Design and Synthesis of the Gravity Compensator with a Non-zero Length Spring

Yang Zhang [1, 2, 3] and Vigen Arakelian [1, 2, *]

[1] LS2N/ECN UMR 6004, F-44321 Nantes, France
[2] INSA-Rennes/Mecaproce, F-35708 Rennes, France
[3] Productions Core Laboratory, Flanders Make, 8500 Kortrijk, Belgium; Email: yang.zhang@flandersmake.be (Y.Z.)
*Correspondence: vigen.arakelyan@insa-rennes.fr or vigen.arakelyan@ls2n.fr (V.A.)

***Abstract*—It is well known that conventional linear spring directly jointed with a rotating link cannot ensure a complete gravity balancing. In this case, the application of zero length springs is suggested. The zero length spring corresponds to a spring with special coils ensuring zero elastic force for zero length of the spring. This type of spring is often used in theoretical solutions or in academic studies but rarely in real robotic structures. The use of traditional linear springs leads to inclusion of the auxiliary mechanism. The associated mechanisms are different and each of them requires its study and improvement. The applied methods for their study are various. They depend on the structure of the auxiliary mechanism and locations of balancing forces. This paper deals with design and synthesis of the gravity compensator including an auxiliary two-link dyad added to the rotating link. The balancing spring is installed vertically at the base. The choice of a vertical installation is due not only to constructive practicality but also to the fact that the mass of the spring system does not affect the balancing conditions of the rotating link. The aim of the study is to propose an analytically tractable solution permitting to synthesize such link lengths of the additional dyad that will provide a more optimal generation of the balancing moment. The efficiency of the suggested mechanism design is illustrated via numerical simulations. It is shown that by using a non-zero length spring a quasi-perfect balancing has been achieved.**

***Keywords*—gravity compensation, static balancing, mechanism synthesis, non-zero length spring, input torque cancellation**

I. Introduction

A mechanism is statically balanced if its potential energy is constant for all possible configurations. It means that zero actuator forces are required when the mechanism is in slow motion, i.e., a motion for which the forces of gravity are much greater than the forces of inertia. The static balancing of rotating links, which are often elements of serial or parallel manipulators, is well known. Different approaches and solutions devoted to this problem have been developed and documented [1–3]. The nature of the compensation force used for balancing of rotation links may be various:

- the gravitational force of a counterweight arranged on the opposite side of the rotating link [4–7] or via an auxiliary linkage [8–10];
- the elastic force of a spring, jointed directly with links [11–27] or via an auxiliary mechanism, which can be a cable and pulley arrangement [28–33], a linkage [34–39], a cam mechanism [40–43] or a gear train [44–49];
- the active force developed by a pneumatic or hydronic cylinder [50–56].

The literature review shows that the use of counterweights is a simple and not expensive solution. However, it leads to an increase in the overall dimensions of manipulators, as well as to a substantial increase in inertia of links. Increasing the inertia of links is undesirable because this creates additional dynamic loads on the actuators that are not taken into account in static balancing, i.e., in balancing of gravitational forces. Balancing by means of springs is more optimal from the point of view of the simple design and the low inertia of links. However, it is known that a conventional linear spring directly joined with a rotating link cannot fully balance its gravitational forces. This requires the use special springs called “zero free length springs” [57]. These springs are often applied in theoretical solutions, but they are very rare in real robotic structures. The use of conventional linear springs leads to application of auxiliary mechanical systems mentioned above. The use of a cable and pulley arrangement is relatively easy to study. However, its practical implementation reveals a number of inconveniences, which are not obvious at first sight. For example, the arrangement of the spring with respect to the rotating link. The condition of maximum elongation of the spring that should be not more than 25% of its initial length leads to the design concept, in which the spring must often be installed along the rotating arm and the number of pulleys should be at least two. Considering the diameter of the cable with protection, the diameter of the pulley turns out to be relatively large, which leads to a perturbation of the theoretical conditions of the balancing. Furthermore, the frictional forces between the pulley and the cable are also not low. All these factors lead to an unbalanced moment of more than 10–15% of the moment of gravitational forces. In design concepts with auxiliary linkages, it is

