of 15 m/s, 5 mm and 0.5 L, respectively. Fig. 5 shows that when a crack is located near to the pipe's mid-span, the first natural frequency is reduced, and this drop is more pronounced when the fracture depth is growing. The combined impact of crack depth and position on the first three frequencies are shown in Figs. 6-8, respectively for outlet/inlet diameter ratio equal to 90%, and fluid velocity 5 m/s. It should be mentioned that Alfaqs *et al*. [35] Discussed same pipe model but for different boundary conditions. Moreover, current study investigates the influence of crack depth and position combination on natural frequencies contrary to previous studies [18, 33, and 34]. With a constant input velocity of 5 m/s, the effect of fracture position on the first, second, and third natural frequencies in the tapered pipe under consideration is depicted in Figs. 6-8 for various crack depths of 0, 5, 7, 10, 12, and 15 mm, respectively. It is evident that as the fracture depth increases, the rigidity of the structure will decrease, which will have an adverse effect on all natural frequencies taken into account at certain points. When a crack is created, the system's overall stiffness is decreased, which causes a drop in the system's natural frequencies and deterioration of the related mode shapes. It is clearly observed that minimum first natural frequency was obtained at crack position 0.7 L and crack depth of 15 mm as depicted in Fig. 6. However, for the second and third natural frequency minimum frequency occurred at a crack position 0.5 L and crack depth of 15mm. It should be said that all natural frequencies are not affect for crack depths 3 and 5mm for regardless of crack positions.

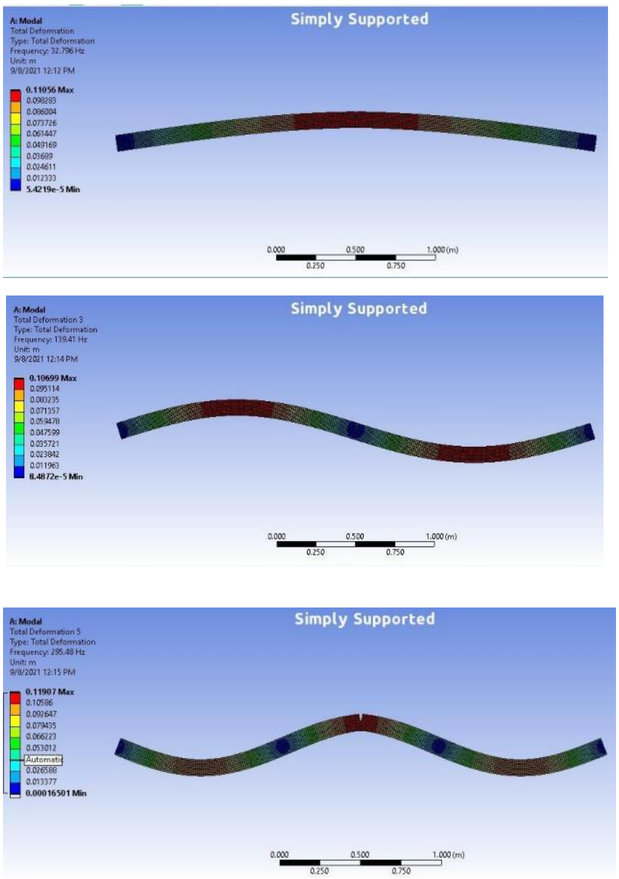


Figure 5. Mode shapes for first three fundamental frequencies for the tapered hinged pipe conveying fluid.

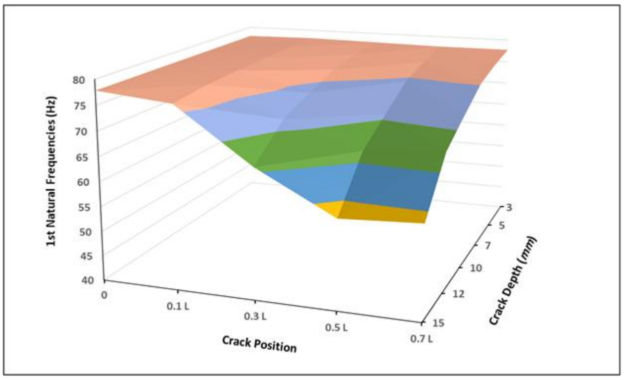


Figure 6. Effect of crack depth and crack position on the first circular frequency of tapered simply supported pipe for fluid flow velocity 10 m/s.