Manuscript received November 30, 2022; revised February 6, 2023; accepted March 3, 2023.

doi: 10.18178/ijmerr.12.3.131-136

mainly neglected the weight of springs, which can significantly affect the balancing conditions [58]. Furthermore, the balancing solutions are often considered by simplified schemes, in which the constructive features are not taken into account, such as the spring guide with variable length, the protective cylinder around of the spring, etc. In other words, the kinematic scheme turns out to be simple and accessible. However, our observations show that in practice, it remains complicated with various additional restrictions.

It is obvious that the adding of an auxiliary linkage to a rotating link increases the number of free parameters, which can be used for optimization of the gravity balancing. However, the problem is not whether this is feasible, but the problem is how to do it more effectively and the approaches for solving it can be various. It is understandable that the use of numerical calculation methods will lead to the minimization or cancellation of the unbalanced moment due to gravitational forces. However, it is more attractive to find an analytically tractable solution that can be easily applied.

In this study, the goal is to propose a synthesis approach to design a gravity compensator a non-zero length spring installed vertically at the base. The structural solution is carried out by adding a two-link rigid body prismatic-revolute dyad. The choice of a vertical installation is due not only to constructive practicality but also to the fact that the mass of the spring does not affect the balancing conditions of the rotating link.

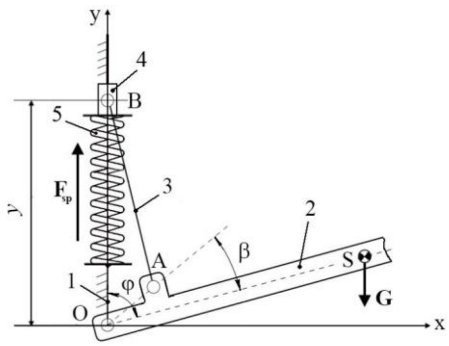


Figure 1. Design of the gravity compensator with a non-zero length spring.

In the proposed mechanism, shown in Fig. 1, the axis B moving by the rotation link 2 through the rod 3, controls the elastic force of the spring 5 mounted between the slider 4 and the frame 1. The aim of the study is to synthesize such link lengths of the additional dyad that will provide a more optimal generation of the balancing moment. For this purpose, two solutions are considered: an analytically tractable solution and a solution based on the optimization algorithm.

At first glance, for someone who does not have enough experience in the design of balancing mechanisms, there are not too many differences between the proposed compensator and the inverted slider crank mechanism (see Figure 8 in [57]). However, these are two different mechanical structures. Indeed, the inverted slider crank mechanism with a non-zero length spring can fully balance a rotating link. Nevertheless, in the inverted slider crank mechanism, the balancing spring is mounted on the moving link. The novelty of this research lies in the fact that a new balancing scheme is proposed. In the proposed balancing scheme, the spring is installed vertically on the frame. Such structural solutions are proposed for the first time and have not been discussed in previous works. It has advantages since the weight of the spring does not affect the balancing conditions. Another new aspect of this study is that an analytical solution is proposed. By using the geometric synthesis of a crank-slider mechanism formed by addition of an articulated dyad to the rotating link to be balanced, the gravity compensation conditions are achieved.

II. Gravity Blancing by Means of Non-zero Length Springs-Conservation of Potential Energy

It is known that for an equilibrium system, the total potential energy needs to be conserved. Hence, let us first analyze the potential energies in the proposed system.

The potential energy of the gravity of the rotating link 2 can be expressed as:

$$P_G = Gs(1+\cos\varphi) \tag{1}$$

where, G is the gravitational force of the rotating link 2; φ is the rotating angle of the link 2 and $s = l_{OS}$ is the distance between the center of mass of the link 2 and its center of rotation.

The potential energy of the spring is:

$$P_{sp} = \frac{1}{2}k\Delta y^2 \tag{2}$$

where, Δy is the deformation of the spring and k is its stiffness coefficient.

The whole potential energy of the system is:

$$P = P_G + P_{sp} \tag{3}$$

As it was mentioned in the previous section, the system will be balanced if:

$$\frac{dP}{d\varphi} = 0 \tag{4}$$

This means, for all the configurations of the system, the whole potential energy will remain as a constant.

Hence for a C=const, we can have:

$$Gs(1+\cos\varphi) + \frac{1}{2}k\Delta y^2 = C \tag{5}$$

and for a given C determine

$$\Delta y = \pm\sqrt{\frac{2C - 2Gs(1+\cos\varphi)}{k}} \tag{6}$$

and

$$y = \Delta y + l_0 \tag{7}$$

where l_0 is its initial length of the spring.

Thus, by imposing C and $\varphi_i (i=1,...,n)$, where n is the number of given positions of the rotating link, the displacements y_i with respect to each input angle φ_i can be determined.

III. Synthesis of the Balancing Mechanism for Prescribed Positions of the Rotating Link

For the balancing mechanism showed in Fig. 1, there are three main parameters r, l and β, where, $r = l_{OA}$ is the distance between the centers of joints O and A; y is the position of the axis B with respect to O; $l = l_{AB}$ is the length of the rod 3; β is the angle between the axes created by the line passing through the centers O and A, and the line passing through the centers O and S. In this section, with the prescribed positions, geometric synthesis is used to determine the parameters r, l, and β.

For the mechanism OAB, we have

$$(y_i - r\cos(\varphi_i - \beta))^2 + r^2 \sin(\varphi_i - \beta)^2 = l^2 \tag{8}$$

or

$$y_i^2 - 2y_i r\cos(\varphi_i - \beta) + r^2 = l^2 \tag{9}$$

Eq. (9) can be rewritten as:

$$y_i^2 - 2y_i\, r(\cos\varphi_i \cos\beta + \sin\varphi_i \sin\beta) = l^2 - r^2 \tag{10}$$

or

$$y_i^2 - 2y_i r\cos\varphi_i \cos\beta - 2y_i r\sin\varphi_i \sin\beta = l^2 - r^2 \tag{11}$$

Let's introduce new unknowns, which will allow one to transform the non-linear system of Eq. (11) to a linear system of equations:

$$z_1 = r\cos\beta \tag{12}$$

$$z_2 = r\sin\beta \tag{13}$$

$$z_3 = l^2 - r^2 \tag{14}$$

Now, the obtained linear system of equations is the following:

$$y_i^2 - 2y_i \cos\varphi_i z_1 - 2y_i \sin\varphi_i z_2 - z_3 = 0 \tag{15}$$

or

$$a_i z_1 + b_i z_2 + z_3 = c_i \tag{16}$$

where, $a_i = 2y_i\cos\varphi_i$, $b_i = 2y_i\sin\varphi_i$ and $c_i = y_i^2$.

Thus, the unknowns r, l, and β can be determined by:

$$\beta = \arctan\left(\frac{z_2}{z_1}\right) \tag{17}$$

$$r = \frac{z_2}{\sin\beta} = \frac{z_1}{\cos\beta} \tag{18}$$

$$l = \sqrt{z_3 + r^2} \tag{19}$$

IV. Deisgn Parameter Optimization of the Balancing Mechanism

The moment produced by the gravity of the rotating link 2 is:

$$M_G = -Gs\sin\varphi \tag{20}$$

From OAB triangle can be written:

$$r^2 + y^2 - l^2 = 2ry\cos(\varphi - \beta) \tag{21}$$

Rearranging Eq. (21) gives the following relationship:

$$y^2 - 2ry\cos(\varphi-\beta) + r^2\left[\cos^2(\varphi-\beta) + \sin^2(\varphi-\beta)\right] = l^2 \tag{22}$$

and

$$\left[y - r\cos(\varphi-\beta)\right]^2 = l^2 - r^2\sin^2(\varphi-\beta) \tag{23}$$