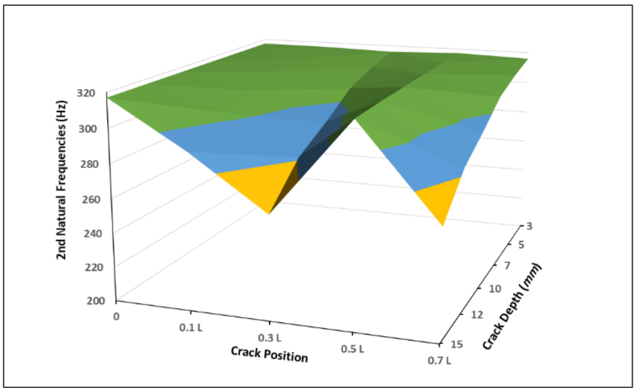


Figure 7. Effect of crack depth and crack position on the second circular frequency of tapered simply supported pipe for fluid flow velocity 10 m/s.

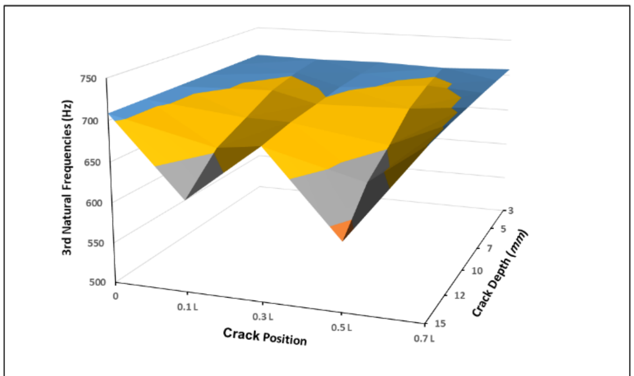


Figure 8. Effect of crack depth and crack position on the third circular frequency of tapered simply supported pipe for fluid flow velocity 10 m/s.

As depicted in Figs. 9–11, the impact of input fluid velocity in a simply supported tapered pipe on the fundamental, second, and third natural frequencies is examined along the pipe taken into consideration for crack depths 0, 5, 7, 10, 12, and 15 mm at a crack position at 0.3 L. It should be noted that regardless of the relative fracture position, raising the fluid's input velocity causes a modest drop in the pipe's natural frequencies. The least fundamental natural frequency found at a fluid velocity of 90 m/s is 64.8 rad/s, as shown in Fig. 9, although it is clear that for a relative length of 0.3 L, the first natural frequency is significantly impacted by the crack's existence. In general, increasing both the fluid velocity and crack depth contributes to decrease the natural frequencies.

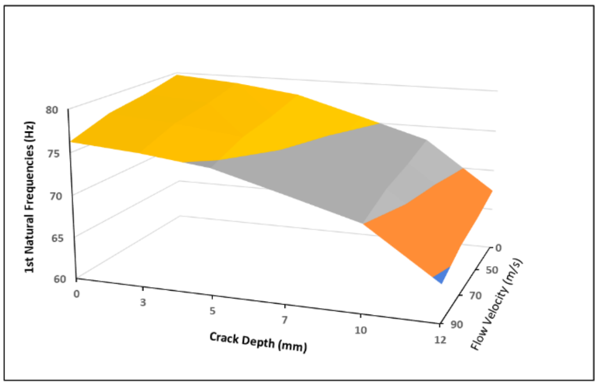


Figure 9. Effect of crack depth and fluid velocity on the first circular frequencies of tapered simply supported pipe for crack relative position 0.3 L.

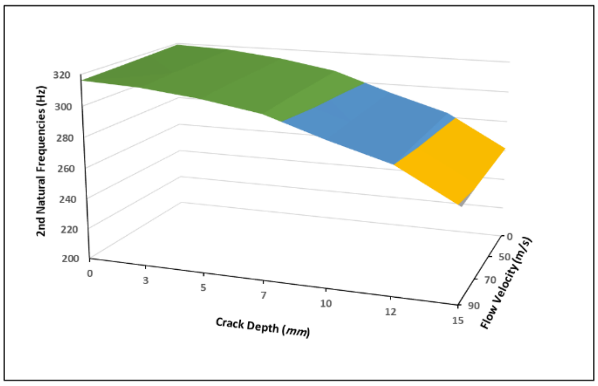


Figure 10. Effect of crack depth and fluid velocity on the second circular frequencies of tapered simply supported pipe for crack relative position 0.3 L.