Hence the displacement of the slider 4 can be obtained by:

$$y = r\cos(\varphi-\beta) + \sqrt{l^2 - r^2\sin^2(\varphi-\beta)} \tag{24}$$

Thus, spring force F_{sp} will be:

$$F_{sp} = k(y - l_0) \tag{25}$$

Therefore, the component of spring force F_{sp} acting on the rod 3 is:

$$F_{ab} = F_{sp}\cos\theta \tag{26}$$

where, $\theta = \angle OBA$ and it be obtained by the law of cosines:

$$\theta = \arccos\left[\frac{y^2 + l^2 - r^2}{2yl}\right] \tag{27}$$

The moment generated by the spring force through the rod 3 is:

$$M_{sp} = F_{ab} r\sin\psi \tag{28}$$

where, $\psi = \angle BAO$:

$$\psi = \arcsin\left[\frac{y\sin(\varphi-\beta)}{l}\right] \tag{29}$$

Thus, the unbalanced moment of the whole system is:

$$M_{ub} = -(M_{sp} + M_G) \tag{30}$$

The total unbalanced moment for all the prescribed positions is:

$$M_{ub}^{total} = \sum_{\varphi=\varphi_i} M_{ub}(\varphi, l, r, \beta) \tag{31}$$

From the previous part, it can be seen that the values of l, r and β depend on input angles and their corresponding slider's positions. And the slider's positions depend on the input angles and the constant potential energy C. Hence the total unbalanced moment M_{ub}^{total} can be described as a function of input angles and constant potential energy. Taking into account that the input angles are prescribed, the only variable of the function M_{ub}^{total} is C.

For minimizing the total unbalanced moment, the following condition should be satisfied for given configurations of the gravity balancer:

$$\frac{\partial M_{ub}^{total}}{\partial C} = 0 \tag{32}$$

By solving Eq. (32), the constant potential energy C can be calculated and then the optimal parameters *l, r* and *β* of the gravity balancer can be determined.

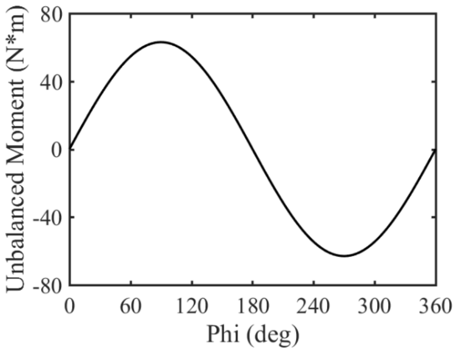
V. Illustrative Examples

A. Balancing via 3 Prescribed Positions of the Rotating Link

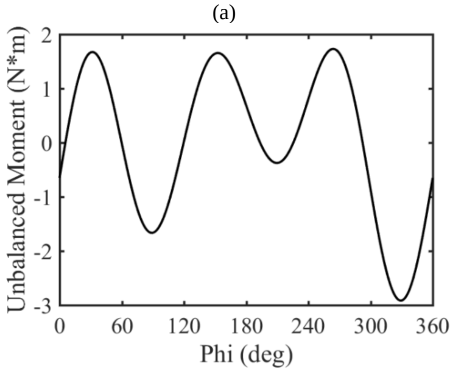
The initial parameters of the rotating link and the spring are the following: k=1000 N/m, l_0=1 m, G=75 N and s=0.84 m. Three prescribed positions of the rotating link 2 are the following: φ_1=30°, φ_2=60° and φ_3=120°.

From Eq. (32), the constant potential energy has been determined: *C*=184. Thus, by solving the system of linear Eq. (16) and considering relationships in Eqs. (17)–(19), the three unknown parameters have been determined: r=0.132 m; l=0.526 m and β= −0.662°.

With these parameters, the moment of the gravitational forces before balancing and the remained moment after balancing are shown in Fig. 2(a) and Fig. 2(b).



(a)



(b)

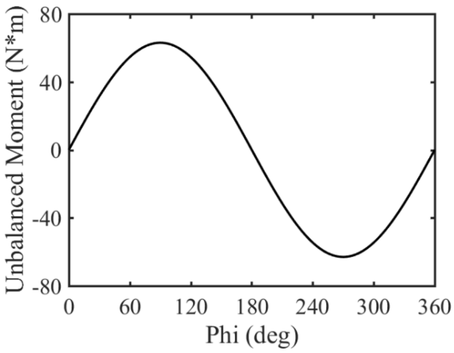
Figure 2. Moment of the gravitational forces before balancing (a) and the remained moment after balancing (b).

As we can see from these figures, via the proposed gravity compensation solution, the maximum value of the unbalanced moment has been reduced from 63 Nm to 2.92 Nm, i.e., 95.37%.

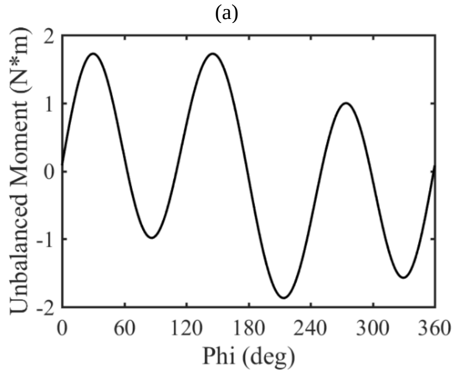
B. Balancing via n Prescribed Positions of the Rotating Link

The initial parameters of the rotating link and the spring are the following: k=1000 N/m, l_0=1 m, G=75 N and s=0.84 m. The prescribed locations of the rotating link 2 were given for n=120 positions between angles 30° and 150°, i.e., with a step of one degree.

In this case: C=195 and the following parameters of the crank-slider mechanism have been obtained: r=0.125m; l=0.502m and β=0.08°.



(a)



(b)

Figure 3. Moment of the gravitational forces before balancing (a) and the remained moment after balancing (b).

With these parameters, the moments of the gravitational forces before and after balancing have shown in Fig. 3(a) and Fig. 3(b).

The obtained results show that the maximum value of the unbalanced moment quite small, about 1.87 Nm, which is better than the three given position case. The reduction of the moment of the gravitational forces is 97.03%.

Thus, one can state with certainty that a quasi-perfect balancing has been achieved.

VI. Conclusion

In this study, an analytically tractable synthesis approach to design of the gravity compensator is proposed, which is a symbiosis of the geometrical synthesis of the proposed linkage and the energetic equilibrium of moments due to the gravitational and elastic forces. The particularity of the suggested structural concept is that the balancing spring is installed vertically on the frame and connected with the rotating link by means of an articulated dyad forming a crank-slider mechanism. Upon rotation of the link moves the end of the spring and changes the length of the spring creating a variable balancing moment. Geometric synthesis of the linkage allows one to control the optimal displacements of the spring providing an efficient balancing moment. By selecting three prescribed positions of the rotating link, which leads to the three positions of the spring, the balancing can be reached by solving a system of three linear equations. When the prescribed positions are more than three, an optimization algorithm for balancing is proposed. The efficiency of the suggested mechanism design is illustrated via numerical simulations. By selecting three prescribed positions of the rotating link has been reached a balancing up to 97.8%, with 120 prescribed positions of the rotating link 98.6%. The numerical example has showed that the analytical solution based on the solving of linear equations leads to a sufficiently high level of balancing. The increase in the prescribed positions led to the improvement of the balancing about 0.8%. Such a simple and easily applicable balancing approach, which allow one to carry out a quasi-perfect balancing, can find a wide application in industrial robots.

Conflict of Interest

The authors declare no conflict of interest.

Author Contributions

Both authors contributed to the design and analysis of this research. Yang Zhang wrote the manuscript; Vigen Arakelian revised and refined the article; all authors had approved the final version.

References

[1] V. Arakelian and S. Briot, *Balancing of Linkages and Robot Manipulators: Advanced Methods with Illustrative Examples*, Springer, 2015, p. 291.