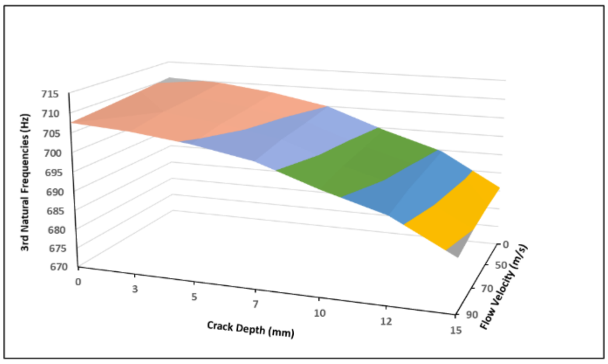


Figure 11. Effect of crack depth and fluid velocity on the third circular frequencies of tapered simply supported pipe for crack relative position 0.3 L.

Figs. 12–14 examine how fluid velocity affects the first three fundamental circular frequencies at various crack positions for fluid flow velocities 0, 50, 70, and 70 m/s when the crack depth is 7 mm. It is clearly observed that minimum first natural frequency 72.4 rad/s is recorded at a crack position 0.5 L for fluid flow velocity 90 m/s as shown in Fig. 12. However, Figs. 13–14 present the combined impact of crack location and fluid velocity on the first three fundamental circular frequencies, respectively, with crack depth of 7 *mm*. Significant impact of crack location can be observed especially when it is close to the nodal point; however, the effect of flow velocity is relatively minimal.

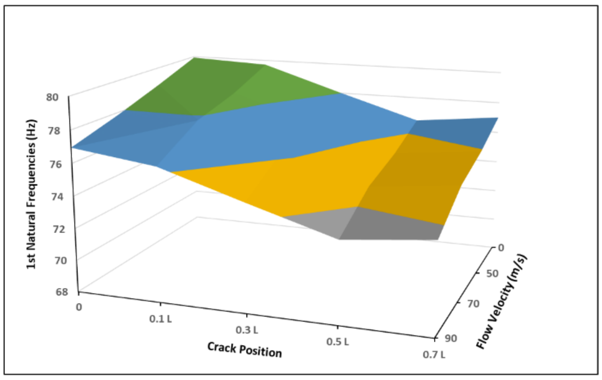


Figure 12. Effect of crack position and fluid velocity on the first fundamental circular frequencies of tapered simply supported pipe for crack depth 7mm.

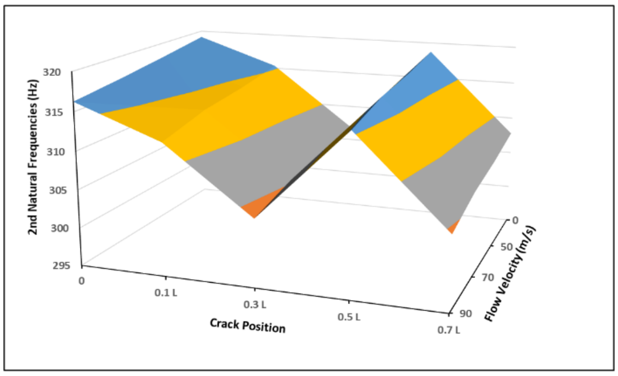


Figure 13. Effect of crack position and fluid velocity on the second fundamental circular frequencies of tapered simply supported pipe for crack depth 7mm.

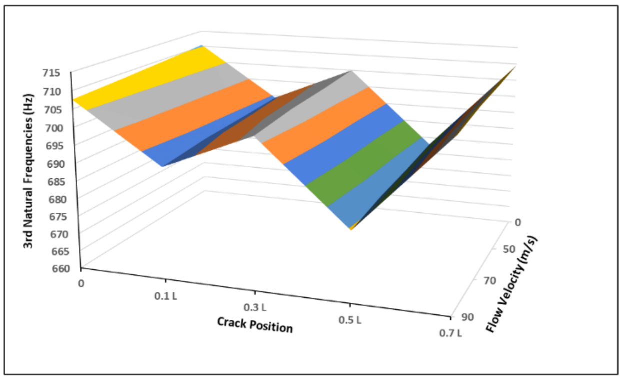


Figure 14. Effect of crack position and fluid velocity on the third fundamental circular frequencies of tapered simply supported pipe for crack depth 7mm.