[2] V. Arakelian, *Gravity Compensation in Robotics. Mechanisms and Machine Science*, vol. 115, Springer, Cham., 2022, p. 273.

[3] A. Agrawal and S. K. Agrawal, "Design of gravity balancing leg orthosis using non-zero free length springs," *Mechanism and Machine Theory*, vol. 40, no. 6, pp. 693–709, June 2005.

[4] A. Bayer and G. Merk, "Industrial robot with a weight balancing system," EP Patent 2 301 727, August 24, 2011.

[5] C. M. Gosselin, "Gravity compensation, static balancing and dynamic balancing of parallel mechanisms," *Smart Devices and Machines for Advanced Manufacturing*, L. Wang and J. Xi, Ed. Springer, 2008, pp. 27–48.

[6] J. Wang and C. M. Gosselin, "Static balancing of spatial four-degree-of-freedom parallel mechanisms," *Mech. and Mach. Theory*, vol. 35, pp. 563–592, 2000.

[7] T. Laliberté, C. M. Gosselin, and M. Jean, "Static balancing of 3-DOF planar parallel mechanisms," *IEEE/ASME Trans. J. Mechatronics*, vol. 4, no. 4, pp. 363–377, 1999.

[8] E. F. Fukushima, P. Debenest, Y. Tojo, *et al.*, "Teleoperated buggy vehicle and weigh balanced arm system for mechanization of mine detection and clearance tasks," *in Proc. HUDEM*, Tokyo, 2005, pp. 58–63.

[9] A. Russo, R. Sinatra, and F. Xi, "Static balancing of parallel robots," *Mech. and Mach. Theory*, vol. 40, pp. 191–202, 2005.

[10] D. Zang, F. Gao, X. Hu and Z. Gao, "Static balancing and dynamic modeling of a three-degree-of-freedom parallel kinematic manipulator," in *Proc. Int. Conf. Robot. Autom*., Shanghai, China, 2011, pp. 3211–3217.

[11] D. A. Streit and E., Shin, "Equilibrators for planar linkages," *ASME Trans. J. Mech. Design*, vol. 115, no. 5, pp. 604–611, 1993.

[12] Y. Ono and T. Morita, "An underactuated manipulation method using a mechanical gravity canceller," *J. of Robotics and Mechatronics*, vol. 106, no. 6, pp. 563–569, 2004.

[13] Y. Perrot, "Ressort d'équilibrage, pour mécanisme articulé, notamment pour bras de robot," FR Patent 2 847 958, June 4, 2004.

[14] B. Soethoudt and J. L. Herder, "Synthesis of perfect spring balancers with higher-order zero-free-length springs," in *Proc ASME 2007 IDETC/CIE conference*, Las Vegas, Nevada, USA, 2007.

[15] P. Y. Lin, W. B. Shieh, and D. Z. Chen, "Design of perfectly statically balanced one-DOF planar linkage with revolute joints only," *ASME Trans. J. Mech. Design*, vol. 131, 2009.

[16] P. Y. Lin, W. B. Shieh, and D. Z. Chen, "A stiffness matrix approach for the design of statically balanced planar articulated manipulators," *Mech. and Mach. Theory*, vol. 45, pp. 1877–1891, 2010.

[17] C. Cho and S. Kang, "Design of a static balancing mechanism with unit gravity compensators," in *Proc. Int. Conf. Robot. Autom.*, San Francisco, 2011, pp. 1857–1861.

[18] B. M. Wisse, W. D. Van Dorsser, R. Barents, *et al.*, "Energy-free adjustment of gravity equilibrators using the virtual spring concept," in *Proc. of the IEEE 10th International Conference on Rehabilitation Robotics*, Noordwijk, The Netherlands, 2007, pp. 742-750.

[19] Q. Lu, C. Ortega, and O. Ma, "Passive gravity compensation mechanisms: Technologies and applications," *Recent Patents on Engineering*, vol. 5, no. 1, pp. 32–44, 2011.

[20] P. Y. Lin, W. B. Shieh, and D. Z. Chen, "Design of statically balanced planar articulated manipulator with spring suspension," *IEEE Trans. Robot.*, vol. 28, no. 1, pp. 12–21, 2012.

[21] S. D. Deepak and G. K. Ananthasuresh, "Static balancing of a four-bar linkage and its cognates," *Mech. and Mach. Theory*, vol. 48, pp. 62–80, 2012.

[22] Y. Zhang, V. Arakelian, and J. P. Le Baron, "Linkage design for gravity balancing by means of non-zero length springs," in *ROMANSY 22 - Robot Design, Dynamics and Control*, 2019, pp. 163–170.

[23] V. Arakelian and Y. Zhang, "An improved design of gravity compensators based on the inverted slider-crank mechanism," *J. Mechanisms Robotics*, vol. 11, no. 3, 2019.

[24] V. L. Nguyen, "A design approach for gravity compensators using planar four-bar mechanisms and a linear spring," *Mechanism and Machine Theory*, vol. 172, 2022.

[25] A. Alamdari, R. Haghighi, and V. Krovi, "Gravity-balancing of elastic articulated-cable leg-orthosis emulator author links open overlay pane," *Mechanism and Machine Theory*, vol. 131, pp. 351–370, 2019.

[26] S. D. Deepak and G. K. Ananthasuresh, "Perfect static balancing of linkages by addition of springs but not auxiliary bodies," *ASME Trans. J. Mechanisms and Robotics*, vol. 4, 2012.

[27] S. H. Kim and C. H. Cho, "Design of gravity compensators using the Stephenson and Watt mechanisms," *Mechanism and Machine Theory*, vol. 102, pp. 68–85, 2016.

[28] I. Ebert-Uphoff and K. Johnson, "Practical considerations for the static balancing of mechanisms of parallel architecture," *Journal of Multi-body Dynamics, Part K*, vol. 216, pp. 73–85, 2002.

[29] G. Endo, H. Yamada, A. Yajima, *et al.*, "A weight compensation mechanism with a non-circular pulley and a spring: Application to a parallel four-bar linkage arm," *SICE Journal of Control, Measurement, and System Integration*, vol. 3, no. 2, pp. 130–136 2010.

[30] K. Kobayashi, "New design method for spring balancers," *ASME Journal of Mech. Des.*, vol. 123, pp. 494–500, 2001.

[31] N. Schmit and M. Okada, "Simultaneous optimization of robot trajectory and nonlinear spring to minimize actuator torque," in *Proc. Int. Conf. Robot. Autom.*, Saint Paul, Minnesota, USA, 2012, pp. 2806–2811.

[32] D. W. Bailey, "Spring counterbalance for rotating load," US Patent 8 220 765, 2012.