### *B. Numerical Results*

Numerical analysis was carried out using finite difference method used by Nagaraju and Ramana [36] to find the first three natural frequencies for simply supported pipe transporting a fluid with defined speed and an outlet diameter equal to 90% of the inlet diameter. Tables II, III, IV, and V compare numerical and finite element results for modal analysis at different crack relative positions and crack depths. It is clearly observed that good agreement is found between numerical and finite element results for all natural frequencies considered at different crack position and relative crack depths.

TABLE II. NUMERICAL AND FINITE ELEMENT RESULTS FOR FIRST, SECOND, AND THIRD NATURAL FREQUENCIES AT CRACK POSITION 0.1 L AND RELATIVE CRACK DEPTH 0.5 IN THE BEAM

| Frequency number | Numerical Value (Hz) | Ansys Value (Hz) | Error (%) |
|---|---|---|---|
| 1 | 80 | 79.86 | 0.18% |
| 2 | 550.27 | 543.86 | 1.16% |
| 3 | 1591.63 | 1559.6 | 2.01% |

TABLE III. NUMERICAL AND FINITE ELEMENT RESULTS FOR FIRST, SECOND, AND THIRD NATURAL FREQUENCIES AT CRACK POSITION 0.5 L AND RELATIVE CRACK DEPTH 0.5 IN THE BEAM

| Frequency number | Numerical Value (Hz) | Ansys value (Hz) | Error (%) |
|---|---|---|---|
| 1 | 90.6 | 90.087 | 0.57% |
| 2 | 524.89 | 527.11 | 0.42% |
| 3 | 1612.85 | 1587.6 | 1.57% |

TABLE IV. NUMERICAL AND FINITE ELEMENT RESULTS FOR FIRST, SECOND, AND THIRD NATURAL FREQUENCIES AT CRACK POSITION 0.9 L AND RELATIVE CRACK DEPTH 0.5 IN THE BEAM

| Frequency number | Numerical Value (Hz) | Ansys value (Hz) | Error (%) |
|---|---|---|---|
| 1 | 92.92 | 91.906 | 1.09% |
| 2 | 578.88 | 571.06 | 1.35% |
| 3 | 1594.67 | 1573.5 | 1.33% |

TABLE V. NUMERICAL AND FINITE ELEMENT RESULTS FOR FIRST, SECOND, AND THIRD NATURAL FREQUENCIES AT CRACK POSITION 0.1 L AND RELATIVE CRACK DEPTH 0.7 IN THE BEAM

| Frequency number | Numerical Value (Hz) | Ansys value (Hz) | Error (%) |
|---|---|---|---|
| 1 | 64.09 | 63.44 | 1.01% |
| 2 | 519.7 | 519.79 | 0.02% |
| 3 | 1565.58 | 1547.3 | 1.17% |

## IV. CONCLUSION

The effects of flow velocity and existence of crack are considered for a tapered simply supported pipe containing moving fluid using finite element method. However, the pipe is modeled as a Bernoulli like beam theory using ANSYS software. The findings revealed that the existence of a crack causes a drop in the stiffness of the systems, which results in a fall in the fundamental natural frequencies. This loss is more noticeable when the fracture is located distant from the nodal points for each mode. Furthermore, it is proved that increasing flow velocity has an effect on reducing natural frequencies. On the other hand, good agreement was found between numerical and FE results for the first three natural frequencies when compared.

## Conflict of Interest

All authors declare that they have no conflicts of interest

## Author Contributions

Ibrahim Al-Adwan is the lead author who organized the whole research. Ahmad Awwad and Mohammad Ghaith are the results analysts. Fadi Alfaqs is the corresponding author and manuscript writer. Zaid Haddadin, Abdulah Wahbe, Mahmoud Hamam, Mahmoud Qunees, Mohammad Al Khatib, Mohammad Bsaileh, Abd Al-Aziz Jaber, and Ahmad Aqra'a conducted the finite element modeling. All authors had approved the final version.

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