[33] R. Barents et al., "Spring-to-spring balancing as energy-free adjustment method in gravity equilibrators," in *Proc ASME IDETC/CIE Conference*, San Diego, California, USA, 2009.
[34] M. C. Cui, S. X. Wang, and J. M. Li, "Spring gravity compensation using the noncircular pulley and cable for the less-spring design," in *Proc. the 14th IFTOMM World Congress*, Taipei, Taiwan, 2015.
[35] R. Saravanan, et al., "Optimum static balancing of an industrial robot mechanism," *J. Engineering Applications of Artificial Intelligence*, vol. 21, pp. 824–834, 2008.
[36] J. Wang and C. M. Gosselin, "Passive mechanisms with multiple equilibrium configurations," *Trans. of CSME*, vol. 28, pp. 139–151, 2004.
[37] S. Segla, "Optimization of static balancing of a robot with parallelogram mechanism," in *Proc. 9th International Conference on the theory of machines and mechanisms*, 2004, pp. 699–704.
[38] A. Fattah and S. K. Agrawal, "Gravity-balancing of classes of industrial robots," in *Proc. Int. Conf. Robot. Autom.*, Orlando, Florida, 2006, pp. 2872–2877.
[39] I. Simionescu and L. Ciupitu, "The static balancing of the industrial arms. Part I: Discrete balancing," *Mech. and Mach. Theory*, vol. 35, pp. 1287-1298, 2000.
[40] W. B. Shieh, B. S. Chou, "A novel spring balancing device on the basis of a Scotch Yoke mechanism," in *Proc. the 14th IFTOMM World Congress*, Taipei, Taiwan, 2015
[41] H. S. Kim and J. B. Song, "Low-cost robot arm with 3-DOF counterbalance mechanism," in *Proc. Int. Conf. Robot. Autom.*, Karisruhe, Germany, 2013, pp. 4168–4173.
[42] I. Simionescu and L. Ciupitu, "The static balancing of the industrial arms. Part II: Continuous balancing," *Mech. and Mach. Theory*, vol. 35, pp. 1299–1311, 2000.
[43] P. Pracht, P. Minotti, and M. Dahan, "Synthesis and balancing of cam-modulated linkages," in *Proc. ASME Design and Automation Conference*, vol. 10, pp. 221–226, 1987.
[44] Y. F. Gvozdev, "Manipulator," SU Patent 1 777 993, 1992.
[45] Y. F. Gvozdev, "Manipulator," SU Patent 1 537 512, 1990.
[46] H. P. Greene, "Load compensator for spring counter-weighting mechanism," US Patent 5 400 721, 1995.
[47] C. H. Kuo and Y. X. Wu, "Perfect static balancing using Cardan-gear spring mechanisms," *Mechanism and Machine Theory*, vol. 181, 2023.
[48] C. Cho, W. Lee, and S. Kang, "Design of a static balancing mechanism with unit gravity compensators," in *Proc. IEEE/ASME Int. Conf. on Advanced Intelligent Mechatronics*, San Francisco, USA, 2011, pp. 1857–1862.
[49] C. Cho, W. Lee, J. Lee, and S. Kang, "A 2-dof gravity compensator with bevel gears," *Journal of Mechanical Science and Technology*, vol. 26, no. 9, pp. 2913–2919, 2012.
[50] R. Yamamoto, A. Hirakawa, and O. Horikawa, "Load balancer with automatic lifting force compensation," in *Proc. ABCM Symposium in Mechatronics*, vol. 4, 2010, pp. 580–589.
[51] F. Wildenberg, "Compensating system for a hexapod," US Patent 6 474 915, 2002.
[52] Y. Segawa, M. Yamamoto, and A. Shimada, "Parallel link mechanism," JP Patent 2000120824, 2000.
[53] N. Lauzier, C. Gosselin, T. Laliberte, *et al.*, "Adaptive gravity compensation of decoupled parallel and serial manipulators using a passive hydraulic transmission," *Journal of Mechanical Engineering Science*, vol. 223, no. 12, pp. 2871–2879, 2009.
[54] M. A. Lacasse, *et al.*, "On the design of a statically balanced serial robot using remote counterweights," in *Proc. Int. Conf. Robot. Autom.*, Karlsruhe, Germany, 2013, pp. 4174–4179.
[55] T. Laliberté, C. Gosselin, and D. Gao, "Closed-loop actuation routings for cartesian scara-type manipulators," in *Proc. ASME Int. DETC/CIE Conf.*, 2010
[56] C. Baradat, V. Arakelian, S. Briot, *et al.*, "Design and prototyping of a new balancing mechanism for spatial parallel manipulators," *ASME Trans. J. Mechanical Design*, vol. 130, no. 7, 2008.
[57] V. Arakelian, "Gravity compensation in robotics," *Advanced Robotics*, vol. 30, no. 2, pp. 79–96, 2016.
[58] V. Arakelian and S. Ghazaryan, "Improvement of balancing accuracy of robotic systems: application to leg orthosis for rehabilitation devices," *Mechanism and Machine Theory*, vol. 43, no. 5, pp. 565–575, 2008.